Test 3 Solutions MATH $1004D$, Fall, 2019

Last name: First name: Student no.:

This test has two parts, the first part has 4 multiple choices questions (3 marks each) and the second part has 4 long answers. Calculator is NOT allowed

A1: Let $f(x) = e^{3x}\ln(x^2 + x + e)$. Then $f'(0)$ is:

(a) 1 (b) $e + 3$ (c) $1 + \frac{e}{3}$ (d) $3 + \frac{1}{e}$ (e) None.

Ans (d)

A2: Let $g(x) = e^x \log_3 e^x$. Then $g'(x)$ is:

(a) $\frac{e^x}{\ln 3}$ (b) $\frac{e^x + 1}{\ln 3}e^x$ (c) $\frac{e^x}{(x+1)\ln 3}$ (d) $\frac{x+1}{\ln 3}e^x$ (e) None.

Ans (d)

A3: The $\lim_{x\to 0^+} x^x$ is

(a) 0 (b) 1 (c) e (d) $+\infty$ (e) None.

Ans (b)

A4: Let $g(x) = x^x$. Then $g'(x)$ is:

(a) $(1+x)x^x$ (b) doesn't exist (c) x^{x-1} (d) $(1+\ln x)x^x$ (e) None.

Ans (d)

B1-[5 marks]: Find the derivative of $(\sin x)^x$

$$
\begin{aligned}
&y = (\sin x)^x \Longrightarrow y = e^{x\ln(\sin x)} \Longrightarrow y' = (x\ln(\sin x))'e^{x\ln(\sin x)}\\
&\Longrightarrow y' = (1 \times \ln(\sin x) + x\frac{\cos x}{\sin x})e^{x\sin x}\\
&\Longrightarrow y' = (\ln(\sin x) + x\cot x)(\sin x)^x
\end{aligned}
$$

B2-[5 marks]: Find the derivative of $3^x \log_2(2x)$

$$
\begin{aligned}
(3^x \log_2(2x))' &= (3^x)'(\log_2(2x)) + (3^x)(\log_2(2x))'\\
&= (3^x \ln 3)(\log_2(2x)) + (3^x)(\frac{1}{\ln 2}\frac{2}{2x})\\
&= (3^x \ln 3)(\log_2(2x)) + (3^x)(\frac{1}{(\ln 2)x})
\end{aligned}
$$

B3-[4 marks]: Find $\int (4x+5)^{10}\,dx$

$$
\begin{aligned}
&u = 4x + 5 \Longrightarrow du = (4x+5)'dx = 4dx\\
&\int (4x+5)^{10}dx = \frac{1}{4}\int 4u^{10}dx = \frac{1}{4}\int u^{10}du = \frac{1}{4}(\frac{1}{11})u^{11} + C\\
&\Longrightarrow \int (4x+5)^{10}dx = \frac{1}{44}(4x+5)^{11} + C
\end{aligned}
$$

B4-[4 marks]: Find $\int x^2 \cos(x^3+10)\,dx$

$$u = x^3 + 10 \Longrightarrow du = (x^3+10)'dx = 3x^2dx$$
$$\int x^2\cos(x^3+10)dx = \frac{1}{3}\int 3x^2\cos(u)dx = \frac{1}{3}\int \cos u\,du = \frac{1}{3}\sin u + C$$
$$\Longrightarrow \int x^2\cos(x^3+10)\,dx = \frac{1}{3}\sin(x^3+10) + C$$