University of Ottawa
Department of Mathematics and Statistics

MAT1322

Calculus II

Midterm test 2

March 4, 2019

Instructor: Vadim Kaimanovich

Duration: 75 minutes

Read the following information before starting the test:

- Verify that your copy of the test contains 5 pages, including this one.
- Write your name and student number on this page.
- Work the problems in the space provided. Use the back-pages and the blank sheet attached at the end for rough work. Do not use any other paper. Before submitting the test remove the rough work page 5.
- Show all work, clearly and in order, if you want to get full credit. Points may be taken off if it is not clear how you arrived at your answer (even if your final answer is correct).
- Please keep your written answers brief; be clear and to the point. Points may be taken off for rambling and for incorrect or irrelevant statements.
- Circle your final answers.

The Faculty of Science requires that you read and sign the following statement:

Cellular phones, calculators or other electronic devices and course notes are not allowed during this test. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the test.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

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Problem	1(2)	2(2)	3(2)	4(2)	5(2)	6(2)	Total(12)
Points							

1. Determine whether the sequence $a_n = \ln(3n^2 - 2) - \ln(n^2+1)$ converges or diverges; if it converges, find its limit L.

A. converges, $L = 3$ B. diverges C. converges, $L = \ln 3$ D. converges, $L = \ln(-3)$
E. converges, $L = -\ln 3$ F. diverges, $L = \ln 3$ G. converges, $L = \infty$ H. none of the above

$$a_n = \ln(3n^2-2) - \ln(n^2+1) = \ln \frac{3n^2-2}{n^2+1}$$

Since $\dfrac{3n^2-2}{n^2+1} = \dfrac{3-\frac{2}{n^2}}{1+\frac{1}{n^2}} \to 3$,

$$L = \lim_{n\to\infty} a_n = \ln 3$$

2. Write a formula for the n-th entry of the sequence $a_1 = 1/3, a_2 = -2/9, a_3 = 4/27$, $a_4 = -8/81, \ldots$ and find $S = \sum_{n=1}^{\infty} a_n$.

A. $a_n = (-1)^{n-1}2^n/3^n, S = 2/5$ B. $a_n = (-1)^n 2^{n-1}/3^n, S = 1/5$ C. $a_n = -2^n/3^n, S = 1/5$
D. $a_n = (-1)^{n-1}2^n/3^n, S = 1/5$ E. $a_n = (-1)^{n-1}2^{n-1}/3^n, S = 1/5$ F. none of the above

Since $a_{n+1} = \left(-\frac{2}{3}\right) a_n$ for any $n \geq 1$,

$$a_n = a_1 \left(-\frac{2}{3}\right)^{n-1} = \frac{1}{3}\cdot\left(-\frac{2}{3}\right)^{n-1} = (-1)^{n-1}\frac{2^{n-1}}{3^n}$$

$$S = a_1 + a_2 + \ldots = \frac{1}{3}\left(1 + \left(-\frac{2}{3}\right) + \left(-\frac{2}{3}\right)^2 + \ldots\right)$$

$$= \frac{1}{3}\cdot\frac{1}{1-\left(-\frac{2}{3}\right)} = \frac{1}{3}\cdot\frac{1}{\frac{5}{3}} = \frac{1}{5}$$

$$a_n = a_1 \cdot r^{(n-1)}$$

$$S = \frac{a_1}{1-r}$$

3. Which of the following series converge? (I) $\sum_{k=1}^{\infty} \frac{k \sin k}{1+k^3}$, (II) $\sum_{n=0}^{\infty}(-1)^n \frac{n}{n+2}$ (III) $\sum_{n=1}^{\infty} \frac{2n^2+n}{3n^3-2}$

A. all these series B. (II) and (III) C. none of these series D. (I) and (III) E. (I) only
F. (II) only G. (III) only H. none of the above

comon series

(I) $\left|\frac{k \sin k}{1+k^3}\right| \le \frac{k}{1+k^3} \le \frac{1}{k^2}$, whence (I) converges by the comparison criterion (as $\sum \frac{1}{k^2}$ is p-series with $p=2$)

$p \; \frac{1}{k^p} \; p>1$

(II) $\left|(-1)^n \frac{n}{n+2}\right| = \frac{n}{n+2} \to 1 \neq 0 \Rightarrow$ (II) diverges ?

alternating = 0 convergent

(III) Compare with the series of $b_n = \frac{1}{n}$. Then

$\frac{a_n}{b_n} = \frac{2n^2+n}{3n^3-2} \cdot n \to \frac{2}{3}$. Since $\sum b_n$ diverges (as a p-series with $p=1$), $\sum a_n$ also diverges

$0 < C < \infty$

divergent

limit $\frac{n^2}{n^3}$ $\frac{1}{n}$ diver

$b_n = n^3$

$\frac{a_n}{b_n} = \frac{n^2}{n^3}$

4. Choose one of the following descriptions of the series $\sum_{n=1}^{\infty}(-1)^n \frac{2^n \cos n}{4^n - n^2}$

A. absolutely divergent B. geometric divergent C. alternating conditionally convergent
D. alternating absolutely convergent E. absolutely conditionally convergent F. alternating divergent
G. absolutely convergent H. geometric convergent I. none of the above

$|a_n| = \frac{2^n |\cos n|}{4^n - n^2} \le \frac{2^n}{4^n - n^2} = b_n$; the series $\sum b_n$ converges by the ratio test, since $\boxed{\frac{b_{n+1}}{b_n}} \to \frac{1}{2} < 1$, therefore $\sum |a_n|$ is convergent by the comparison test, so that $\sum a_n$ is absolutely convergent; it is not alternating as $\cos n$ changes sign.

$|a_n| = \frac{2^n |\cos n|}{4n - n^2}$ $0 \le \cos n \le 1$ ratio test

$\le \frac{2^n}{4^n - n^2} = b_n$

5. How many terms n of the series $\sum_{n=1}^{\infty} \frac{1}{n^3}$ should one take so that the error R_n be less than 0.01?

A. 10 B. 11 C. ∞ D. 15 E. 14 F. 8 G. 7 H. none of the above

$a_n = \frac{1}{n^3} = f(n)$ for $f(x) = \frac{1}{x^3}$ which is non-negative and monotone, whence $R_n \le \int_n^\infty f(x)\,dx = \int_n^\infty \frac{dx}{x^3} = \frac{1}{2n^2}$, so that n has to satisfy the inequality $\frac{1}{2n^2} \le 0.01 = \frac{1}{100}$, or $n^2 \geq 50$. The minimal n with this property is $n = 8$.

$$\sum_{n=1}^{\infty} \frac{1}{n^3} = \int_n^\infty \frac{dx}{x^3} = \int_n^\infty n^{-3} = \frac{1}{-3+1} n^{-2}$$

6. Estimate the error when the series $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \ldots$ is approximated by its first 50 terms.

A. $\frac{1}{50}$ B. $-\frac{2}{101}$ C. $\frac{2}{103}$ D. $\frac{1}{103}$ E. 0 F. $\frac{2}{101^2}$ G. none of the above

This series is alternating with the n-th term $a_n = (-1)^{n+1} \frac{2}{2n+1}$. Since $|a_n|$ is monotone,

$$|R_{50}| \le |a_{51}| = \frac{2}{103}$$

extra page for calculations (please remove it when submitting the test!)