Sample for Test 1

Math 184 - 2011W Term 2 - 3 Feb 2012

Section 201 (Warren Code)

Last Name:

First Name:

Student Number:

Read all of the following information before starting the exam:

- **This is a sample for Test 1, but these directions are as they will appear on the actual test.**
- You will have about 48 minutes to write this exam - please pay attention to the announced time remaining.
- Calculators are not permitted on this exam. You may leave your answer in a calculator-ready form: a calculator that can do arithmetic and exponents, but not one that can solve equations or take derivatives for you.
- Show all your work in order to receive full credit. If multiple steps are involved in arriving at an answer, a response consisting solely of the answer will not receive credit.
- This test has 7 (some with multiple parts) problems worth a total of 50 points. Make sure that you have all pages of this exam.

1. (*8 points*) Use the limit definition of the derivative (you will receive no credit for any other method) to compute $f'[x]$ for

$$f[x] = \frac{1}{x^2 + 1}$$

Correct use of notation is required for full points.

2. (*8 points*) Is the following function $f[x]$ continuous for all real x? For full credit, you must clearly justify your answer.

$$f[x] = \begin{cases} \dfrac{1}{2}, & \text{if } x \le 3, \\ \dfrac{\sqrt{x+1} - 2}{x - 3}, & \text{if } x > 3, \end{cases}$$

3. (*14 points*) Calculate the items indicated. Be clear about your steps, but you may use any method you like, and do not need to simplify as long as you have completed the derivative (and evaluation, if required).

a. Find $g'[x]$ if $g[x] = \dfrac{e^x}{x^2+1}$

b. Find the equation of the tangent line to $f[x] = x\cos[x]$ at $x = \pi$. You may use any of the following values if you like: $\sin[0] = 0$, $\sin[\pi] = 0$, $\sin\left[\frac{\pi}{2}\right] = 1$, $\cos[0] = 1$, $\cos[\pi] = -1$, $\cos\left[\frac{\pi}{2}\right] = 0$.

c. Find $C''[1]$ if $C[r] = 2 - r - \sqrt{r} + e^2$. (Note: it says $C''[1]$, not $C'[1]$).

d. The average ticket price for a concert at the opera house was \$50. The average attendance was 4000 people. When the ticket price was raised to \$52, attendance declined to an average of 3800 people per performance. Write a linear demand relation for this situation relating the ticket price to the quantity of tickets sold.

4. (*4 points*) Sketch the graph of a function $f[x]$ which is defined for all real numbers x, but is **not** continuous at $x = 1$ and **not** continuous at $x = -2$, but for which the limit exists at $x = -2$.

5. (*3 points*) Which of the following statements best describes the relationship between continuity and differentiability for a function? Choose only one, and assume that the functions are defined for all real numbers.

(A) A function must be both differentiable and continuous at $x = a$ or neither; you cannot have one without the other.

(B) If a function is differentiable at $x = a$ then it must also be continuous at $x = a$.

(C) If a function is continuous at $x = a$ then it must also be differentiable at $x = a$.

(D) There is at least one function that is continuous for some $x = a$ where it is not differentiable, and at least one other function that is differentiable at some $x = a$ where it is not continuous.

(E) Two of the above statements mean the same thing, and are both the correct choice.

6. (*3 points*) Is continuity of a function required for the conclusion of the Intermediate Value Theorem (that the function takes all intermediate values) to be true? In other words, is it possible for a function to take all intermediate output values over an interval of input values where it is not always continuous? Explain your reasoning, with supporting sketches if appropriate.

7. (*10 points*) For this problem, assume that the postage for sending a first-class letter in Canada is \$0.50 for the first ounce (up to and including 1 oz) plus \$0.15 for each additional ounce (up to and including each additional ounce). For instance, both a 0.5 ounce and a 1 ounce letter cost \$0.50 to send, while both a 1.5 ounce and a 2 ounce letter cost \$0.65 to send.

Graph the function $p[w]$ that gives the postage p for sending a letter that weighs w ounces if $0 < w \leq 5$.

Interpret the limits $\lim_{w \to 1^+} p[w]$ and $\lim_{w \to 1^-} p[w]$ in terms of the costs of sending a letter (i.e. relate this notation back to the word problem).