Final Examination
MATH-1171-FA: Calculus I
Department of Mathematical Sciences
Instructor: Christopher Chlebovec
December 8, 2018 from 6:00 PM - 9:00 PM
RB 1042

INSTRUCTIONS: The duration of the exam is 3 hours. This exam contains 12 pages (including this cover page). Check to see if any pages are missing. **Books, notes, calculators, cell phones or other aids are <u>not</u> permitted.**

- **Give your answers in the space provided.** If you need more space, use the back of the pages; clearly indicate when you have done this.
- **Organize your work**, in a reasonably neat and logical way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Justify your answers.** Unsupported answers will not receive credit.

Question	Points	Score
1	17	
2	5	
3	18	
4	10	
5	5	
6	5	
7	5	
8	15	
9	5	
10	15	
Total:	100	

FIRST NAME: _____

LAST NAME: _____

STUDENT NUMBER: _____

1. Evaluate the following limits. If the limit does not exist, explain why.

(a) (2 points) $\lim_{x \to 5^+} \dfrac{x^2 - 2}{x - 5}$

(b) (5 points) $\lim_{x \to 0} \dfrac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}}$

(c) (5 points) $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} |x| - 2 & \text{if } x < 3 \\ 3 & \text{if } x = 3 \\ \sqrt{x^2 - 8} & \text{if } x > 3 \end{cases}$

(d) (5 points) $\lim_{x\to\infty} \dfrac{\cos(x)-1}{x^2}$

2. (5 points) Let

$$f(x) = \begin{cases} \dfrac{x^2-5x+6}{\sqrt{x-2}} & \text{if } x > 2 \\ A & \text{if } x = 2. \end{cases}$$

Find the value of the constant A that makes f continuous on $[2,\infty)$.

3. Differentiate the following functions.

(a) (3 points) $f(x) = x^{\pi} + 2x^{\frac{3}{5}} + \sqrt{6x}$

(b) (5 points) $g(\theta) = (\theta^3 + 1)\cot(\theta + 1)$

(c) (5 points) $h(t) = \sin\left(\dfrac{t^{100} + t}{t^5 + 2}\right)$

(d) (5 points) $F(x) = \displaystyle\int_{x^3}^{x} \frac{\sin^5(t)}{t^2 + 1}\, dt$

4. (a) (5 points) Find the derivative of $f(x) = \dfrac{1}{1+5x}$ from the definition.

(b) (5 points) Suppose $x^2 + 4y^2 = 17$, where x and y are functions of t. If $\frac{dy}{dt} = 1$, find $\frac{dx}{dt}$ when $x = 1$ and $y = -2$.

5. (5 points) Find the critical numbers and state the local extreme values of

$$f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}.$$

6. (5 points) Show that the equation

$$\sin(x) = 2x - 1$$

has exactly one real root. (Hint: Use the Intermediate Value Theorem and Rolle's Theorem.)

7. (5 points) A balloon leaves the ground 100 feet away from an observer and rises vertically at a rate of 40 feet per minute. At what rate is the angle of inclination of the observer's line of sight increasing at the instant when the ballon is exactly 100 feet above the ground?

8. Evaluate the following integrals.

(a) (5 points) $\displaystyle\int \frac{x^{\frac{7}{2}} + 2x + 4}{x^4}\, dx$

(b) (5 points) $\displaystyle\int_0^{\frac{\pi^2}{4}} \frac{\sin(\sqrt{x})}{\sqrt{x}}\, dx$

(c) (5 points) $\int t(t+1)^{100}\,dt$

9. (5 points) Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on a closed interval:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{2}{n}\sqrt{1+\frac{2i}{n}}$$

10. (15 points) Find the following information and then graph the function $f(x) = x\sqrt{2 - x^2}$.
    (a) The domain, intercepts, symmetry, and asymptotes of f.
    (b) The intervals on which f is increasing, and the intervals on which f is decreasing.
    (c) The local extreme values of f.
    (d) The intervals on which f is concave up and the intervals on which f is concave down.
    (e) The inflection points of f.