SOLUTIONS
Assignment# 1
Physics 204

Q1 A vector has x-component of -25.0 units and a y-component of 40.0 units. Find the magnitude and direction of this vector. Ans: (47.2 units)

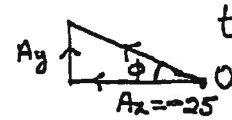
Solution: If $\hat{i}$ and $\hat{j}$ are unit vectors along x and y-axes respectively, then any vector can be written as

$\vec{A} = A_x \hat{i} + A_y \hat{j}$. A_x, A_y are the components of $\vec{A}$ along x and y-axes.

Given: $A_x = -25.0$ units

$A_y = 40.0$ units

$\therefore \vec{A} = -25.0\hat{i} + 40.0\hat{j}$



$\tan\phi = \frac{40}{25} = 1.6$

$\phi = 58^0$

magnitude of $\vec{A} \rightarrow |\vec{A}| = \sqrt{(-25.0)^2 + (40)^2} = \sqrt{2225} = 47.17$ units.

Q2. A displacement vector lying in the x-y plane has a magnitude of 50.0m, and is directed at an angle of 120° to the positive x-axis. What are the rectangular components of this vector?

Solution: If $\vec{R}$ is the given vector

x-Component, $R_x = R\cos\theta$

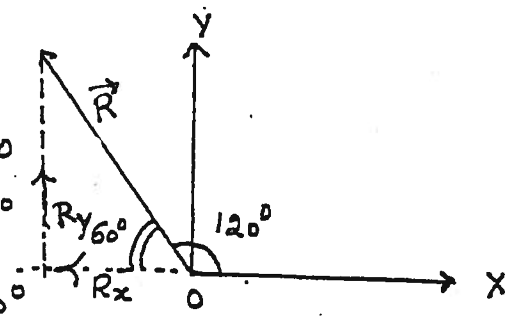
where $\theta = 120^0$, $R = 50$

$\therefore R_x = 50\cos 120^0 = -50\cos 60^0$

or $R_x = -25$ m

$R_y = 50\sin 120^0 = 50\sin 60^0$

or $R_y = 43.30$ m



$\therefore \vec{R} = (-25\,m)\hat{i} + (43.3)\hat{j}$

Q3 Consider two vectors $\bar{A} = 3\hat{i} - 2\hat{j}$ and $\bar{B} = -\hat{i} - 4\hat{j}$. Calculate (a) $\bar{A} + \bar{B}$ (b) $\bar{A} - \bar{B}$ © $|\bar{A} + \bar{B}|$ (d) $|\bar{A} - \bar{B}|$

Solution:

(a) $\vec{A} + \vec{B} = (3\hat{i} - 2\hat{j}) + (-\hat{i} - 4\hat{j}) = 2\hat{i} - 6\hat{j}$

(b) $\vec{A} - \vec{B} = (3\hat{i} - 2\hat{j}) - (-\hat{i} - 4\hat{j}) = 4\hat{i} + 2\hat{j}$

(c) $|\vec{A} + \vec{B}| = |(2\hat{i} - 6\hat{j})| = \sqrt{2^2 + (-6)^2} = \sqrt{40} = 6.32$

(d) $|\vec{A} - \vec{B}| = |4\hat{i} + 6\hat{j})| = \sqrt{4^2 + 2^2} = \sqrt{20} = 4.47$

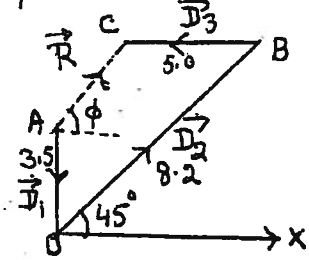
Q4 A particle undergoes the following displacements: 3.50 m south, 8.20 m northeast, and 5.0 m west. What is the resultant displacement?

Solution: If $\vec{D_1}$, $\vec{D_2}$, and $\vec{D_3}$ are the three displacement vectors.

$\vec{AO} \rightarrow \vec{D_1} = 0\hat{i} - 3.5\hat{j}$

$\vec{OB} \rightarrow \vec{D_2} = (8.2\cos 45^\circ)\hat{i} + (8.2\sin 45^\circ)\hat{j}$

$\vec{BC} \rightarrow \vec{D_3} = -5.0\,\hat{i} + 0\hat{j}$



The resultant displacement $\vec{R}$ is given by:

$$\vec{R} = \vec{D_1} + \vec{D_2} + \vec{D_3}$$
$$= (0\hat{i} - 3.5\hat{j}) + \left(\frac{8.2}{\sqrt{2}}\hat{i} + \frac{8.0}{\sqrt{2}}\hat{j}\right) + (-5\hat{i} + 0\hat{j})$$
$$= (0 + 5.8 - 5.0)\hat{i} + (-3.5 + 5.8 + 0)\hat{j}$$
$$= 0.8\hat{i} + 2.3\hat{j}$$

Magnitude of $\vec{R} \rightarrow R = \sqrt{(.8)^2 + (2.3)^2} = 2.44$

$$\phi = \tan^{-1}\frac{2.3}{0.8} = 71^\circ$$

Q5 A jet airliner moving initially at 300 km/hr to the east moves into a region where the wind is blowing at 100km/hr in a direction 30.0° north of east. What is the new speed and direction of the aircraft?

$\vec{AB} = 300\,km/hr$
$\vec{BC} = 100\,km/hr$

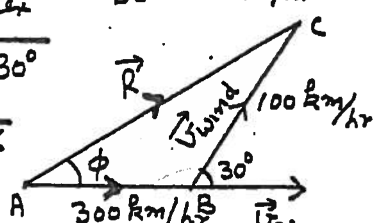
Solution: The resultant speed of the airliner given as

$R = \sqrt{(300)^2 + (100)^2 + 2 \times 300 \times 100 \cos 30^\circ}$

Cosine Law

$= \sqrt{90{,}000 + 10{,}000 + 60{,}000 \times 0.866}$

$\therefore$ New Speed $= 389.8\ km/hr.$



Direction of $\vec{R} \rightarrow \tan\phi = \dfrac{100\sin 30^\circ}{300 + 100\cos 30^\circ} = \dfrac{50}{300 + 86.6} = 0.13$

$\phi = 7.4^\circ$

Q6 Given the displacement vector $\bar{A} = (3\hat{i} - 4\hat{j} + 4\hat{k})$ m and $\bar{B} = (2\hat{i} + 3\hat{j} - 7\hat{k})$ m. Find the magnitudes of vectors (a) $\bar{C} = \bar{A} + \bar{B}$ and (b) $\bar{D} = 2\bar{A} - \bar{B}$, also express each in terms of its rectangular components.

Solution: (a) $\vec{C} = \vec{A} + \vec{B} = (3\hat{i} - 4\hat{j} + 4\hat{k}) + (2\hat{i} + 3\hat{j} - 7\hat{k}) = (5\hat{i} - \hat{j} - 3\hat{k})$

x-Comp = 5 m , y-Comp = −1 m , z-Comp = −3 m

(b) $\vec{D} = 2\vec{A} - \vec{B} = 2(3\hat{i} - 4\hat{j} + 4\hat{k}) - (2\hat{i} + 3\hat{j} - 7\hat{k})$

$= (6\hat{i} - 8\hat{j} + 8\hat{k}) - (2\hat{i} + 3\hat{j} - 7\hat{k})$

$= 4\hat{i} - 11\hat{j} + 15\hat{k}$

x-Comp = 4 m
y-Comp = −11 m
z-Comp = 15 m.

Q7 If $\bar{A} = (6\hat{i} - 4\hat{j})$ units, $\bar{B} = (-8\hat{i} + 3\hat{j})$ units. Determine a and b so that $a\bar{A} + b\bar{B} + c = 0$

Solution: Given: $\vec{A} = (6\hat{i} - 4\hat{j})$, $\vec{B} = (-8\hat{i} + 3\hat{j})$

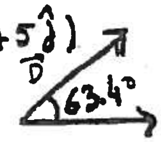
$$a\vec{A} + b\vec{B} + c = 0 \quad (1)$$
$$a(6\hat{i} - 4\hat{j}) + b(-8\hat{i} + 3\hat{j}) + c = 0$$
$$\therefore 6a\hat{i} - 4a\hat{j} - 8b\hat{i} + 3b\hat{j} + c = 0$$
$$\text{or } (6a - 8b)\hat{i} + (3b - 4a\hat{j}) + c = 0 \quad (2)$$ Therefore each term in eq (2) must be equal to zero ∴

$$6a - 8b = 0 \text{ or } 3a = 4b \text{ or } a = \tfrac{4}{3}b$$
$$3b - 4a = 0 \text{ or } 4a = 3b \text{ or } a = \tfrac{3}{4}b$$
→ Not possible unless $a = 0$, $b = 0$
$$c = 0$$

$\therefore a = 0,\ b = 0,\ c = 0$ to satisfy (1) (trivial Solution)

Q8 Consider the displacement vectors $\bar{A} = (3\hat{i} + 3\hat{j})$m, $\bar{B} = (\hat{i} - 4\hat{j})$m, and $\bar{C} = (-2\hat{i} + 5\hat{j})$m. Determine the magnitude and direction of the vector (a) $\bar{D} = \bar{A} + \bar{B} + \bar{C}$, (b) the magnitude and direction of $\bar{E} = -\bar{A} - \bar{B} + \bar{C}$

Solution: (a) $\vec{D} = \vec{A} + \vec{B} + \vec{C} = (3\hat{i} + 3\hat{j}) + (\hat{i} - 4\hat{j}) + (-2\hat{i} + 5\hat{j})$
$= 2\hat{i} + 4\hat{j}$

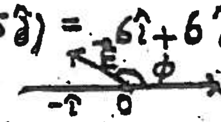


Magnitude of $\vec{D}$, $|\vec{D}| = \sqrt{4 + 16} = 4.47$

$\tan\phi = \frac{4}{2} = 2$ or $\phi = 63.4^\circ$ to x-axis

(b) $\vec{E} = -\vec{A} - \vec{B} + \vec{C} = -(3\hat{i} + 3\hat{j}) - (\hat{i} - 4\hat{j}) + (-2\hat{i} + 5\hat{j}) = -6\hat{i} + 6\hat{j}$

$|\vec{E}| = \sqrt{(-6)^2 + 6^2} = \sqrt{72} = 8.49$



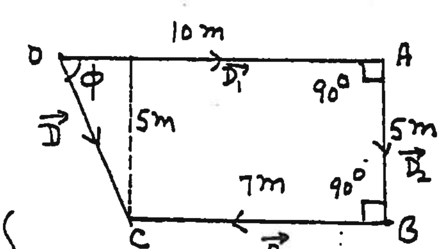
$\tan\phi = \frac{6}{-6} = -1$, $\phi = 135^\circ$

Q9 Indiana Jones is trapped in a maze. To find his way out, he walks 10.0 m to the right, makes a 90° right turn, walks 5.0 m, makes another 90° right turn, and walks 7.0 m. What is his displacement from his initial position?

Ans: $(3\hat{i} - 5\hat{j})$

Solution:

If $\vec{D}$ is the displacement of Jones from initial position



$|\vec{D}| = \sqrt{5^2 + 3^2} = \sqrt{34} = 5.83$m

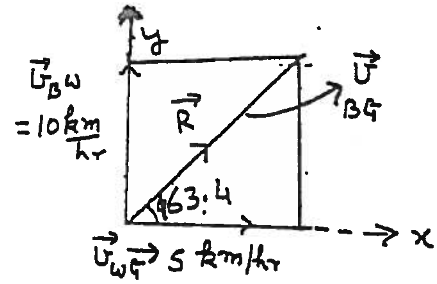
$\tan\phi = \frac{-5}{3}$ or $\phi = -59^\circ$ (South east)

$\overrightarrow{OA} \rightarrow \vec{D}_1 = 10\hat{i} + 0\hat{j}$
$\overrightarrow{AB} \rightarrow \vec{D}_2 = 0\hat{i} - 5\hat{j}$
$\overrightarrow{BC} \rightarrow \vec{D}_3 = -7\hat{i} + 0\hat{j}$
$\vec{R} \rightarrow \overrightarrow{OC} = 3\hat{i} - 5\hat{j}$

Resultant → $\vec{R} = 3\hat{i} - 5\hat{j}$

Q10 A boat heading due north crosses a wide river with a speed of 10.0 km/hr relative to the water. The river has a uniform speed of 5.0 km/hr due east relative to earth. Determine the velocity of the boat relative to a stationary ground observer.

Ans: ($5\hat{i}+10\hat{j}$, v_{BG}=11.2 km/s, ϕ=63.4°)



__Solution:__ If $\vec{v}_{BW}$ is the velocity of boat with respect to water, and $\vec{v}_{WG}$ be the velocity of water with respect to ground,

$$\vec{v}_{BW} = (0\hat{i} + 10\hat{j})\ \frac{km}{hr}$$

$$\vec{v}_{WG} = (5\hat{i} + 0\hat{j})\ \frac{km}{hr}$$

If $\vec{v}_{BG}$ is the velocity of the boat with respect to ground,

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$$

$$= (0\hat{i} + 10\hat{j}) + (5\hat{i} + 0\hat{j}) = (5\hat{i} + 10\hat{j})$$

$$\therefore\ v_{BG} = \sqrt{5^2 + 10^2} = \sqrt{125} = 11.18\ \frac{km}{hr}, \quad \tan\phi = \frac{10}{5}$$

$$\phi = 63.4^\circ$$

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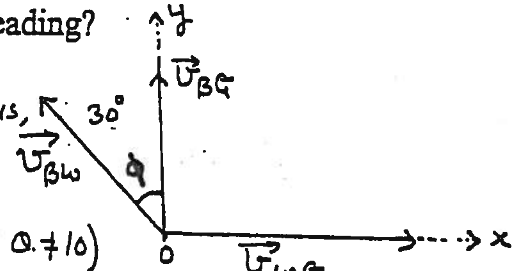
in Q.10

Q11 A boat travels with a speed of 10.0 km/hr. relative to water and is to travel due north. In what direction should it be heading?



__Solution:__ If ϕ is the angle which the direction of the boat makes with y-axis,

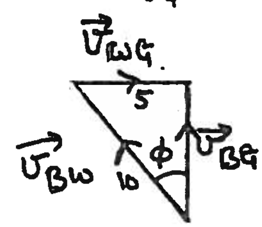
$$\vec{v}_{BW} = -10\sin\phi\ \hat{i} + 10\cos\phi\ \hat{j} \qquad (\phi = ?)$$

$$\vec{v}_{BG} = v_{BG}\ \hat{j} \qquad \to \text{(from Sol. of Q.10)}$$

$$\vec{v}_{WG} = 5\hat{i} + 0\hat{j}$$

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$$

$$\text{or } \vec{v}_{BW} = \vec{v}_{BG} - \vec{v}_{WG}.$$



$$\therefore\ -10\sin\phi\ \hat{i} + 10\cos\phi\ \hat{j} = v_{BG}\ \hat{j} - 5\hat{i}$$

From above

$$-10\sin\phi = -5 \quad \text{and} \quad v_{BG} = 10\cos\phi$$

$$= 8.66\ \text{km/hr}$$

$$\therefore\ \phi = \tfrac{1}{2}$$

$$\text{or } \phi = 30^\circ$$

Therefore the boat must head in a direction making an angle of 30° with the y-axis in north-west direction.

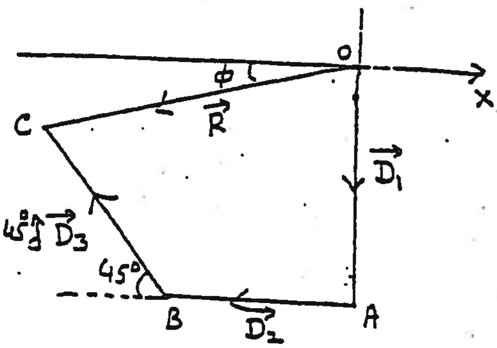
Q12 A motorist drives south at 200 m/s for 3.0 minutes, the turns west and travels at 25.0 m/s for 2.0 minutes, and finally travels northwest at 30.0 m/s for 1.0 min. For this 6.0-minute trip, find (a) the resultant vector displacement, (b) the average speed, and © the average velocity. Ans: ($-4272\hat{i} - 34727\hat{j}$, 113m/s, 97.2 m/s)

Solution: (a) If $\vec{D_1}$, $\vec{D_2}$ and $\vec{D_3}$ are the displacement vectors, $\boxed{\vec{D} = \vec{v} \times t}$

$\vec{OA} \rightarrow \vec{D_1} = -200 \times 180\,\hat{j}$

$\vec{AB} \rightarrow \vec{D_2} = -25 \times 120\,\hat{i}$

$\vec{BC} \rightarrow \vec{D_3} = -30 \times 60 \cos 45^\circ\,\hat{i} + 30 \times 60 \times \cos 45^\circ\,\hat{j}$



The resultant displacement $\vec{R}$ is,

$\vec{CB} \rightarrow \vec{R} = \vec{D_1} + \vec{D_2} + \vec{D_3}$

$$= (-36{,}000)\hat{j} - 3000\,\hat{i} - \frac{1800}{\sqrt{2}}\,\hat{i} + \frac{1800}{\sqrt{2}}\,\hat{j}$$

$$= -\left(3000 + \frac{1800}{\sqrt{2}}\right)\hat{i} + \left(-36000 + \frac{1800}{\sqrt{2}}\right)\hat{j}$$

$$\therefore\ \vec{R} = -4272\,\hat{i} - 34727.2\,\hat{j}$$

$R = \sqrt{(4272)^2 + (34727)^2} = 34988\ m.$

$\tan\phi = \frac{34727}{4272}$ or $\phi = 83^\circ$

Total time $= 180 + 120 + 60 = 360$ min

(b) Average Speed $= \frac{200 \times 180 + 25 \times 120 + 30 \times 60}{(180 + 120 + 60)} = \frac{40800}{360} = 113\ m/s.$

(c) Average velocity $= \frac{34988}{360}$ at an angle of 83° to $-x$ direction

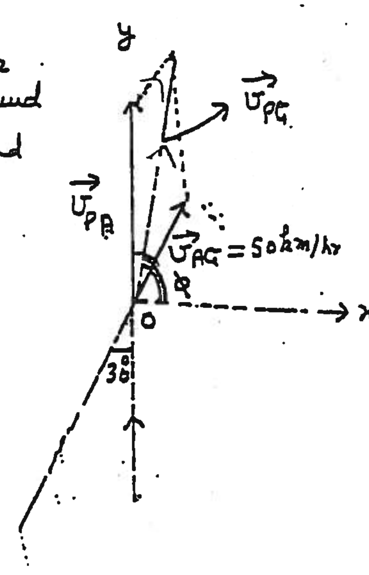
$\vec{v}_{ave} = 97.2\ m/s.$ at $\angle 83^\circ$ to x-axis

Q13 The velocity of an aircraft relative the air is 360 km/hr north. The wind is blowing 50 km/hr from the south and west with the wind velocity making an angle of 30° with north. Find the velocity of the aircraft relative to the ground.

Solution: If $\vec{v}_{PA} \rightarrow$ velocity of plane relative to air

$\vec{v}_{PG} \rightarrow$ velocity of plane relative to ground

$\vec{v}_{AG} \rightarrow$ velocity of air relative to ground



$\vec{v}_{PA} = 0\hat{i} + 360\hat{j}$

$\vec{v}_{AG} = 50 \sin 30^\circ\,\hat{i} + 50 \cos 30^\circ\,\hat{j}$

$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG} = 360\hat{j} + (50 \times \frac{1}{2}\,\hat{i} + 50 \times \frac{\sqrt{3}}{2}\,\hat{j})$

$= (25\hat{i} + 403.3\hat{j})$

$\therefore\ v_{PG} = \sqrt{25^2 + 403.3^2} = 404\ km/hr$

$\tan\phi = 403.3/25$ or $\phi = 86.5^\circ$ North East

Q14. If $\bar{A} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\bar{B} = -3\hat{i} - 2\hat{j} + 4\hat{k}$, find $\bar{A}.\bar{B}$

Solution:

$$\vec{A} = 2\hat{i} - \hat{j} + 4\hat{k}$$
$$\vec{B} = -3\hat{i} - 2\hat{j} + 4\hat{k}$$
$$\therefore \vec{A}.\vec{B} = (2\hat{i} - \hat{j} + 4\hat{k}).(-3\hat{i} - 2\hat{j} + 4\hat{k})$$

using $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
$\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$, we have

$$\vec{A}.\vec{B} = (-6+2+16) = 12$$

Q15. If $\bar{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, and $\bar{B} = -3\hat{i} + 2\hat{j} - 3\hat{k}$, find $\bar{A} \times \bar{B}$

Solution:

$$\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$$
$$\vec{B} = -3\hat{i} + 2\hat{j} - 3\hat{k}$$
$$\vec{A} \times \vec{B} = (2\hat{i} + 3\hat{j} - 4\hat{k}) \times (-3\hat{i} + 2\hat{j} - 3\hat{k})$$

using $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
$\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$, we have

$$\vec{A} \times \vec{B} = \{-6\hat{i}\times\hat{i} + 4\hat{i}\times\hat{j} - 6\hat{i}\times\hat{k}\} + \{-9\hat{j}\times\hat{i} + 6\hat{j}\times\hat{j} - 9\hat{j}\times\hat{k}\}$$
$$+ \{+12\hat{k}\times\hat{i} - 8\hat{k}\times\hat{j} + 12\hat{k}\times\hat{k}\}$$
$$= \{0 + 4\hat{k} + 6\hat{j}\} + \{+9\hat{k} + 0 - 9\hat{i}\} + \{12\hat{j} + 8\hat{i} + 0\}$$
$$\vec{A} \times \vec{B} = -\hat{i} + 18\hat{j} + 13\hat{k}$$

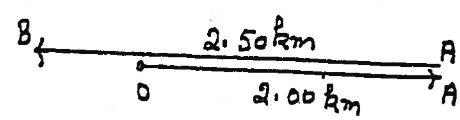
By method of determinant

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -4 \\ -3 & 2 & -3 \end{vmatrix} = i(-9+8) - \hat{j}(-6-12) + \hat{k}(4+9)$$
$$= \hat{i} + 18\hat{j} + 13\hat{k}$$

Q16. You drive your car 2.00km down a road, then 2.50 km in the opposite direction, completing the excursion in 180 seconds. (a) Determine the initial and final position vectors and the change in position vector during the time interval.. (b) Find the average velocity. © Find the average speed.. (d) Explain why the average speed of the car is not equal to the magnitude of the average velocity.

Solution:

(a) Initial position vector $= \vec{OA}$
Final position vector $= \vec{AB}$



Change in position vector $= \vec{AB} - \vec{OA}$
$= 2.50 - 2.00 = 0.50$ km in the direction AB

(b) Average velocity $\vec{v}_{Ave} = \dfrac{0.50\,(\text{dir } \vec{AB})}{180} = \dfrac{1}{360}\,\text{km/s}$ in the direction AB.
$= 0.003\ \text{km/s}$ in the direction AB

(C) Average speed $= \dfrac{2.00 + 2.50}{180} = \dfrac{4.50\,\text{km}}{180\ \text{s}} = 0.025\ \text{km/s}$

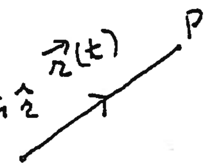
(D) In case of average speed we take into account the total distance traveled regardless of the direction, but in case of average velocity the two distances traveled are in opposite directions, therefore the average velocity is much less

Q17. The one-dimensional position vector of a sports car is given by $\vec{r}(t) = \left[5.0 - 6.0\,t + 3.0\,t^2 + 7.0\,t^3\right]$.

(a) Find the velocity $\vec{v}(t)$ as a function of time. (b) Find the velocity at the instant when t=2.0 s.

(c) Determine at what time the velocitv is zero.

Solution: Given



(a) $\vec{r}(t) = 5.0 - 6.0t + 3.0t^2 + 7.0t^3$ in the direction $\hat{r}$

$\vec{v}(t) = \dfrac{d\vec{r}(t)}{dt} = (-6.0 + 6.0t + 21.0t^2)\,\hat{r}$

$\hat{r} \rightarrow$ unit vector in the direction of $\hat{r}(t)$

(b) $\vec{v}(t = 2.0) = (-6.0 + 12.0 + 84.0)\,\hat{r}$

$= 90.0\,\hat{r}$ where $\hat{r}$ is a unit vector in the direction of $\vec{r}(t)$

(C) The velocity will be zero when $21.0t^2 + 6.0t - 6.0 = 0$

or $7t^2 + 2t - 2 = 0$

$t = \dfrac{-2 \pm \sqrt{4 + 56}}{14}$

Taking the +ve sign $t = \dfrac{-2 + 7.75}{14} = 0.4\ \text{s}$

Q18. Find the angle between two vectors $\vec{A} = 1.0\,\hat{i} + 2.0\,\hat{j} - 1.0\,\hat{k}$ and $\vec{B} = -1.0\,\hat{i} + 3.0\,\hat{j} + 5.0\,\hat{k}$

Solution:

$\vec{A} = 1.0\hat{i} + 2.0\hat{j} - 1.0\hat{k}$

$\vec{B} = -1.0\hat{i} + 3.0\hat{j} + 5.0\hat{k}$

$\vec{A}\cdot\vec{B} = AB \cos\angle \vec{A},\vec{B} \quad \therefore \cos\angle \vec{A},\vec{B} = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}$

$= \dfrac{(1.0\hat{i} + 2.0\hat{j} - 1.0\hat{k})\cdot(-1.0\hat{i} + 3.0\hat{j} + 5.0\hat{k})}{AB}$

$= \dfrac{(-1 + 6 - 5)}{AB} = \dfrac{0}{AB} = 0$

Q19 Vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes of 5.00. If the sum of $\vec{A}$ and $\vec{B}$ is the vector $6.00\,\hat{j}$, determine the angle between $\vec{A}$ and $\vec{B}$.

Solution: Let $\vec{A} = A_x\hat{i} + A_y\hat{j}$
and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

Since $\vec{A}$ and $\vec{B}$ have equal magnitudes,

$$(A_x^2 + A_y^2) = (B_x^2 + B_y^2) = 25 \quad (1)$$

Also the sum of $\vec{A}$ and $\vec{B}$ is $6.00\hat{j}$

$$\therefore \quad (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j}) = 0\hat{i} + 6.00\hat{j}$$

$$\therefore \quad A_x + B_x = 0 \quad \text{or} \quad A_x = -B_x \quad (2)$$

$$\text{and} \quad A_y + B_y = 6 \quad (3)$$

From (2) and (1) we get

$$(B_x^2 + A_y^2) = (B_x^2 + B_y^2)$$

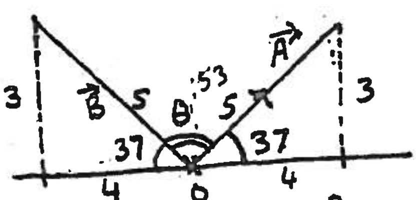
$$\text{or} \quad A_y^2 = B_y^2 \quad \text{or} \quad A_y = B_y \quad (4)$$

From (3) and (4) we get $A_y = B_y = 3$ (5)

From (5), (1) and (2) we get

$$A_x = 4 \qquad B_x = -4$$

$$\therefore \quad \vec{A} = 4\hat{i} + 3\hat{j}$$
$$\vec{B} = -4\hat{i} + 3\hat{j}$$



Angle between $\vec{A}$ and $\vec{B}$ $= \theta = 180 - 74 = 106^\circ$

Q20 Given three vectors $\vec{A} = 2\hat{i} + \hat{j}$, $\vec{B} = \hat{i} + \hat{k}$, and $\vec{C} = 4\hat{j}$, find the following:
(a) $\vec{A}\cdot(\vec{B}+\vec{C})$, (b) $\vec{A}\cdot(\vec{B}\times\vec{C})$, and (c) $\vec{A}\times(\vec{B}\times\vec{C})$

Solution: (a) $\vec{A}\cdot(\vec{B}+\vec{C}) = (2\hat{i}+\hat{j})\cdot[(\hat{i}+\hat{k}) + (4\hat{j})]$

$$= (2\hat{i}+\hat{j})\cdot(\hat{i}+4\hat{j}+\hat{k})$$

$$= 2 + 4 = 6$$

(b) $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{A}\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$

$= \vec{A}\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 0 & 4 & 0 \end{vmatrix}$

$= \vec{A}\cdot\left\{\hat{i}(0-4) - \hat{j}(0-0) + \hat{k}(4-0)\right\}$

$= (2\hat{i}+\hat{j})\cdot(-4\hat{i}+4\hat{k})$

$= -8$

(c) $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{A}\times\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$

$= (2\hat{i}+\hat{j})\times(-4\hat{i}+4\hat{k})$

$= -8\,\hat{i}\times\hat{i} + 8\,\hat{i}\times\hat{k} - 4\,\hat{j}\times\hat{i} + 4\,\hat{j}\times\hat{k}$

$= 0 - 8\hat{j} + 4\hat{k} + 4\hat{i}$

$= 4\hat{i} - 8\hat{j} + 4\hat{k}$