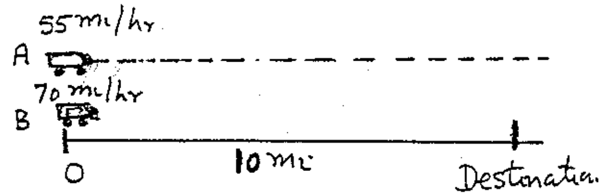
Solutions
Assignment 2
P.204

Q1 Two cars travel in the same direction along a straight highway, one at 55 mi/hr and the other at 70 mi/hr. (a) Assuming that they start at the same point, how much sooner does the faster car arrive at a destination 10 miles away? (b) How far must the faster car travel before it has a 15 minute lead on the slower car?

Solution:



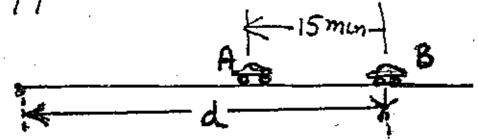
(a) Time taken to travel 10 miles by Car A $= \frac{10}{55} = \frac{2}{11}$ hr

Time taken by Car B $= \frac{10}{70} = \frac{1}{7}$ hr.

Car B arrives sooner by $\left(\frac{2}{11} - \frac{1}{7}\right)$ hr

$$= \frac{14-11}{77} = \frac{3}{77} \text{ hr} = 2.34 \text{ min.}$$

(b) Suppose d is the distance faster Car B travels to have 15 min lead on Car A. Then



15 min $= \frac{1}{4}$ hr.

$$\frac{d}{55} - \frac{d}{70} = \frac{1}{4}$$

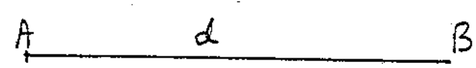
$$\therefore \quad 15d = \frac{70\times 55}{4} \quad \text{or} \quad d = \frac{70\times 55}{15\times 4} = 64.2 \text{ mi}$$

Q2. A person walks first at a constant speed of 5.0 m/s along a straight line from point A to point B, and then back along a line from B to A at constant speed of 3.0 m/s. (a) What is her average speed over the entire trip? (b) Her average velocity over the entire trip.?

Solution:

(a) Average Speed $= \dfrac{\text{Total distance traveled}}{\text{total time}}$

If d is the total distance between A and B, then



$$v_{ave} = \frac{2d}{\frac{d}{5} + \frac{d}{3}} = \frac{2d}{\frac{8d}{15}} = \frac{15}{4} = 3.75 \text{ m/s.}$$

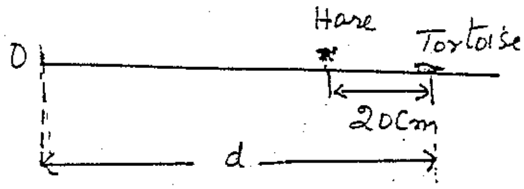
(b) Average Velocity $= \dfrac{\text{Total displacement}}{\text{total time}} = \dfrac{0}{\frac{d}{5} + \frac{d}{3}} = 0$

$\vec{v}_{ave} = 0$

Q3. A speedy tortoise can run at 10.0 cm/s, and a hare can run 20 times as fast. In a race they start at the same time, but the hare stops to rest for 2.0 minutes, and so the tortoise wins the race by 20 cm. (a) How long does the race take? (b) What is the length of the race?

Solution:

Let d meters be the length of the track. The hare travels a distance of (d − 0.2) m and tortoise travels a distance of d.



$v_{tor} = 0.1\ m/s$

$v = 0.1 \times 20 = 2\ m/s$

If t is the time the race takes, then

$d = 0.1 \times t$ (1) (For tortoise), $\therefore t = \frac{d}{0.1}$

and $(d - 0.2) = 2 \times (t - 120)$ (2) (For hare)

From (1) and (2) $d - 0.2 = 2 \times \frac{d}{0.1} - 240$

or $240 - 0.2 = 20d - d$

(b) $\therefore$ The length of track or $d = \frac{239.8}{19} = 12.6\ m$.

(a) From (1) total time taken by the race, $t = \frac{12.6}{0.1} = 126\ s$.

Q4. A particle is moving with a velocity v_0=60 m/s at t=0. Between t=0 and t=15s, the velocity decreases uniformly to zero. What is the average acceleration during this 15-s interval? What is the significance of the sign of your answer?

Solution:

At $t = 0$, $v_0 = 60\ m/s$

At $t = 15\ s$, $v = 0$

Change $\Delta v = v - v_0 = 0 - 60 = -60\ m/s$

Average acceleration $a_{ave} = \frac{\text{Change in velocity}}{\text{total time}}$

$= \frac{-60}{15} = -4\ m/s^2$

The significance of negative sign is that the particle decelerates (slows down)

Q5. A particle moves along x-axis according to the equation $x = 2.0 + 3t - 1.0t^2$, where x is in meters and t in seconds. At t=3.0 s find (a)The position of the particle, (b) its velocity, © its acceleration.

Solution: x is given by

$$x = 2.0 + 3t - 1t^2$$

(a) at $t = 3$, $x = 2.0 + 3 \times 3 - 1 \times 3 \times 3$

$$= 2 + 9 - 9 = 2\text{ m}$$

(b) velocity $\rightarrow \frac{dx}{dt} = 3 - 2t$

At $t = 3$, $\left(\frac{dx}{dt}\right)_{t=3} = 3 - 2 \times 3 = -3\text{ m/s}$

(c) acceleration $\rightarrow a = \frac{dv}{dt} = \frac{d}{dt}(3 - 2t) = -2\text{ m/s}^2$

$$\therefore \quad a = -2$$

Q6. A particle moving in a straight line has a velocity of 80 m/s at t=0. Its velocity at t=20s is 20.0 m/s. (a) What is the average acceleration in this time interval? Can the average velocity be obtained from the information presented?

Solution: At $t=0$, $v_0 = 80\text{ m/s}$

at $t = 20\text{s}$, $v = 20\text{ m/s}$

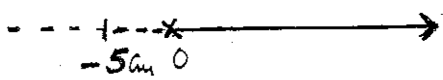
(a) Average Acceleration $\rightarrow a_{ave} = \frac{\text{Change in Velocity}}{\text{total time}} = \frac{20 - 80}{20}\text{ m/s}$

$$= -3\text{ m/s}$$

(b) Since the acceleration may be changing during the motion (if no constant acceleration), there is not enough information about the total distance covered. So, the average velocity cannot be obtained.

Q7. A body moving with uniform acceleration has a velocity of 12.0 cm/s when its x-coordinate t=2.00s later is -5.00 cm. What is the magnitude of its acceleration?

Solution: At $t = 2\text{S}$, $x = -5 = -0.05\text{ m}$



If at $t = 0$, $v = v_0$, then

$$0.12 = v_0 + 2a$$

or $v_0 = 0.12 - 2a \quad (1)$

using the equation $x = v_0 t + \frac{1}{2} a t^2$, we get

$$-.05 = 2v_0 + \frac{a}{2}\cdot(2)^2$$

$$-.05 = 2v_0 + 2a \quad (2)$$

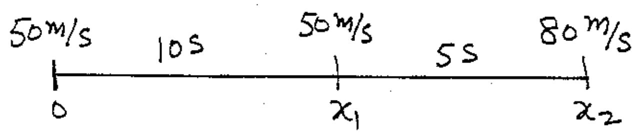
From (1) $\quad .12 = v_0 + 2a \quad (1)$

(2) − (1) gives $\therefore \quad v_0 = -.17\ m/s = 17\ cm/s$

From (1) and $\quad a = \frac{.12 + .17}{2} = \frac{0.29}{2} = 0.145\ m/s^2 = 14.5\ cm/s^2$

Q8. A particle travels in the +ve x-direction for 10s at a constant speed of 50 m/s. It accelerates uniformly to a speed of 80 m/s in the next 5s. Find (a) the average acceleration of the particle in the first 10s. (b) Its average acceleration in the interval t=10s to t=15s, © the total displacement of the particle between t=0 and t=15s and (d) its average speed in the interval t=10s to t=15s.

Solution:



(a) Average acceleration,

$$a_{ave} = \frac{\text{Change in Velocity}}{\text{Time}} = \frac{0}{10} = 0$$

(b) $\quad a_{ave} = \frac{80-50}{5} = \frac{30}{5} = 6\ m/s^2$

(c) Displacement $0x_1 = 50 \times 10 = 500\ m.$

Displacement, $x_2 - x_1 = 50 \times 5 + \frac{1}{2}\cdot 6 \cdot 5^2$

$$= 250 + 75 = 325$$

Total Displacement $= 500 + 325 = 825\ m.$

(d) Average Speed between $t=10$ to $t=15$ $= \frac{\text{Total distance}}{\text{Time}} = \frac{325}{5} = 65\ m/s.$

Q9. The initial speed of a body is 5.20 m/s. What is its speed at 2.50s if it accelerates uniformly at (a) 3.0 m/s^2 and (b) at -3.00 m/s^2.

Solution: Initial Speed, $v_0 = 5.2\ m/s$

Acceleration, $a = 3.0\ m/s^2$

(a) using $v = v_0 + at$ we get $\quad v = 5.2 + 3 \times 2.5 = 12.7\ m/s$

(b) Similarly, for $a = -3\ m/s^2$ $\quad v = 5.2 - 3 \times 2.5 = -2.3\ m/s$

Q10. A jet plane lands with a velocity of 100m/s and can accelerate at a maximum rate of -5.0 m/s^2
as it comes to rest. (a) From the instant it touches the runway, what is the minimum time needed before it stops? (b) Can the plane land at the small airport where the runway is 0.80 km long?

Solution: Acceleration of the plane $a = -5.0\, m/s^2$

(a) Minimum time needed before it comes to stop can be found by using, $v = v_0 + at$,

or $0 = 100 - 5.0 \times t$ or $t = 20s$

(b) Distance covered by the plane before it comes to rest is found by using, $x = v_0 t + \frac{1}{2} a t^2$

or $x = 100 \times 20 - \frac{1}{2} \times 5 \times (20)^2$

$= 2000 - 1000 = 1000\, m.$

Length of the runway $= 0.8\, km = 800\, m$.

$\therefore$ The runway is too short for the plane to land.

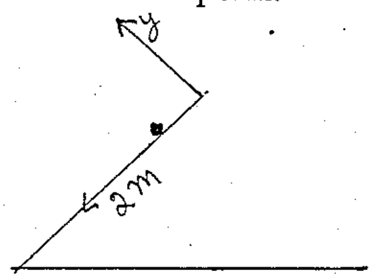
Q11. A particle starts from rest from the top of an inclined plane and slides down with constant acceleration. The inclined plane is 2.00m long, and it takes 3.0s for the particle to reach the bottom. Find (a) the acceleration of the particle, (b) its speed at the bottom of the incline, © the time it takes the particle to reach the middle of the incline, and (d) its speed at the midpoint.

Solution:

(a) using $x = v_0 t + \frac{1}{2} a t^2$

$2 = 0 \times 3 + \frac{1}{2} \times a \times 3^2$

or $a = \frac{4}{9}\, m/s^2$



(b) The speed at the bottom of the plane is found by using $v = v_0 + at$,

or $v = 0 + \frac{4}{9} \times 3 = \frac{4}{3}\, m/s$

(c) The distance to the middle of plane is 1m

$\therefore\ 1 = 0 \times t + \frac{1}{2} \times \frac{4}{9} \times t^2$

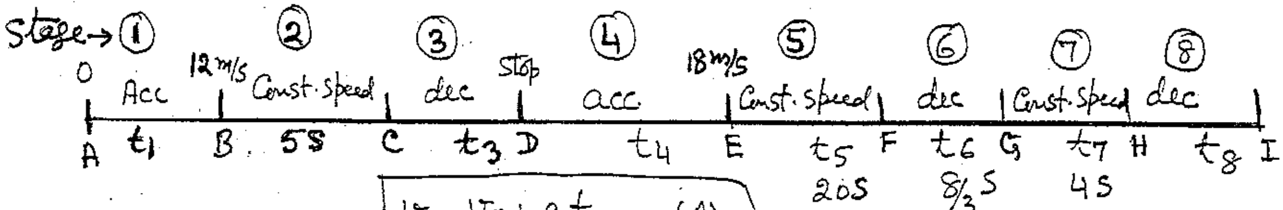
$\therefore\ t^2 = \frac{9}{2}$ or $t = \frac{3}{\sqrt{2}} = 2.12s$.

(d) speed at the mid-point is found by using $v^2 - v_0^2 = 2ax$

i.e $\left.\begin{array}{l} v^2 - 0 = 2 \times \frac{4}{9} \times 1 = \frac{8}{9} \\ v = \frac{2\sqrt{2}}{3} = 0.94\, m/s. \end{array}\right\}$ or $v = 0 + \frac{4}{9} \times 2.12 = 0.94\, m/s$

Q12. A teenager has a car that accelerates at 3.0 m/s^2 and decelerates at 4.5 m/s^2. On a trip to the store, he accelerates from rest to 12m/s, drives at a constant speed for 5.0s, then comes to a momentary stop at the corner. He then accelerates to 18 m/s, drives at a constant speed for 20s, decelerates for 8/3 s, continues for 4.0 s at this speed, and then comes to a stop. (a) How long does the trip take? (b) How far has he traveled © What is his average speed for the trip, (d) How long would it take to walk to the store and back if he walked at 1.5 m/s?

Solution: Acc $= 3.0\ m/s^2$, Dec. $= 4.5\ m/s^2$



$$\boxed{\begin{array}{ll} v = v_0 + at & (A) \\ x = v_0 + \frac{1}{2}at^2 & (B) \\ v^2 - v_0^2 = 2ax & (C) \end{array}}$$

Stage ① using (A), $12 = 0 + 3t_1$ gives $t_1 = 4S$ (Time from A to B)
using (C), $x_1 = 0 + \frac{1}{2}\times 3\times 4^2$ or $x_1 = 24m$ → distance AB
From (A), velocity at B → $v_B = 0 + 4\times 3 = 12\ m/s$

Stage ② → $t_2 = 5S$
$x_2 = 5\times 12 = 60m$ → distance BC

Stage ③ using (A), $0 = 12 - 4.5t_3$ or $t_3 = \frac{8}{3}S$. (Time from C to D)
using (B), $x_3 = 12\times\frac{8}{3} - \frac{1}{2}\times(4.5)\times\left(\frac{8}{3}\right)^2$ or $x_3 = 16m$ → CD

Stage ④ using (A), $18 = 0 + 3t_4$ or $t_4 = 6S$ (Time from D to E)
using (B), $x_4 = 0 + \frac{1}{2}\times 3\times 6^2 = 54\ m$ (Distance DE)

Stage ⑤ $t_5 = 20S$, velocity at E → $v_E = 0 + 6\times 3 = 18\ m/s$
$x_5 = 18\times 20 = 360m$ → (Distance EF)

Stage ⑥ using (A), $v = 18 - 4.5\times\frac{8}{3} = 6\ m/s$, $t_6 = \frac{8}{3}S$.
using (B), $x_6 = 18\times\frac{8}{3} - \frac{1}{2}\times(4.5)\times\left(\frac{8}{3}\right)^2 = 32m$. (FG)

Stage ⑦ $t_7 = 4S$ velocity at G → $v_G = 18 - 8/3\times 4.5 = 6\ m/s$
$x_7 = 4\times 6 = 24m$ → (Distance GH)

Stage ⑧ using (A), $0 = 6 - 4.5t_8$ or $t_8 = \frac{4}{3}S$ (Time from H to I)
using (B), $x_8 = 6\times\frac{4}{3} - \frac{1}{2}\times(4.5)\times\left(\frac{4}{3}\right)^2 = 4\ m$ (HI)

(a) total time $= t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 + t_8 = 4 + 5 + \frac{8}{3} + 6 + 20 + \frac{8}{3} + 4 + 4/3$
$= 45.7S$.

(b) total distance $= x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8$
$= 24 + 60 + 16 + 54 + 360 + 32 + 24 + 4$
$= 574\ m$.

(c) Average speed $= \dfrac{\text{Total distance}}{\text{time}} = \dfrac{574}{45.66} = 12.6\ m/s.$

(d) midpoint $= \dfrac{574}{2} = 287m$, the midpoint lies in stage ⑤ of the motion

$\therefore$ speed at midpoint $= 18\ m/s$

(e) Time taken to walk the distance and return to the starting point $= \dfrac{574 \times 2}{1.5} = 765.3s$
$= 12.8\ min.$

Q13. A river has a steady speed of 0.5 m/s. A student swims upstream a distance of 1.0 km and returns to the starting point. If the student can swim at a speed of 1.20 m/s in the still water, how long does the trip take? Compare this with the time the trip would take if the water were still.

Solution:

velocity of water with respect to ground $\vec{v}_{WG} = 0.5\,\hat{i}$

.5 m/s $\longrightarrow x$

velo. of student relative to water,

$\vec{v}_{SW} = 1.2\,\hat{i}$ (down stream)

$= -1.2\,\hat{i}$ (upstream)

$t_1 \rightarrow$ Time taken to swim upstream in the negative x-direction $= \dfrac{1000}{1.2 - .5} = \dfrac{1000}{.7} = 1428.6s.$

$t_2 \rightarrow$ Time taken to swim downstream $= \dfrac{1000}{1.2 + .5} = \dfrac{1000}{1.7} = 588.2s.$

Total time taken to swim 1 km and back to the starting point $= t_1 + t_2 = 1428.6s + 588.2s = 2016.8s$
$= 33.6\ min$

If the water were still, the time taken for the trip is

$= \dfrac{2 \times 1000}{1.2} = 1666.7s.$
$= 27.8\ min$

Q14. A pilot of an airplane notes that the compass indicates a heading due west. The airplane's speed relative to the air is 150 km/hr. If there is a wind of 30.0 km/hr toward the north, find the velocity of the airplane relative to the ground.

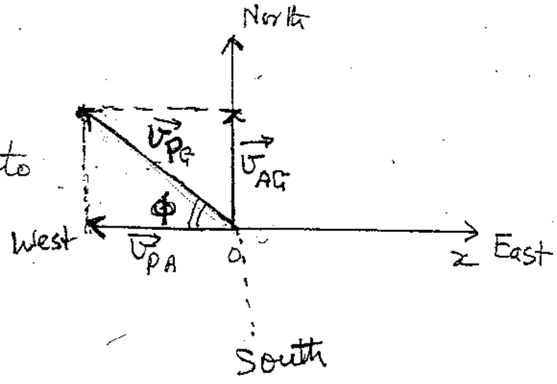
Solution:

speed of plane relative to air $v_{PA} = 150\ km/hr$

velocity of wind relative to ground $v_{AG} = 30\ km/hr$, North

$\vec{v}_{PA} = -150\,\hat{i} + 0\,\hat{j}$ (1)

$\vec{v}_{AG} = 0\,\hat{i} + 30\,\hat{j}$ (2)



If $\vec{v}_{PG}$ is the velocity of plane relative to ground

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

$$= -150\,\hat{i} + 30\,\hat{j} \quad \text{(From (1) and (2))}$$

$$v_{PG} = \sqrt{(150)^2 + (30)^2} = 153 \text{ mi/hr}$$

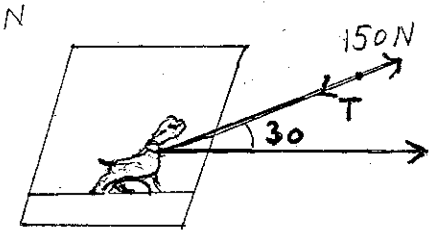
$$\tan\phi = \frac{30}{150} \quad \text{or} \quad \phi = 11.3^\circ$$

$$\therefore \quad \vec{v}_{PG} = 153 \text{ mi/hr},\ 11.3^\circ \text{ north of west}$$

Q15. Your dog Sirus does not want to budge from her favorite rug. You pull on her leash with a force of magnitude 150N at an angle of 30° with the floor. Consider the leash to be an ideal string. (a) What is the force of the leash on her? (b) What is the force of the leash on you? (c) What is the force of friction between the rug and the floor if Sirus still does not budge in spite of this force?

Solution:

(a) The force of the leash on Sirus = 150 N

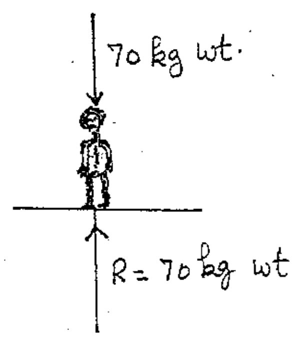


(b) The for of leash on you is due to the tension in the leash which is equal to → 150 N.

(c) **Force of friction** between the rug and floor if Sirus does not budge $= 150 \cos 30^\circ = 150 \times \frac{\sqrt{3}}{2} = 129.9\text{ N}$

Q16. Your mass is 70.0 kg, and you are standing at rest on level ground. (a) Make a second law force diagram indicating all the forces acting on you. (b) What is your weight? (c) What is the normal force of ground on you?

Solution:



(b) weight = 70 kg.wt
or
$= 70 \times 9.8$ N
$= 686$ N

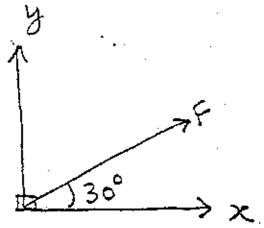
(c) Normal reaction of ground on you

$= 686$ N.

Q17. A total force of 10.0 kg mass produces an acceleration of magnitude 3.00 m/s^2 at an angle of 30 ° to the horizontal. (a) Find the magnitude and direction of the total force on the mass. (b) What are the horizontal and vertical components of the force?

Solution: mass = 10.0 kg.

acceleration = 3.00 m/s^2



(a) ∴ Force required, $F = ma$

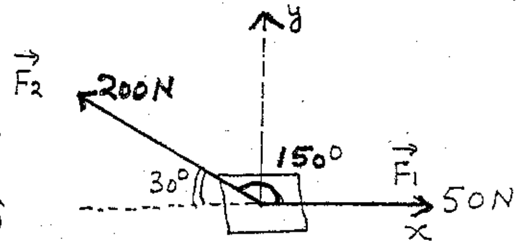
$= 10 \times 3 = 30\,N$

Since the acceleration takes place in the same direction as the force, the direction of force is at an ∠30° to the horizontal

(b) $F_x = 30 \cos 30° = 26\,N$ $F_y = 30 \sin 30 = 15\,N$.

Q18. The only two forces acting on a 5.0 kg mass are 50 N in the positive x-direction and 200N acting at an angle of 150° with the direction of 50N force. (a) Determine the total force acting on the mass. (b) Find the magnitude of the total force. (c) Find the acceleration of the mass. (d) Determine the magnitude of acceleration.

Solution:



(a) $\vec{F_1} = 50\hat{i} + 0\hat{j}$

$\vec{F_2} = -200 \cos 30° \hat{i} + 200 \sin 30° \hat{j}$

$= -200 \times 0.866\,\hat{i} + 200 \times .5\,\hat{j}$

$= -173.2\,\hat{i} + 100\,\hat{j}$

∴ Total force $\vec{F_T} = \vec{F_1} + \vec{F_2}$

$= 50\hat{i} + (-173.2\hat{i} + 100\hat{j})$

$\vec{F_T} = -123.2\hat{i} + 100\hat{j}$

(b) $F_T = \sqrt{(123.2)^2 + (100)^2} = 158.7\,N$

(c) $\vec{a} = \frac{-123.2\hat{i} + 100\hat{j}}{5}$

$= -24.6\,\hat{i} + 20\hat{j}$

(d) Acceleration $a = \frac{158.7}{5.0} = 31.7\ m/s^2$

or from (c) → $a = \sqrt{(24.6)^2 + (20)^2}$

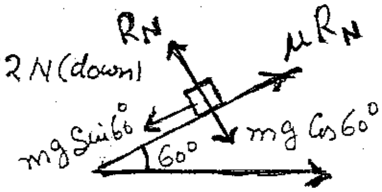
$= 31.7\,m/s^2$

Q19. A downhill skier is sliding down a steep slope that makes an angle 60° with the horizontal direction. The mass of the fully equipped skier is 80 kg, the coefficient of kinetic friction for the skis on the snowy slope is 0.09. (a) Calculate the magnitude of his acceleration. (b) If his teammate of mass 90 kg follows him down the slope, is his acceleration the same? (c) For what slope angle is his acceleration zero?

Solution:

$R_N = mg \cos 60° = 80 \times 9.8 \times \frac{1}{2} = 392\,N$ (down)

∴ $\mu R_N = 392 \times 0.09 = 35.3\,N$ (up the plane)



Net Force down the plane $= mg \sin 60° - 35.3\,N$

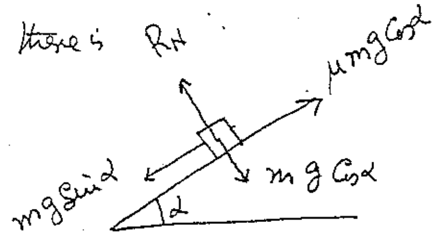
$= 80 \times 9.8 \times 0.866 - 35.3 = 643.6\,N$

(Continued)

(a) Acceleration $= \dfrac{643.6}{80} = 8.0\ m/s^2$

(b) The acceleration of the teammate will be the same.

(c) If α is the slope angle at which the acceleration will be zero, the acceleration will be zero when there is no force acting on the skier



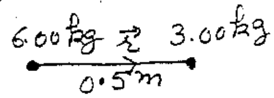
i.e when $mg \sin\alpha = \mu mg \cos\alpha$

or $\tan\alpha = \mu = 0.09$

$\therefore \alpha = 5.1^\circ$

Q20. A 6.00 kg mass is located 50 cm from a 3.00 kg mass. Assume they are both point-like masses. (a) Find the gravitational force of the 6.00 kg mass on the 3.00 kg mass. (b) What is the gravitational force of the 3.00 kg mass on the 6.00 kg mass?

Solution: Force $\vec{F}$ of 6.00 kg mass on 3.00 kg mass is



$$\vec{F} = \frac{(-6.67\times10^{-11})(6.00)(3.00)}{(.5)^2}$$

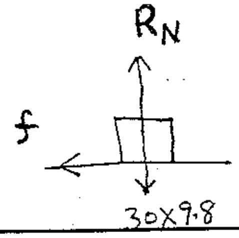
$= -4.8\times10^{-9}\ \hat{r}$ N (towards 6.00 kg mass) attractive

(b) The gravitational force of 3.00 kg mass on 6.00 kg

$\vec{F} = 4.8\times10^{-9}\ \hat{r}$ N (towards 3.0 kg mass)

Q21. You find that a horizontal force of magnitude 250N is able to slide a 30 kg mass across a horizontal surface with an acceleration of 5.00 m/s^2. What is the coefficient of kinetic friction for the mass on the surface?

Solution: Force required to produce an acceleration of 5.00 m/s^2



$F = ma = 30\times5 = 150$ N.

$\therefore$ Force of friction $= 250 - 150 = 100$ N.

$\therefore$ Coefficient of kinetic friction $\mu_k = \dfrac{100}{30\times9.8} = 0.34$

Q22. The coefficient of friction for a file cabinet on the floor are μ_s=0.45 and μ_k=0.30. You push horizontally with a force of magnitude 75N on a 100kg file cabinet and are unable to move it. What is the force of static friction by the floor on the cabinet?

Solution: The force of static friction by the floor = 75 N.