Test 5 MATH 1004D, Fall, 2019

Last name: First name: Student no.:

This test has two parts, the first part has 4 multiple choices questions (3 marks each) and the second part has 3 long answers. Calculator is NOT allowed

A1: Evaluate $\int_0^{\sqrt{\pi}} x \sin x^2 \, dx$

a) $\frac{1}{2}$ b) π c) $\frac{\pi}{3}$ d) 1 (e) None.

Ans (d)

A2: Find $\int_1^e \frac{\ln x}{x} \, dx$

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{e^2}$ (d) e^2 (e) None.

Ans (a)

A3: What is $\int x^2 e^{5x} \, dx$?

(a) $\left(-\frac{1}{5}x^2 + \frac{2}{5}x - \frac{2}{125}\right) e^{5x} + C$

(b) $\left(\frac{1}{5}x^2 - \frac{2}{5}x + \frac{2}{125}\right) e^{5x} + C$

(c) $\left(\frac{1}{5}x^2 - \frac{2}{25}x + \frac{3}{125}\right) e^{5x} + C$

(d) $\left(-\frac{1}{5}x^2 + \frac{2}{5}x - \frac{3}{125}\right) e^{5x} + C$

(e) None

Ans (e)

A4: Find $\int \frac{1}{x(x+1)} \, dx$

(a) $\ln|x| + \ln|x+1| + C$ (b) $\ln|x| + 2\ln|x+1| + C$ (c) $\ln\left|\frac{x}{x+1}\right| + C$ (d) $\ln\left|\frac{x+1}{x}\right| + C$

(e) None.

Ans (c)

B1-[6 marks]:Evaluate $\int_0^1 \sin^{-1} x \, dx$.

$$\begin{array}{cc} u = \sin^{-1} x & dv = dx \\ \Downarrow & \Downarrow \\ du = \dfrac{dx}{\sqrt{1-x^2}} & v = x \end{array}$$

and $\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} \, dx$

Put $1 - x^2 = t \Longrightarrow -2x \, dx = dt \Longrightarrow \int \frac{x}{\sqrt{1-x^2}} \, dx = \frac{-1}{2} \int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{1-x^2}$

Therefore, $\int \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C$ and

$$\int_0^1 \sin^{-1} x \, dx = x \sin^{-1} x + \sqrt{1-x^2} + C|_0^1 = \frac{\pi}{2} - 1$$

B2-[6 marks]: Evaluate $\int e^{4x}\cos 3x\,dx$.

$$I = \int e^{4x}\cos 3x\,dx$$

$sign$	$derivative$	$integral$
$+$	e^{4x}	$\cos 3x$
$-$	$4e^{4x}$	$\frac{1}{3}\sin 3x$
$+$	$16e^{4x}$	$-\frac{1}{9}\cos 3x$

$$I = \frac{1}{3}e^{4x}\sin 3x + \frac{4}{9}e^{4x}\cos 3x - \frac{16}{9}\int e^{4x}\cos 3x\,dx$$

$$\Longrightarrow I = \frac{1}{3}e^{4x}\sin 3x + \frac{4}{9}e^{4x}\cos 3x - \frac{16}{9}I$$

$$(1+\frac{16}{9})I = \frac{1}{3}e^{4x}\sin 3x + \frac{4}{9}e^{4x}\cos 3x$$

$$I = \frac{9}{25}(\frac{1}{3}e^{4x}\sin 3x + \frac{4}{9}e^{4x}\cos 3x) + C$$

B3-[6 marks]: Evaluate $\int \frac{x-3}{x(x-2)(x+1)}\,dx$

$$\frac{x-3}{x(x-2)(x+1)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1} \quad (*)$$

Multiply both sides of (*) by x then, put $x = 0$:

$$\Longrightarrow \frac{x-3}{(x-2)(x+1)}|_{x=0} = A + 0 + 0 \Longrightarrow A = \frac{3}{2}$$

Multiply both sides of (*) by $(x-2)$ then, put $x = 2$:

$$\Longrightarrow \frac{x-3}{x(x+1)}|_{x=2} = 0 + B + 0 \Longrightarrow B = \frac{-1}{6}$$

Multiply both sides of (*) by $(x+1)$ then, put $x = -1$:

$$\Longrightarrow \frac{x-3}{x(x-2)}|_{x=-1} = 0 + 0 + C \Longrightarrow C = \frac{-4}{3}$$

$$\int \frac{x-3}{x(x-2)(x+1)}\,dx \quad = \int \frac{\frac{3}{2}}{x}\,dx + \int \frac{\frac{-1}{6}}{x-2}\,dx + \int \frac{\frac{-4}{3}}{x+1}\,dx$$

$$= \frac{3}{2}\ln|x| - \frac{1}{6}\ln|x-2| - \frac{4}{3}\ln|x+1| + C$$