Physics 204
Solutions
Assignment #4

Q1 A horizontal force of 150 N is used to push a 40.0 kg-box 6.0 m on a rough, horizontal surface. If the box moves with a constant speed, find (a) the work done by the 150 N-force, (b) the energy lost due to friction, © the coefficient of kinetic friction. Ans: ~~(900J, 900J, 0.38)~~

Solution: Force $F = 150\,N$, Mass $m = 40\,kg$

Since the box moves with a constant speed the force F is necessary to work against the force of friction.

(a) work done by F → $W = \vec{F}\cdot\vec{d} = F\,d\,\cos 0 = Fd$

$= 150 \times 6 = 900\,J$

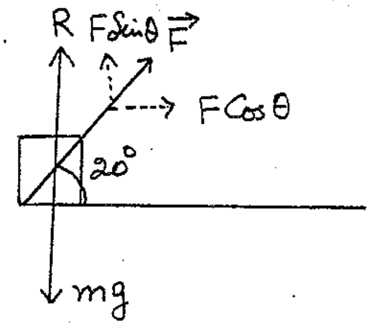
(b) All this work (900J) is spent to overcome the force of friction. Therefore, the energy lost due to friction = 900 J

(c) A force of 150N is being applied to maintain the constant speed of the box.

$\therefore \mu_k m g = F$ or $\mu_k = \frac{F}{mg} = \frac{150}{40 \times 9.8} = 0.38$

Q2 A 15 kg-block is dragged over a rough, horizontal surface by a 70 N force acting 20° above the horizontal. The block is displaced 5.0 m, and the coefficient of kinetic friction is 0.3. Find the work done by (a) the 70 n force, (b) the normal force, and © the force of gravity. (d) What is the energy lost due to friction? Ans: (329 J,0, 0, 185 J)

Solution: $m = 15\,kg$, $F = 70\,N$, $d = 5\,m$



(a) Work done → $w = F d \cos\theta = (70)(5)(\cos 20°)$

$= 328.9\,J$

(b) The normal forces R and F Sin θ will do no work because Cos 90° = 0

$\therefore$ work done by normal force = 0

(c) The gravitational force (mg) is also ⊥ d. Therefore the work done by weight of the body (mg) will be zero.

(d) Energy lost due to friction = work done against friction

$= \mu_k (mg - F\sin\theta)\,d = (0.3)(15 \times 9.8 - 70 \times 0.34)(5)$

$= 184.8\,J$

Q3 If you push a 40.0 kg crate at a constant speed of 1.40 m/s across a horizontal floor (μ_k=0.25), at what rate (a) is work being done on the crate by you and (b) is the energy dissipated by the frictional force?

Ans: (137 J, 137 J)

Solution: $m = 40.0\,kg$ speed (Constant) $\rightarrow v = 1.40\,m/s$.

The 40kg crate moves a distance of 1.4 m in one second.

$\therefore$ Power $\rightarrow$ Rate at which the work is done

$$= \underbrace{(40)(9.8)(0.25)}_{\text{frictional force}} \times (1.4) = 137.2\,J/s \rightarrow W$$

Since the crate is moving with a constant velocity, all the work done against the force of friction. Therefore, the energy dissipated by the frictional force = 137.2 J

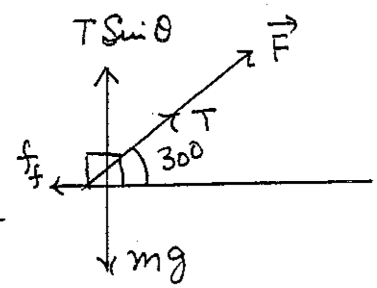
$\therefore$ Rate of dissipation = 137.2 J/s = W

Q4 A car loaded with bricks has a total mass of 18.0 kg and is pulled at constant speed by a rope. The rope is inclined at 30° above the horizontal, and the cart moves 20.0 m on a horizontal surface. The coefficient of kinetic friction between ground and cart is 0.5. (a) What is the tension on the rope? (b) How much work is done on the cart by the rope? © What is the energy lost due to friction? Ans: (79.0 N, 1368 J, 1368 J)

Solution: $m = 18.0\,kg$, $d = 20\,m$, $\mu = 0.3$

The tension in the rope is equal to the force applied on the rope. Since the cart is pulled with a constant speed, all the work done by T is spent in overcoming the force of friction.

work done by tension = work done on the force of friction



(a) $T \cos 30° = (0.5)(18 \times 9.8 - T \sin 30°)$

or $0.866\,T = (0.5)(18)(9.8) - T/4$ or $T = \dfrac{(0.5)(18)(9.8)}{(0.866 + 0.25)} = 79.0\,N$

(b) work done by T on the cart is

$W = Td \cos 30° = (79.0)(20)(0.866) = 1368.3\,J$

(c) All the work done in (b) is spent against the force of friction.

$\therefore$ Energy lost due to friction = 1368.3 J

Q 5 For $\vec{A} = 4\hat{i}+3\hat{j}$ and $\vec{B} = -\hat{i}+3\hat{j}$, find (a) $\vec{A}.\vec{B}$ and (b) the angle between $\vec{A}$ and $\vec{B}$. Ans: $(5, 71.6^{\circ})$

Solution: $\vec{A} = 4\hat{i}+3\hat{j}$, $\vec{B} = -\hat{i}+3\hat{j}$

(a) $\vec{A}\cdot\vec{B} = (4\hat{i}+3\hat{j})\cdot(-\hat{i}+3\hat{j}) = (-4+9) = 5$

(b) $\vec{A}\cdot\vec{B} = AB\cos\theta$ where $\theta \rightarrow \angle$ between $\vec{A}$ and $\vec{B}$.

$\therefore \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$, $A = \sqrt{4^2+3^2} = 5$, $B = \sqrt{(-1)^2+3^2} = \sqrt{10}$

$\therefore \cos\theta = \dfrac{5}{5\times\sqrt{10}} = 0.32$ $\quad\therefore \theta = 71.6^{\circ}$

Q 6 For $\vec{A} = 3\hat{i}+\hat{j}-\hat{k}$, $\vec{B} = -\hat{i}+2\hat{j}+5\hat{k}$, and $\vec{C} = 2\hat{j}-3\hat{k}$, find $\vec{C}\cdot(\vec{A}-\vec{B})$ Ans: (16)

Solution: $\vec{A} = 3\hat{i}+\hat{j}-\hat{k}$, $\vec{B} = -\hat{i}+2\hat{j}+5\hat{k}$

$\vec{C} = 2\hat{j}-3\hat{k}$

$\therefore \vec{C}\cdot(\vec{A}-\vec{B}) = (2\hat{j}-3\hat{k})\cdot\{(3\hat{i}+\hat{j}-\hat{k})-(-\hat{i}+2\hat{j}+5\hat{k})\}$

$= (2\hat{j}-3\hat{k})\cdot(4\hat{i}-\hat{j}-6\hat{k}) = -2+18 = 16$

Q 7 A 7.0 kg bowling ball moves at 3.00 m/s. How fast must a 46 g golf ball move so that the two balls have the same kinetic energy? Ans: (37.0 m/s)

Solution: $m(\text{ball}) = 7\,kg$, $v = 3\,m/s$, $m(\text{golf}) = 45\,g$

Kinetic energy of the ball $\rightarrow T_b = \frac{1}{2}mv^2 = \frac{1}{2}(7)(3)^2 = 31.5\,J$

If v_g is the velocity of golf ball, the kinetic energy of the golf ball will be $\rightarrow K_g = \frac{1}{2}(0.046)v^2\,J$

To have $K_b = K_g$, we have, $31.5 = \frac{0.046}{2}v^2$ or $v = \sqrt{\frac{31.5\times 2}{0.046}}$

$= 37.0\,m/s$

Q 8 A 40 kg box initially at rest is pushed 5.0 m along a rough, horizontal floor with a constant applied horizontal force of 130 N. If the coefficient of friction between box and floor is 0.3, find (a) the work done by the applied force, (b) the energy lost due to friction, © the change in kinetic energy of the box, and (d) the final speed of the box.

Ans: (650 J,588 J,62 J,1.76 m/s

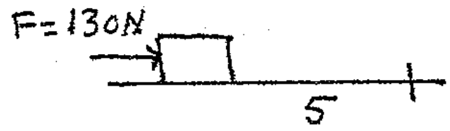
Solution: $m = 40\,kg$ $\quad d = 5.0\,m$ $\quad F = 130\,N$

$\mu = 0.3$

(a) work done by 130 N force

$W = \vec{F} \cdot \vec{d} = 130 \times 5 \times \cos 0$

$= 650 J$



(b) Energy lost due to friction:

frictional force $\to f = (\mu m g) d = (0.3)(40)(9.8)(5) = 588 J$

(c) Since the box is initially at rest, $(650-588)J$ of energy is spent on the kinetic energy of the box.

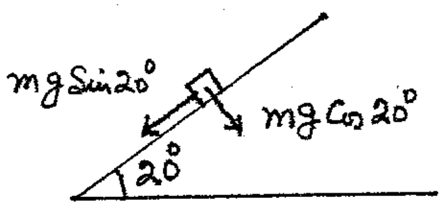
Change in kinetic energy $= 650 - 588 = 62 J$

(d) The final speed v of the box is given by:

$\frac{1}{2} m v^2 = 62$ or $v = \sqrt{\frac{2 \times 62}{40}} = 1.76\ m/s$

Q9 A 4.0 kg block is given an initial speed of 8.0 m/s at the bottom of a 20° incline. The frictional force that retards its motion is 15.0 N. (a) If the block is directed up the incline, how far does it move before stopping? (b)Will it slide back down the incline? Ans: (4.51 m, will not slide back)

Solution: Initial kinetic energy of the block $= \frac{1}{2} m v^2 = \frac{1}{2}(4)(8)^2 = 128 J$



When the block goes up the plane, the forces retarding the motion of the box are:

1. frictional force = 15 N (given)
2. Component of mg down the plane $= mg \sin\theta = 4(9.8) \sin 20° = 13.4 N$

$\therefore$ Total retarding force $= 15 + 13.4 = 28.4 N$

Energy spent in moving the block by a distance d is:

work done $\to W = 28.4 \times d$

The block will come to a stop when the initial kinetic energy of block is spent on the work done in moving the block by d. $\therefore 28.4 d = 128$ or $d = \frac{128}{28.4} = 4.51 m$

(b) It will slide back if $mg \sin\theta > 15 N$

Since $mg \sin\theta = 13.4 < 15 N$, the box will not slide back.

Q 10 Water falls over a section of Niagra falls at a rate of 1.2×10^6 kg/s and falls 50 m. How many 60 W bulbs can be lit with this power? Ans: (9.8×106)

Solution: when 1.2×10^6 kg of water falls through a height of 50m, the potential energy is changed into kinetic energy of water which drives the turbine to produce electrical power.

Power provided by water = mgh per sec.

$= (1.2\times10^6)(9.8)(50)$ W.

Number of bulbs that can be lit $= \dfrac{(1.2\times10^6)(9.8)(50)}{60}$

$= 9.8\times10^6$ bulbs

Q 11 A 1500 kg car accelerates uniformly from rest to 10 m/s in 3.0 s. Find (a) the work done on the car in this time, (b) the average power delivered by the engine in the first 3.0 s, and © the instantaneous power delivered by the engine at t=2.0 s. Ans: (75000 J, 25000 J,33333 W)

Solution: The car starts from rest ($v_0=0$) and after 3 sec attains a speed $v = 10$ m/s. If a is the acceleration of the car

$\therefore \quad 10 = 0 + a\times3 \quad$ or $\quad a = 10/3$ m/s^2

$\therefore$ The force acting on the car, $F = ma = (1500)(\frac{10}{3}) = 5000$N

(a) Distance x moved by the car in 3 sec is given by:

$x = v_0t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(\frac{10}{3})(3)^2 = 15$m.

$\therefore$ work done on the car by the force, $W = (5000)(15) = 75,000$ J

(b) Average power delivered, $P_{ave} = \dfrac{75,000}{3} = 25,000$ W

(c) velocity after 2 sec, $v = 0 + (\frac{10}{3})(2) = \frac{20}{3}$ m/s

$\therefore$ Instantaneous Power $= \vec{F}\cdot\vec{v} = (5000)(\frac{20}{3}) = 33333$ W.

Q 12 A certain horse can maintain 1.0 hp of output for 2.0 hr, how many 70 .0 kg bundles of shingles can the horse hoist (via some pulley arrangement) to the roof of a house 8.0 m tall, assuming 70%efficiency?Ans(:685 bulbs)

Solution: work the horse can do in 2 hours → W is

$W = (746)(120)(60)$ J $= (746)(7200)$ J

Useful work available $= (746)(7200)(0.7)$ J

work done in hoisting a bundle $= (70)(9.8)(8)$J. Number of bundles hoisted $= (746)(7200)(0.7)/(70)(9.8)(8) = 685$ bundles.

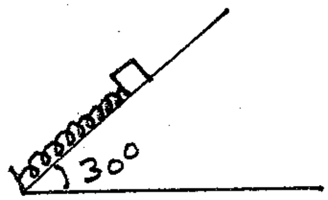
13 A block of mass 12.0 kg slides from rest down a frictionless 30° incline and is stopped by a strong spring with $k=3.0\times10^4$ N/m. The block slides 3.0 m from the point of release to the point where it comes to rest against the spring. When the block comes to rest, how far has the spring been compressed?

Ans: (0.11 m)

Solution: work done on the block

$$W = (12)(9.8)(\sin 30^\circ)(3) = 176.4\,J$$

This work will be stored as energy of the spring. If S is the compression of the spring



$$176.4 = \frac{1}{2}(3\times10^4)\,S^2$$

Constant of spring $= 3\times10^4$

$$\therefore\ S = \sqrt{\frac{176.4}{0.5\times3\times10^4}} = 0.11\,m$$

14 An earth satellite revolves in a circular orbit at a height of 300 km above the earth's surface. (a) What is the speed of satellite, assuming the earth's radius to be 6380 km. (b) What is the time period of the satellite?

Ans: (7.73×10^3 m/s. 90.5 min.)

Solution: (a) Earth's radius $\rightarrow R = 6380\,km = 6.38\times10^6\,m$

Distance of satellite from the Earth's Center $= (6380+300)\,km = 6.68\times10^6\,m$

If m is the mass of the satellite, the equation of motion of the satellite is given by:

$$\frac{mv^2}{r} = G\frac{m\,m_E}{r^2} \qquad \text{Since, } g = \frac{G\,m_E}{r^2}$$

$$\therefore \text{ Speed of satellite} \rightarrow v = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{6.68\times10^6}}$$

$$= 7.73\times10^3\ m/s$$

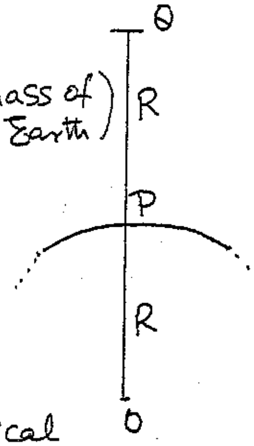
(b) The time period of the satellite is given by

$$T = \frac{2\pi r}{v}$$

$$\therefore\ T = \frac{(2)(\pi)(6.68\times10^6)}{7.73\times10^3} = 5430\ Sec = 90.5\ min$$

Q 15 Use the principle of conservation of mechanical energy to find the velocity with which a body must be projected vertically upward, in the absence of air resistance, (a) to rise to a height above the earth's surface equal to the earth's radius, R, and (b) to escape from the earth.

Ans: (7906 m/s, 11181 m/s)



Solution: Let v_0 be the velocity with which the body must be projected,

The kinetic energy at P → $E_p = -\frac{G m m_E}{R}$ (m_E → mass of the Earth)

∴ Total energy at P → $E_K + E_P = \frac{m}{2} v_0^2 - \frac{G m m_E}{R}$ (1)

At the point Q: $E_p = -\frac{G m m_E}{(R+R)}$, $E_K = 0$.

∴ Total energy at Q $= -\frac{G m m_E}{2R} + 0$ (2)

From the principle of Conservation of mechanical energy, the total energy of a body at P must be the same as at point Q.

∴ From (1) and (2) we have: $\frac{m}{2} v_0^2 - \frac{G m m_E}{R} = -\frac{G m m_E}{2R}$

$$\therefore \frac{m}{2} v_0^2 = \frac{G m m_E}{2R} \text{ or } v_0 = \sqrt{\frac{G m_e}{R}}$$

If radius of the Earth is taken as, $R_E = 6.38 \times 10^6$ m

and mass of the Earth, $m_E = 5.98 \times 10^{24}$ kg

Then, $v_0 = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.38 \times 10^6}} = 7906$ m/s

(b) If v_{Esc} is the velocity of escape from the Earth, then from principle of Conservation of energy we have:

$$\frac{1}{2} m v_{esc}^2 - \frac{G m m_E}{R} = 0 - \frac{G m m_E}{(R+\infty)}$$

$$\therefore \frac{m}{2} v_{esc}^2 = \frac{G m m_E}{R} \text{ or } v_{Esc} = \sqrt{\frac{2 G m_E}{R}}$$

For the Earth: $v_{es} = \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.38 \times 10^6}} = 11181$ m/s.

Q 16 A jet airplane engine develops a thrust of 15,000 N. When the plane is flying at 300m/s, what horsepower is developed? Ans: (6.03×10^3 HP)

Solution: Instantaneous power is given by an expression

$$P = F v \quad \text{where,} \quad F = \text{thrust}, \; v = \text{velocity}$$

$\therefore$ Power delivered by the jet airplane engine is:

$$P = (15,000)(300) \text{ W}$$

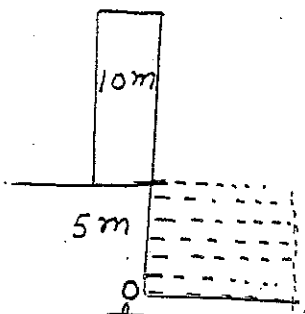
$$\therefore \text{Horse Power} = \frac{(15,000)(300)}{746} = 6.03 \times 10^3 \text{ HP}.$$

Q 17 A 70.0-kg diver steps off a 10.0-m tower and drops straight down into the water. If he comes to rest 5.0 m beneath the surface of water, determine the average resistance force exerted by the water on the diver.

Ans: (2.06×10^3 N)

Solution: Speed of the diver before hitting water is:

$$v^2 - 0 = 2 \times (9.8)(10) \quad (1)$$
$$= 19.6 \, (m/s)^2$$



$\therefore$ Kinetic energy of diver before hitting water $\rightarrow$ $E_K = \frac{1}{2} m v^2 = \frac{1}{2}(70)(196)$

$$= 35 \times (196) \text{ J}$$

Total energy of the diver before hitting water is

$$E = E_K \text{ (kinetic)} + E_p \text{ (Potential)}$$

$$E = [35 \times (196) + 70 \times 9.8 \times 5] \text{ J} \quad (2)$$

we have taken the zero of potential energy at O.

All the energy in (2) is lost to water resistance over a distance of 5 m. If F is the average resistance of water,

$$35 \times 196 + 70 \times 9.8 \times 5 = F \times 5$$

$$\text{or} \quad F = \frac{35 \times 196 + 70 \times 9.8 \times 5}{5} = 2.06 \times 10^3 \text{ N}$$

18 An electric scooter has a battery capable of supplying 120 Wh of energy. If frictional forces and other losses account for 60.0% of the energy usage, what altitude change can a rider achieve when driving in hilly terrain, if the rider and scooter have a combined weight of 800N? Ans: (216 m)

Solution:

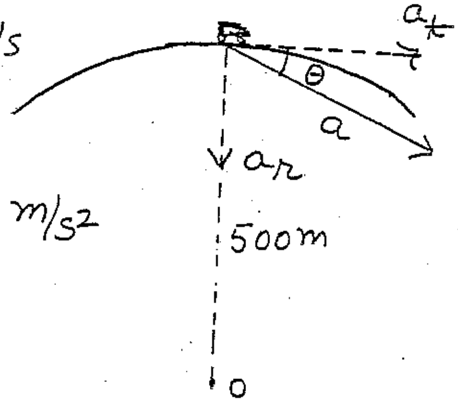
Useful output energy of battery $= 120 \times (40/100)$
$= 48$ wh.

If h is the altitude achievable,

$$48 \times 3600 = 800 \times h$$

$$\therefore h = \frac{48 \times 3600}{800} = 216 \text{ m}.$$

19 A car exhibits a constant acceleration of 0.30 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like a circle of radius 500 m. At the moment the car is on the top of the rise, its velocity vector is horizontal and has a mgnitude of 6.0 m/s. What is the direction of the total acceleration vector for the car at this instant.? Ans: (a=0.309 m?s^2, -13.5^0 to the horizontal



Horizontal velocity at the top = 6.0 m/s

$\therefore$ Centripetal acceleration,

$$a_r = -\frac{v^2}{R} = -\frac{(6)^2}{500} = -0.072 \text{ m/s}^2$$

The tangential acceleration a_t is

$$a_t = 0.3 \text{ m/s}^2$$

Total acceleration $\rightarrow a = \sqrt{a_r^2 + a_t^2}$

$$= \sqrt{(-0.072)^2 + (0.3)^2} = 0.309 \text{ m/s}^2$$

Direction of total acceleration is given by θ, which is:

$\tan\theta = -\frac{0.072}{0.03}$ or $\theta = -13.5^\circ$ to the horizontal. Therefore

$$a = 0.3\hat{t} - 0.072\hat{r}$$