Lecture 1 - Euclid's postulates

(MAT2355 - M. Fraser, based on notes of H. Salmasian)

Euclid: Greek mathematician, ca 300BC

First attempt to axiomatize geometry: (in Euclid's "Elements": 13 volumes)

* to determine foundational statements of geometry from which all other statements/properties can be inferred
* to describe complicated geometric constructions in terms of simpler ones

Later the same axiomatic method was used but starting with different postulates – this gives "non-Euclidean geometries". We'll see some of these in the course.

Euclid's five postulates: Euclid assumes that in geometry we can . . .

P1: draw a unique straight line between any two points P and Q
P2: extend every straight line indefinitely (in both directions)
P3: draw a unique circle with given centre and radius
P4: draw a unique line perpendicular to any line ℓ at any point P on ℓ
P5: draw a unique line parallel to any line ℓ through any point P not on ℓ

Remark: Euclid's axioms had some deficiencies. The complete and accurate axioms of geometry were obtained by Hilbert in 1899 (20 axioms). You may find it interesting also to look at the commentary on Euclid's work that I have linked to in the Aside.

Example of a proof in Euclidean geometry: Prove that any two distinct lines intersect in at most one point.

Proof. Given two distinct lines, denote them ℓ_1 and ℓ_2. Suppose they intersect in some point. We must prove they do not intersect in an additional point. Suppose they did. Let these two points of intersection be denoted P and Q. By postulate P1, there is a unique line through this pair of points. However we have *two* lines – ℓ_1 and ℓ_2 – through the pair. This is a contradiction. Hence our supposition was wrong. It cannot be that ℓ_1 and ℓ_2 intersect in more than one point. □