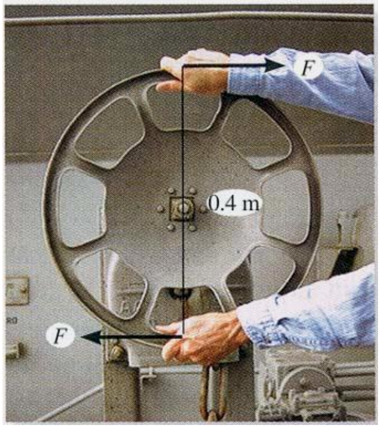
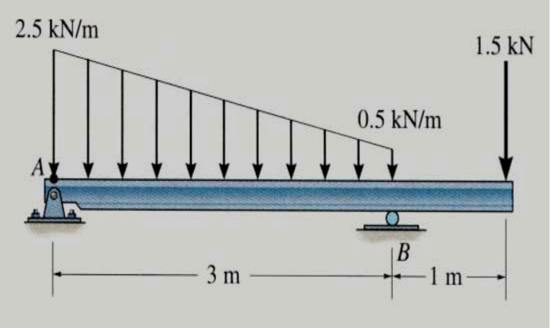
Chapter 4

Moment of a Force





MOMENT OF A FORCE SCALAR FORMULATION, CROSS PRODUCT, MOMENT OF A FORCE VECTOR FORMULATION, & PRINCIPLE OF MOMENTS

Objectives :

Students will be able to:

a) understand and define moment, and,

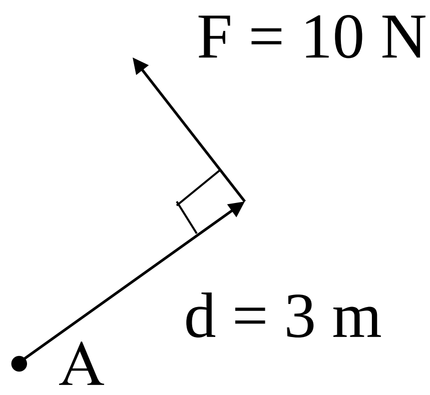
b) determine moments of a force in 2-D and 3-D cases.

READING QUIZ

1. What is the moment of the 10 N force about point A (M_A)?

   A) 10 N·m B) 30 N·m C) 13 N·m

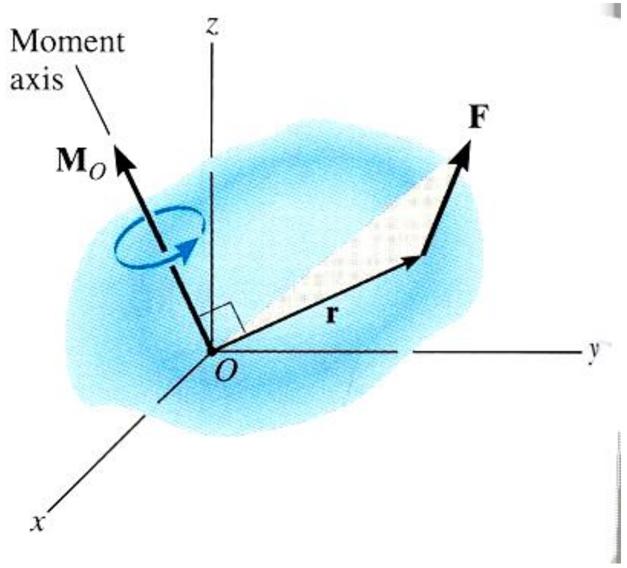
   D) (10/3) N·m E) 7 N·m



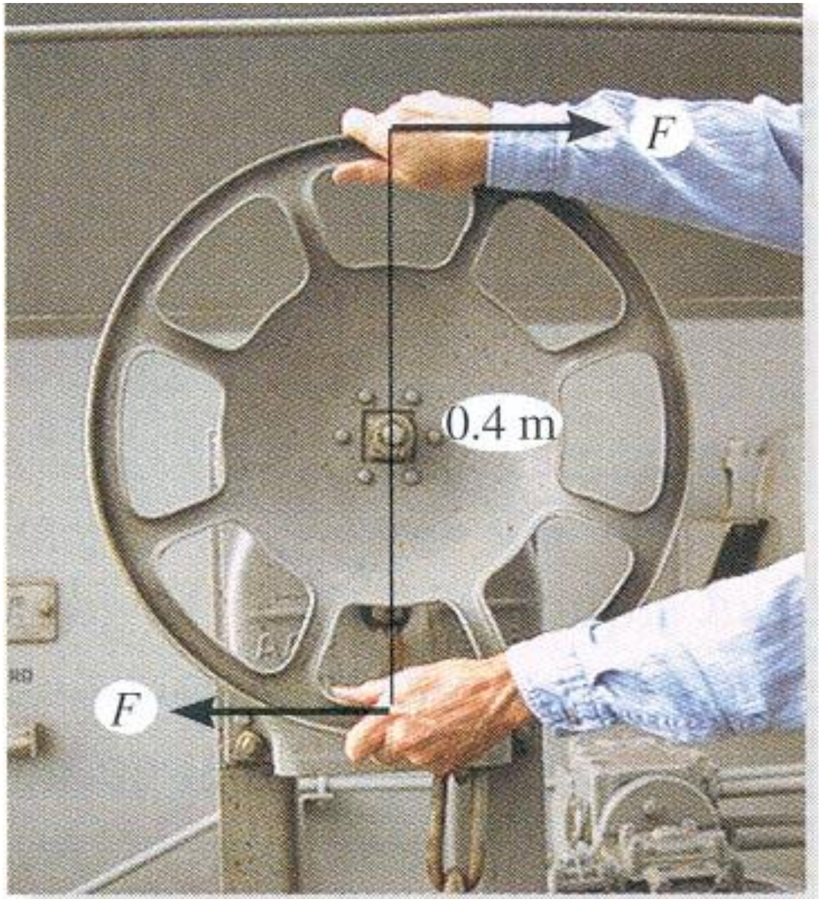
2. Moment of force ***F*** about point O is defined as $\boldsymbol{M_O}$ = ___________ .

   A) ***r*** **x** ***F*** B) ***F*** **x** ***r***

   C) ***r*** **•** ***F*** D) r * F

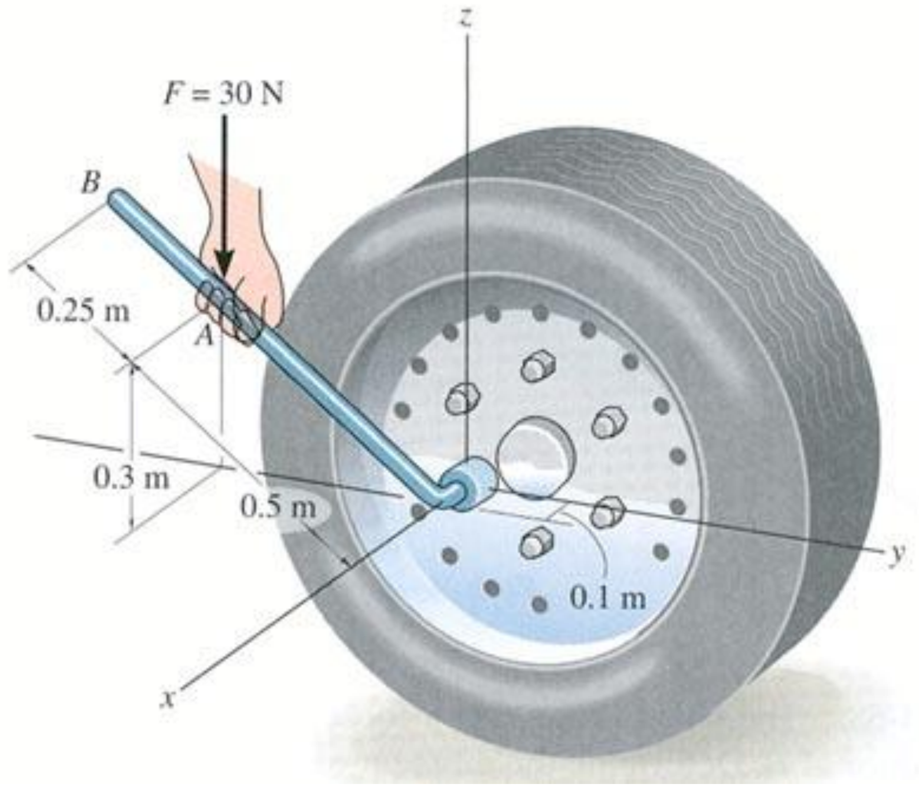


APPLICATIONS



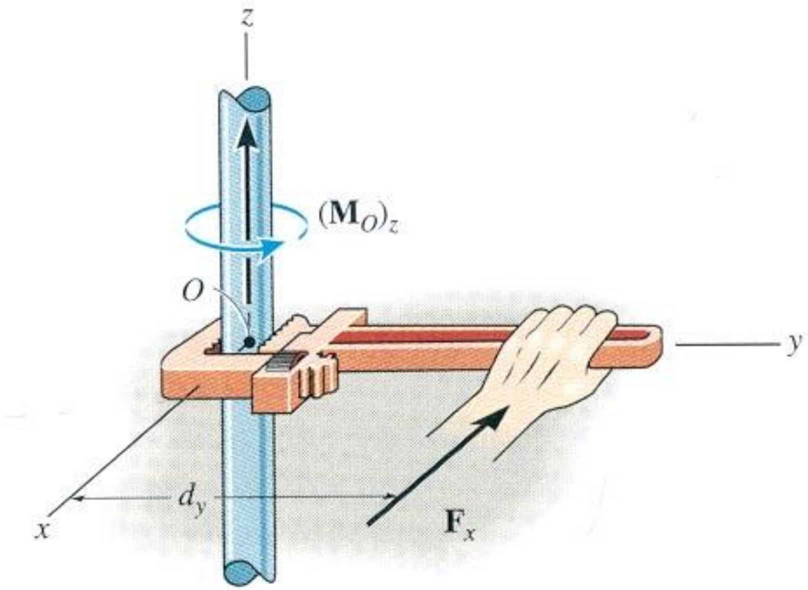
What is the net effect of the two forces on the wheel?

APPLICATIONS
(continued)



What is the effect of the 30 N force on the lug nut?

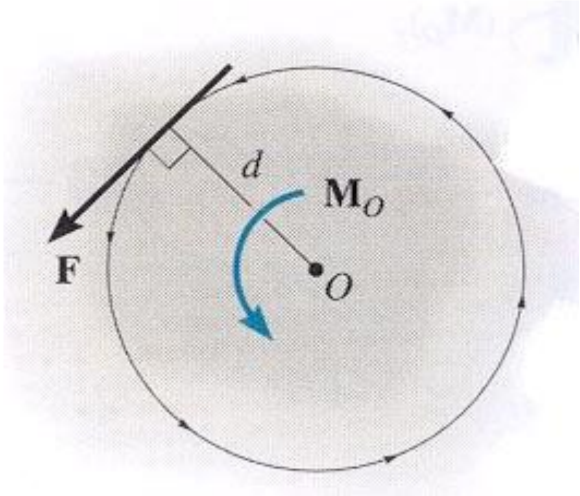
MOMENT OF A FORCE - SCALAR FORMULATION (Section 4.1)



The moment of a force about a point provides a measure of the tendency for rotation (sometimes called a torque).

MOMENT OF A FORCE - SCALAR FORMULATION (continued)

In the 2-D case, the magnitude of the moment is $M_o = d\,F$

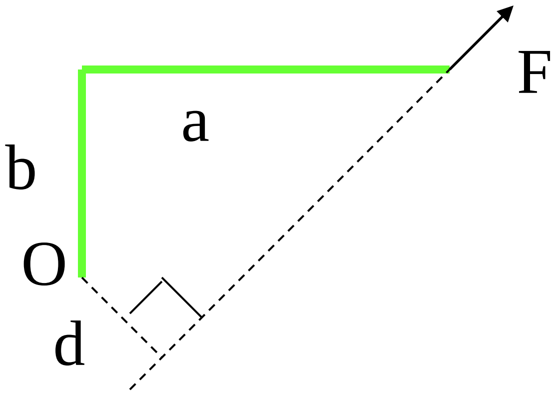


As shown, d is the perpendicular distance from point O to the line of action of the force.

In 2-D, the direction of M_O is either clockwise or counter-clockwise depending on the tendency for rotation.

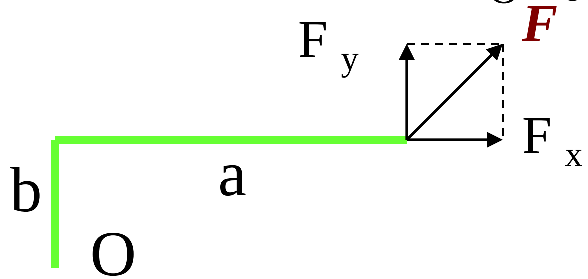
MOMENT OF A FORCE - SCALAR FORMULATION

(continued)



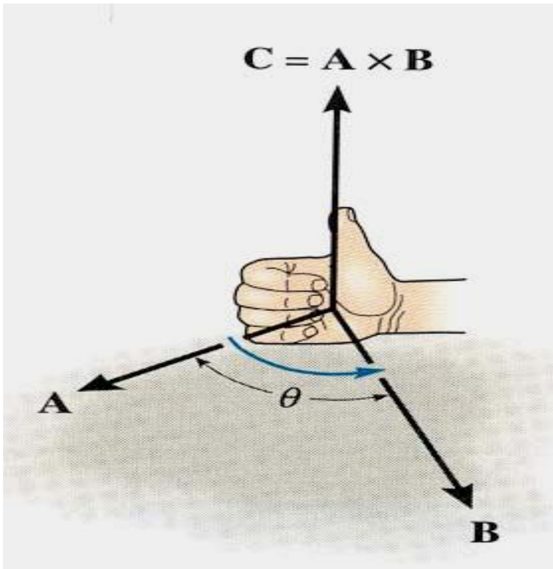
For example, $M_O = F\, d$ and the direction is counter-clockwise.

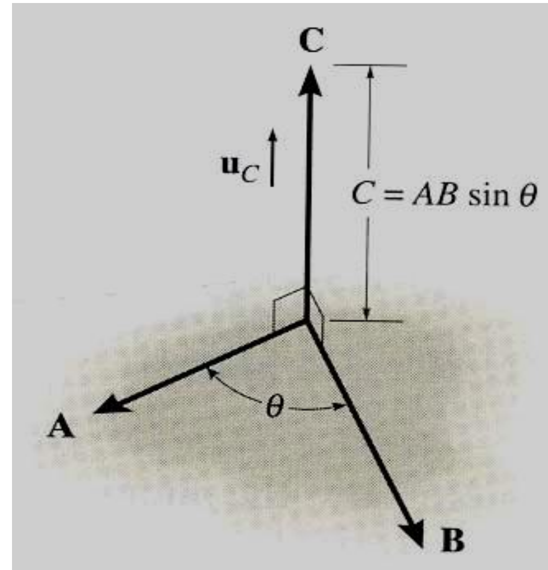
Often it is easier to determine M_O by using the components of ***F*** as shown.



Using this approach, $M_O = (F_Y\, a) – (F_X\, b)$. Note the different signs on the terms! <u>The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive</u>. We can determine the direction of rotation by imagining the body pinned at O and deciding which way the body would rotate because of the force.

CROSS PRODUCT
(Section 4.2)





In general, the cross product of two vectors ***A*** and ***B*** results in another vector ***C*** , i.e., ***C = A × B***. The magnitude and direction of the resulting vector can be written as

$$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B} = \mathrm{A\,B} \sin\theta \, \boldsymbol{U_C}$$

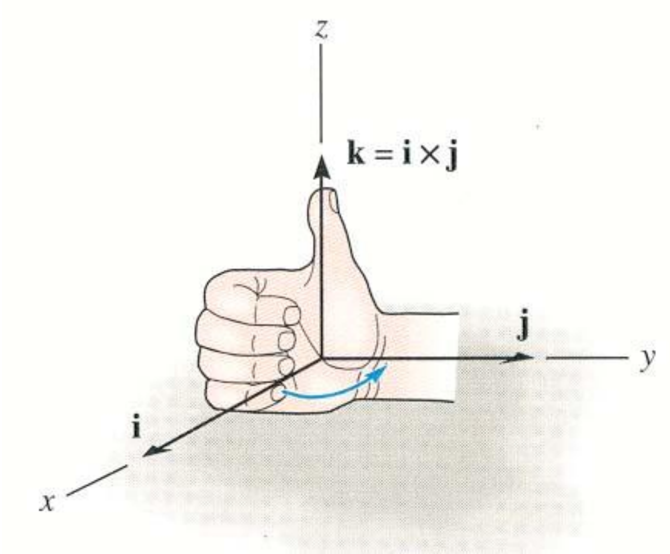
Here $\boldsymbol{U_C}$ is the unit vector perpendicular to both A and B vectors as shown (or to the plane containing the A and B vectors).

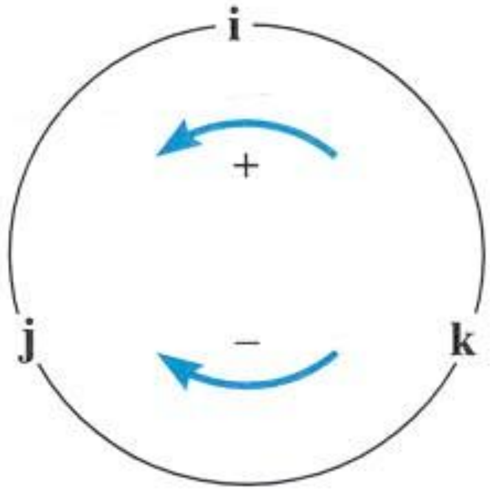
CROSS PRODUCT
(continued)

The right hand rule is a useful tool for determining the direction of the vector resulting from a cross product.

For example: $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$

Note that a vector crossed into itself is zero, e.g., $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$



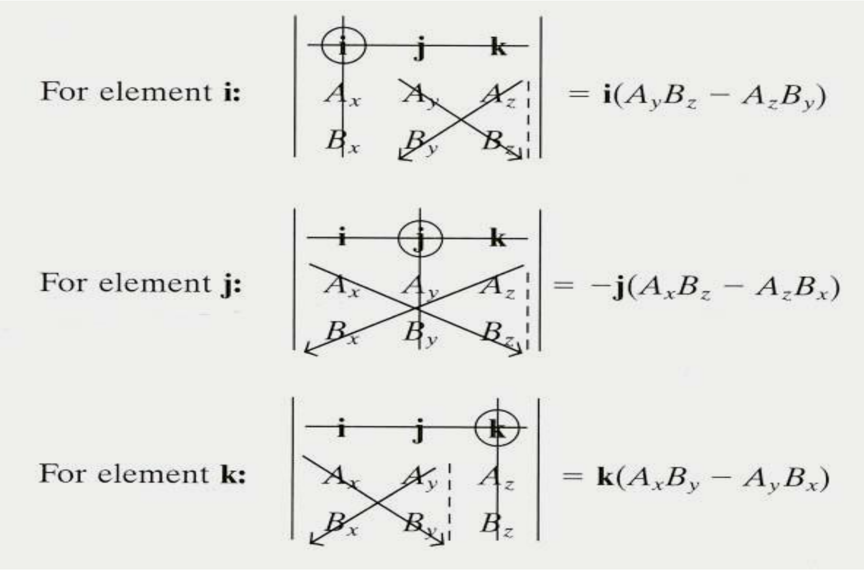


CROSS PRODUCT
(continued)

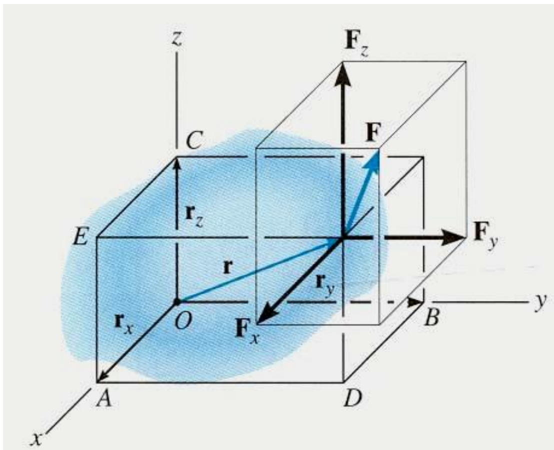
Of even more utility, the cross product can be written as

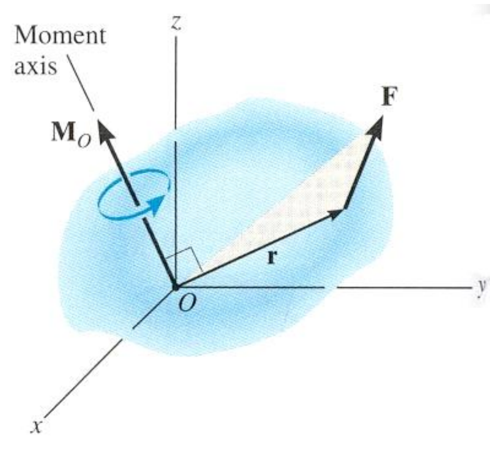
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Each component can be determined using 2 × 2 determinants.



MOMENT OF A FORCE – VECTOR FORMULATION (Section 4.3)





Moments in 3-D can be calculated using scalar (2-D) approach but it can be difficult and time consuming. Thus, it is often easier to use a mathematical approach called the vector cross product.

Using the vector cross product, $\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$.

Here ***r*** is the position vector from point O to any point on the line of action of ***F***.

MOMENT OF A FORCE – VECTOR FORMULATION (continued)

So, using the cross product, a moment can be expressed as

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

By expanding the above equation using 2 × 2 determinants (see Section 4.2), we get (sample units are N - m or lb - ft)

$$\boldsymbol{M_O} = (r_y\ F_Z - r_Z\ F_y)\ \boldsymbol{i} - (r_x\ F_z - r_z\ F_x)\ \boldsymbol{j} + (r_x\ F_y - r_y\ F_x)\ \boldsymbol{k}$$

The physical meaning of the above equation becomes evident by considering the force components separately and using a 2-D formulation.

For element **i**:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{i}(A_yB_z - A_zB_y)$$

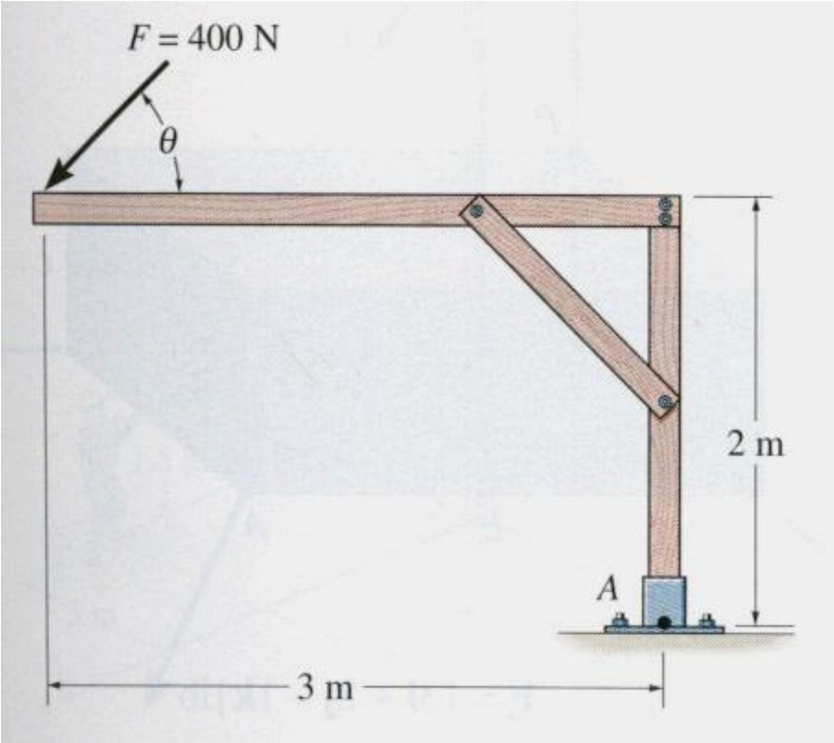
For element **j**:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\mathbf{j}(A_xB_z - A_zB_x)$$

For element **k**:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \mathbf{k}(A_xB_y - A_yB_x)$$

EXAMPLE #1



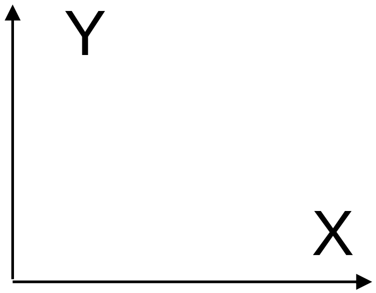
Given: A 400 N force is applied to the frame and $\theta = 20°$.

Find: The moment of the force at A.

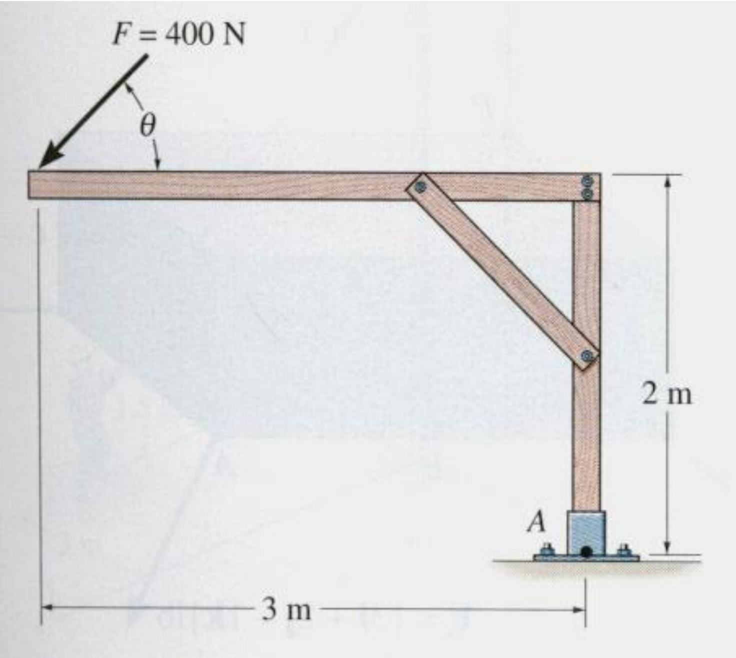
<u>Plan</u>:

1) Resolve the force along x and y axes.

2) Determine M_A using scalar analysis.



EXAMPLE #1 (continued)



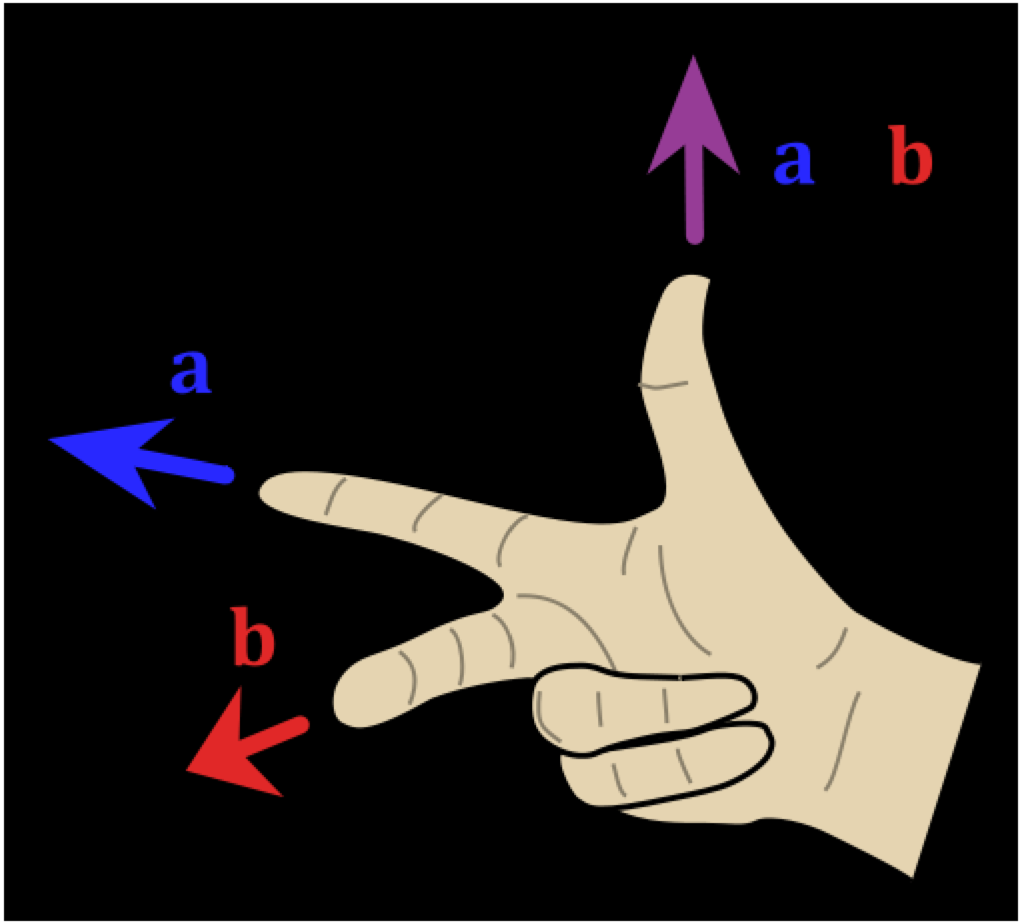
<u>Solution</u>

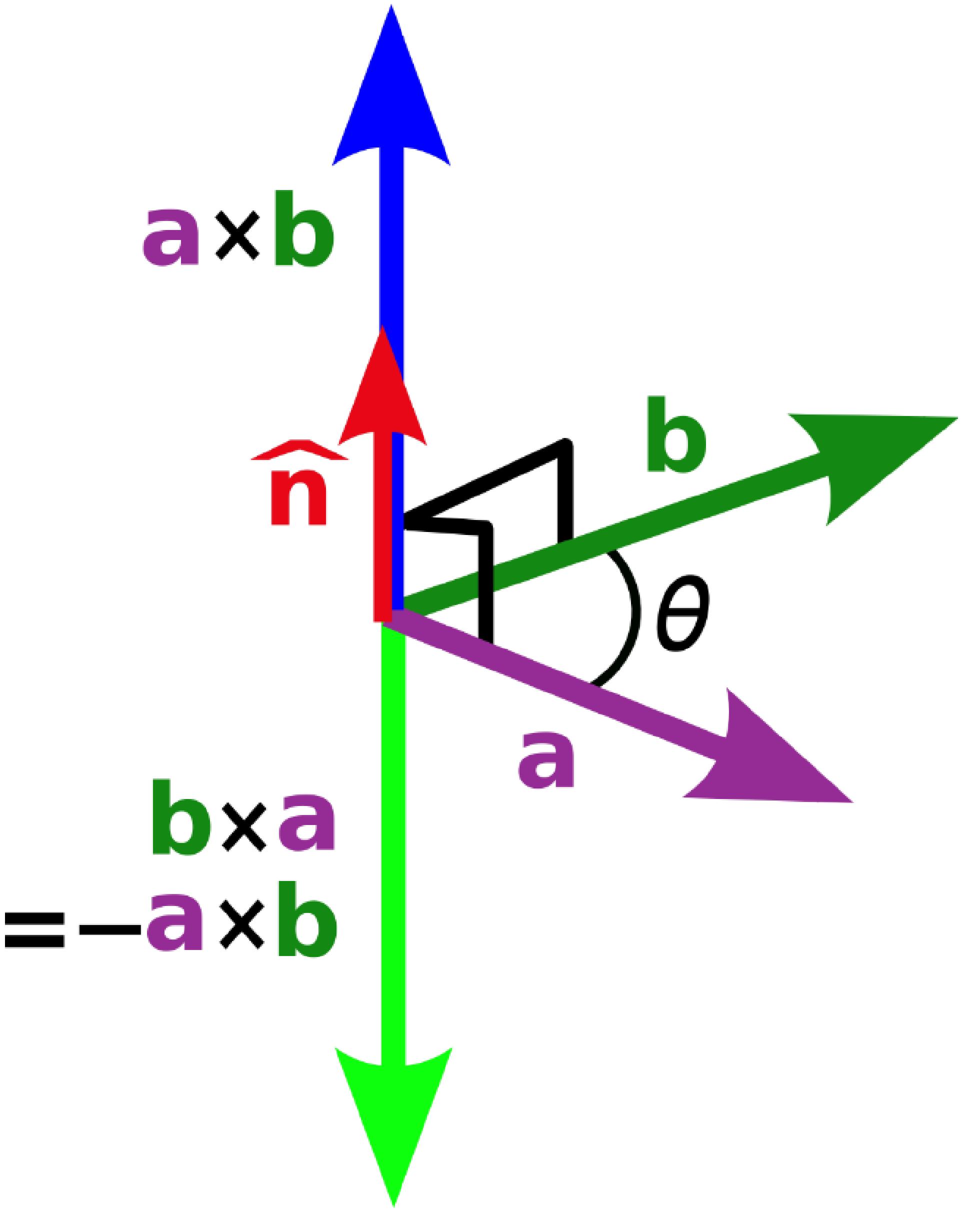
$+\uparrow F_y = -400 \text{ Sin } 20° \text{ N}$

$+\rightarrow F_x = -400 \text{ Cos } 20° \text{ N}$

$+ M_A = \{(400 \cos 20°)(2) + (400 \sin 20°)(3)\} \text{ N}\cdot\text{m}$

$= 1160 \text{ N}\cdot\text{m}$


a b
a
b


a×b
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a
b×a
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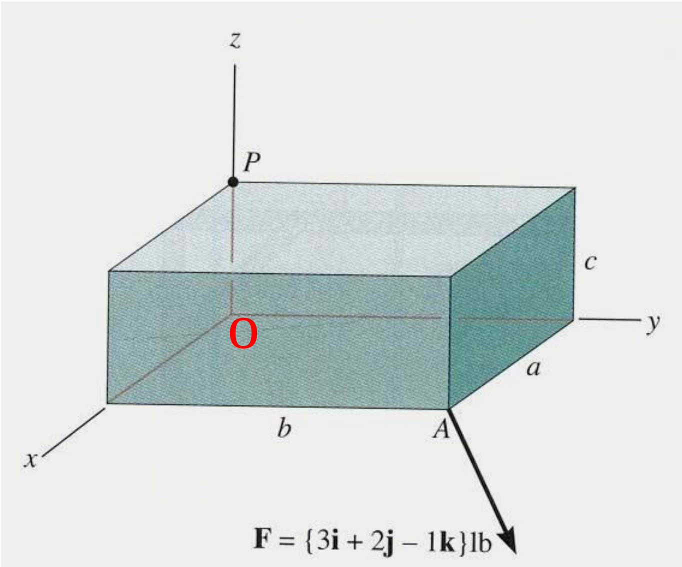


EXAMPLE # 2

Given: a = 3 in, b = 6 in and c = 2 in.

Find: Moment of ***F*** about point O.

Plan:

1) Find $\boldsymbol{r}_{OA}$.

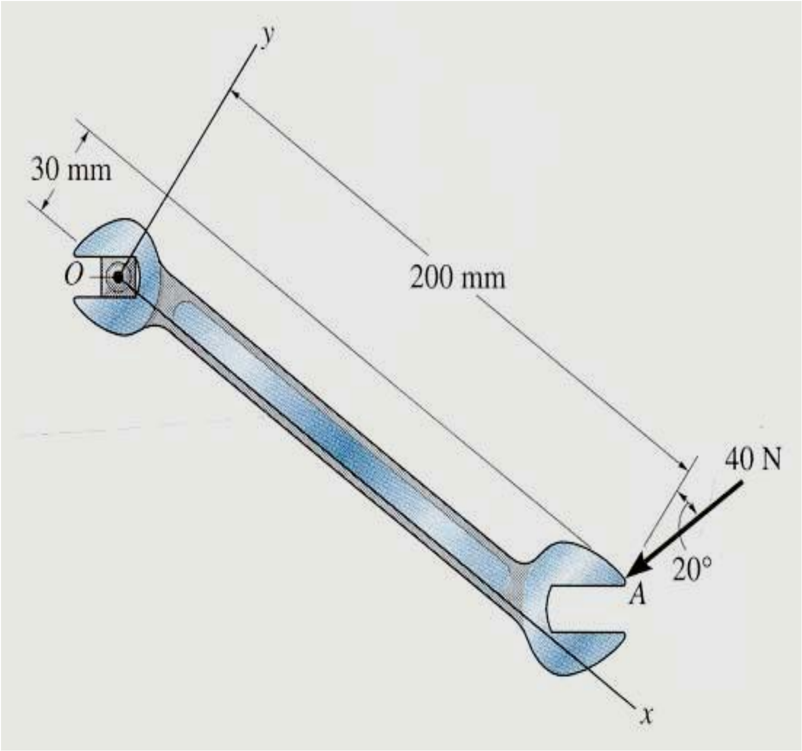
2) Determine $\boldsymbol{M}_O = \boldsymbol{r}_{OA} \times \boldsymbol{F}$.

Solution $\boldsymbol{r}_{OA} = \{3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 0\,\boldsymbol{k}\}$ in

$$M_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & 0 \\ 3 & 2 & -1 \end{vmatrix} = [\{6(-1) - 0(2)\}\,\boldsymbol{i} - \{3(-1) - 0(3)\}\,\boldsymbol{j} + \{3(2) - 6(3)\}\,\boldsymbol{k}]\ \text{lb}\cdot\text{in}$$

$$= \{-6\,\boldsymbol{i} + 3\,\boldsymbol{j} - 12\,\boldsymbol{k}\}\ \text{lb}\cdot\text{in}$$

GROUP PROBLEM SOLVING



Given: A 40 N force is applied to the wrench.

Find: The moment of the force at O.

Plan: 1) Resolve the force along x and y axes.

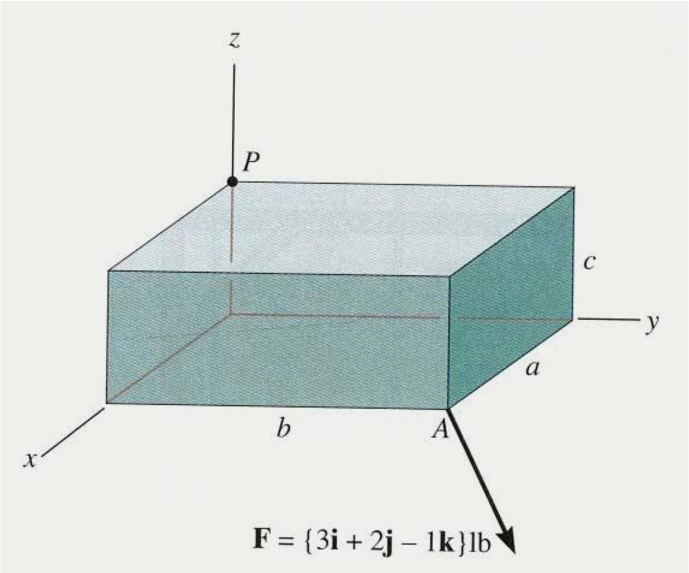
2) Determine M_O using scalar analysis.

Solution: $+\uparrow F_y = -\,40 \cos 20° \text{ N}$

$+\rightarrow F_x = -\,40 \sin 20° \text{ N}$

$$\curvearrowleft + \quad M_O = \{-(40 \cos 20°)(200) + (40 \sin 20°)(30)\} \text{N·mm}$$
$$= -7107 \text{ N·mm} = -7.11 \text{ N·m}$$

GROUP PROBLEM SOLVING



Given: a = 3 in , b = 6 in and c = 2 in

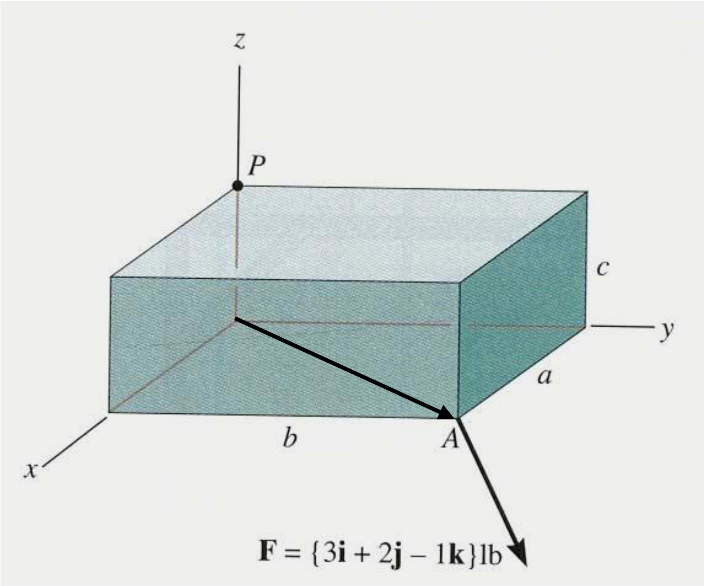
Find: Moment of F about point P

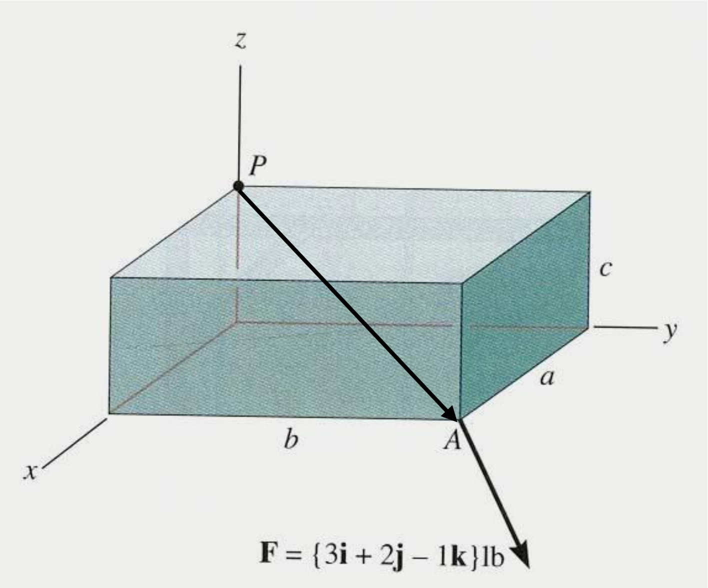
Plan: 1) Find $\boldsymbol{r}_{PA}$.

2) Determine $\boldsymbol{M}_P = \boldsymbol{r}_{PA} \times \boldsymbol{F}$

Solution: $\boldsymbol{r}_{PA} = \{ 3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 2\,\boldsymbol{k} \}$ in

$$\boldsymbol{M}_P = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & -2 \\ 3 & 2 & -1 \end{vmatrix} = \{ -2\,\boldsymbol{i} - 3\,\boldsymbol{j} - 12\,\boldsymbol{k} \} \text{ lb} \cdot \text{in}$$

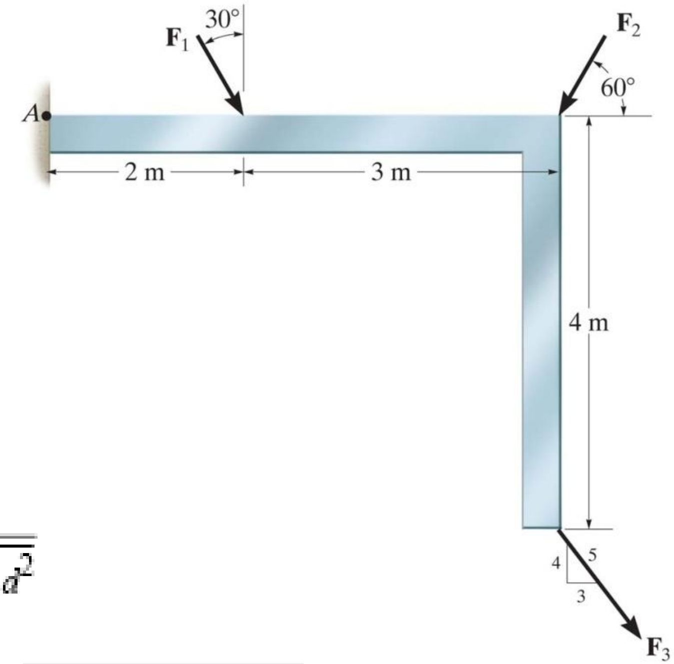




-6 ***i*** + 3 ***j*** – 12 ***k***

-2 ***i*** - 3 ***j*** - 12 ***k***

Determine the resultant moment of the forces about point A. Solve the problem first by considering each force as a whole, and then by using the principle of moments.



$F_1 = 250\ \text{N}$ $\quad a = 2\ \text{m}$

$F_2 = 300\ \text{N}$ $\quad b = 3\ \text{m}$

$F_3 = 500\ \text{N}$ $\quad c = 4\ \text{m}$

$\theta_1 = 60\ \text{deg}$ $\quad d = 3$

$\theta_2 = 30\ \text{deg}$ $\quad e = 4$

Geometry $\quad \alpha = \operatorname{atan}\left(\frac{d}{e}\right) \qquad L = \left(a + b - \frac{d}{e}c\right)\frac{e}{\sqrt{e^2 + d^2}}$

$$M_A = -F_1\left[(a)\cos(\theta_2)\right] - F_2(a+b)\sin(\theta_1) - F_3 L$$

$$M_A = -2.532\ \text{kN}\cdot\text{m}$$

Solution Using Principle of Moments:

$$M_A = -F_1\cos(\theta_2)a - F_2\sin(\theta_1)(a+b) + F_3\frac{d}{\sqrt{d^2+e^2}}c - F_3\frac{e}{\sqrt{d^2+e^2}}(a+b)$$

$$M_A = -2.532 \times 10^3\ \text{N}\cdot\text{m}$$

Applications of the Dot Product and Cross Product

$\vec{\tau} = \vec{r} \times \vec{F}$

Example 1: A 100 N force is applied to a 30 cm wrench at an angle of 75°.

a) Calculate the magnitude of the torque.

b) What is the direction of the torque vector?

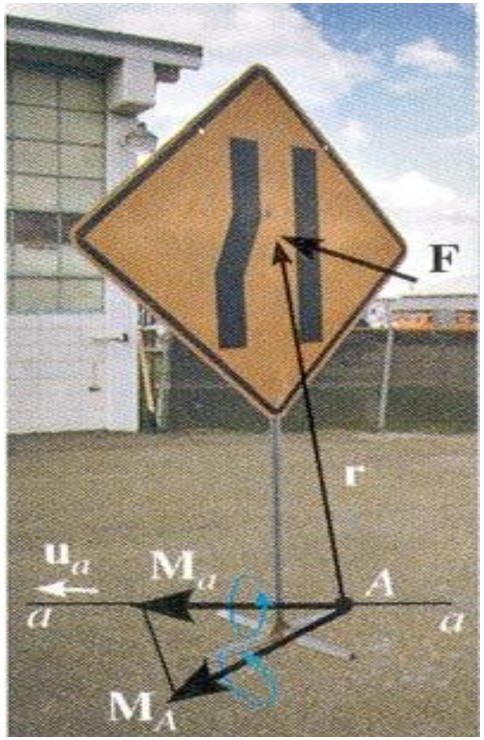
MOMENT ABOUT AN AXIS

Objectives:

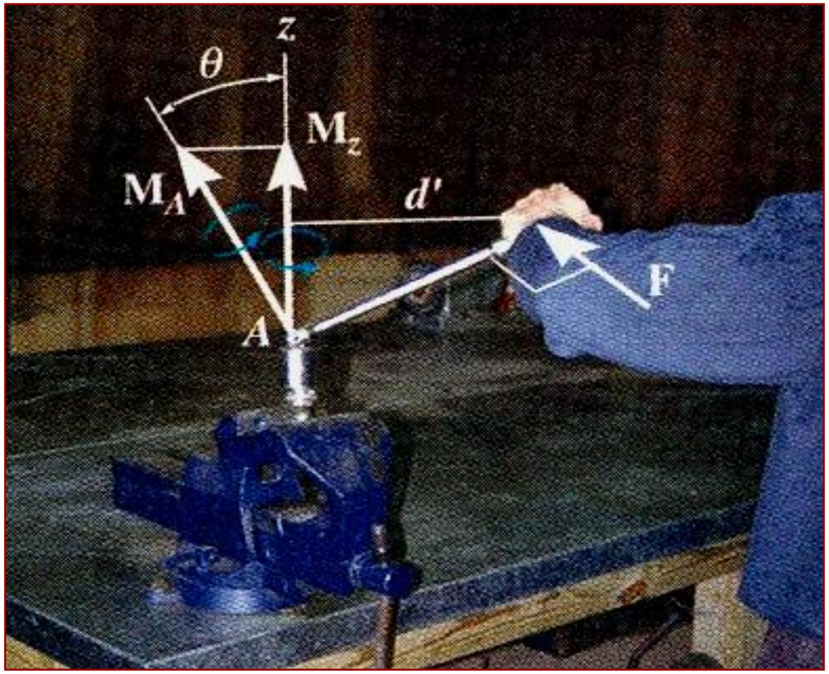
Students will be able to determine the moment of a force about an axis using

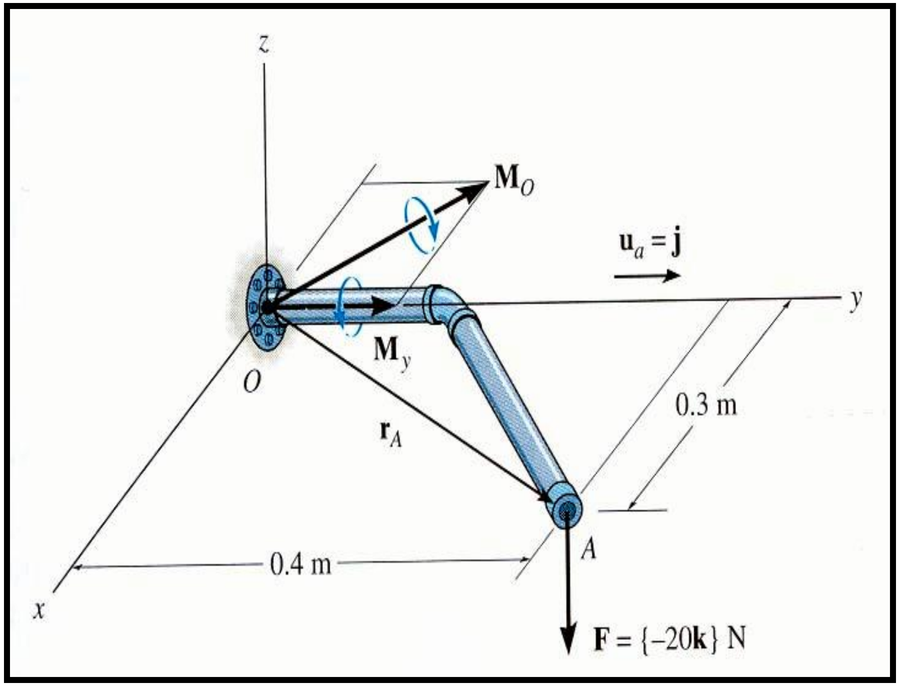
a) scalar analysis, and

b) vector analysis.



APPLICATIONS



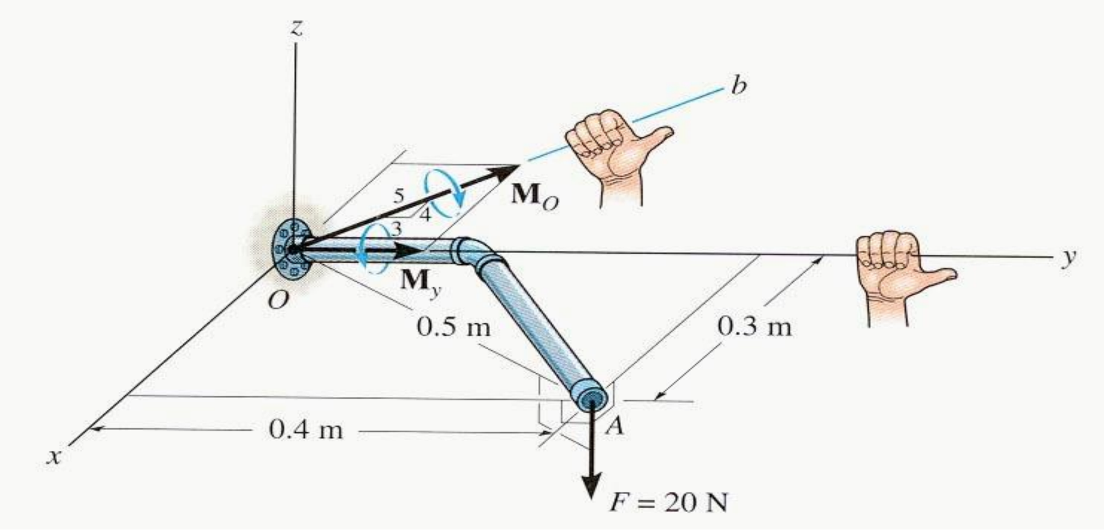


With the force ***F***, a person is creating the moment $\boldsymbol{M_A}$. What portion of $\boldsymbol{M_A}$ is used in turning the socket?

The force ***F*** is creating the moment $\boldsymbol{M_O}$. How much of $\boldsymbol{M_O}$ acts to unscrew the pipe?

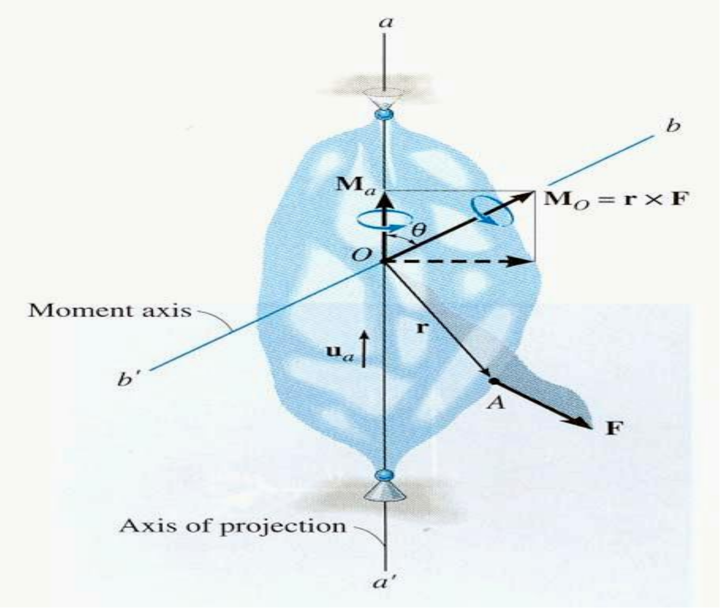
SCALAR ANALYSIS

Recall that the moment of a force about any point A is $M_A = F\, d_A$ where d_A is the perpendicular (or shortest) distance from the point to the force's line of action. This concept can be extended to find the moment of a force about an axis.



In the figure above, the moment about the y-axis would be $M_y = 20\,(0.3) = 6$ N·m. However, this calculation is not always trivial and vector analysis may be preferable.

VECTOR ANALYSIS



Our goal is to find the moment of ***F*** (the tendency to rotate the body) about the axis a'-a.

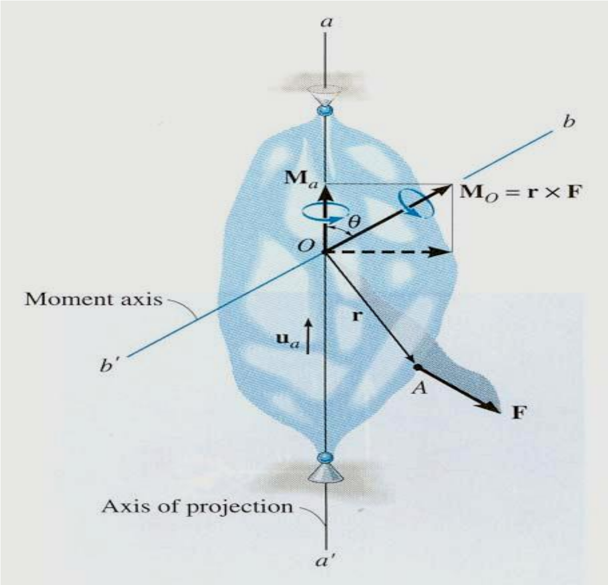
First compute the moment of ***F*** about any arbitrary point O that lies on the a' a axis using the cross product.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

Now, find the component of $\boldsymbol{M_O}$ along the axis a'-a using the dot product.

$$M_a = \boldsymbol{u_a} \cdot \boldsymbol{M_O}$$

VECTOR ANALYSIS (continued)



M_a can also be obtained as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

The above equation is also called the triple scalar product.

In the this equation,

$\boldsymbol{u_a}$ represents the unit vector along the axis a'-a axis,

$\boldsymbol{r}$ is the position vector from any point on the a'-a axis to any point A on the line of action of the force, and

$\boldsymbol{F}$ is the force vector.

$$\vec{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\boldsymbol{M_O} = (r_y\, F_Z - r_Z\, F_y)\, \boldsymbol{i} - (r_x\, F_z - r_z\, F_x)\, \boldsymbol{j} + (r_x\, F_y - r_y\, F_x)\, \boldsymbol{k}$$

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\boldsymbol{A} \bullet \boldsymbol{B} = (A_x B_x + A_y B_y + A_z B_z)$$

$$M_a = \boldsymbol{u_a} \bullet \boldsymbol{M_O}$$

$$\vec{M}_a = (\boldsymbol{u_a} \bullet \boldsymbol{M_O}).\ \boldsymbol{u_a}$$

READING QUIZ

1. When determining the moment of a force about a specified axis, the axis must be along _______________.

   A) the x axis   B) the y axis   C) the z axis

   D) any line in 3-D space   E) any line in the x-y plane

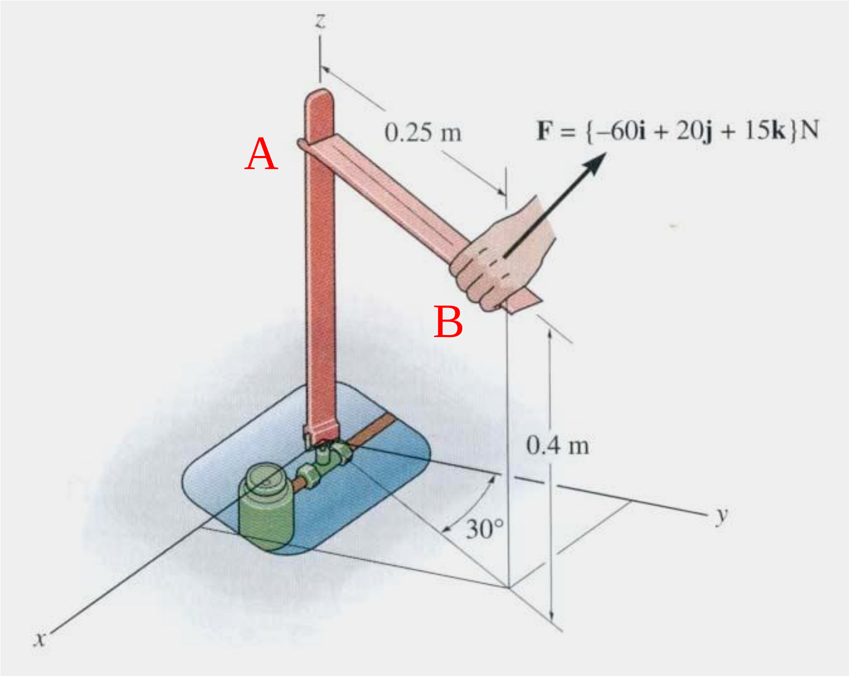
2. The triple scalar product $\boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$ results in

   A) a scalar quantity (+ or -).   B) a vector quantity.

   C) zero.   D) a unit vector.

   E) an imaginary number.

EXAMPLE



Given: A force is applied to the tool to open a gas valve.

Find: The magnitude of the moment of this force about the z axis of the value.

<u>Plan</u>:

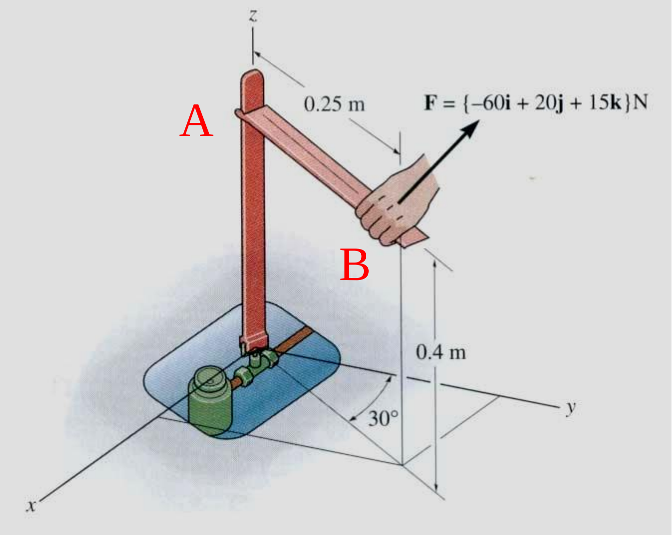
1) We need to use $\boldsymbol{M_z} = \boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$.

2) Note that $\boldsymbol{u} = 1\,\boldsymbol{k}$.

3) The vector ***r*** is the position vector from A to B.

4) Force ***F*** is already given in Cartesian vector form.

EXAMPLE (continued)



$$\boldsymbol{u} = 1\,\boldsymbol{k}$$

$$\boldsymbol{r_{AB}} = \{0.25 \sin 30° \,\boldsymbol{i} + 0.25 \cos 30° \,\boldsymbol{j}\} \text{ m}$$

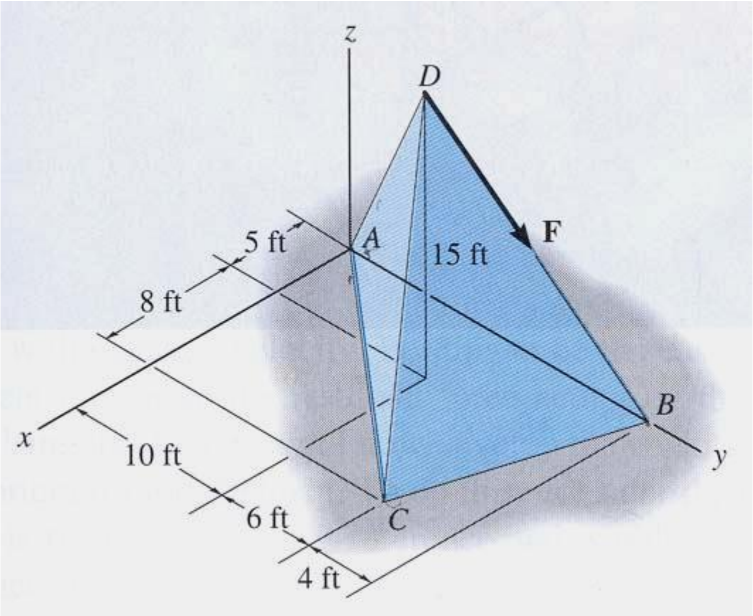
$$= \{0.125\,\boldsymbol{i} + 0.2165\,\boldsymbol{j}\} \text{ m}$$

$$\boldsymbol{F} = \{-60\,\boldsymbol{i} + 20\,\boldsymbol{j} + 15\,\boldsymbol{k}\} \text{ N}$$

$$M_z = \boldsymbol{u} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F})$$

$$M_z = 1\{0.125(20) - 0.2165(-60)\} \text{ N·m}$$
$$= 15.5 \text{ N·m}$$

GROUP PROBLEM SOLVING



Given: A force of 80 lb acts along the edge DB.

Find: The magnitude of the moment of this force about the axis AC.

Plan:

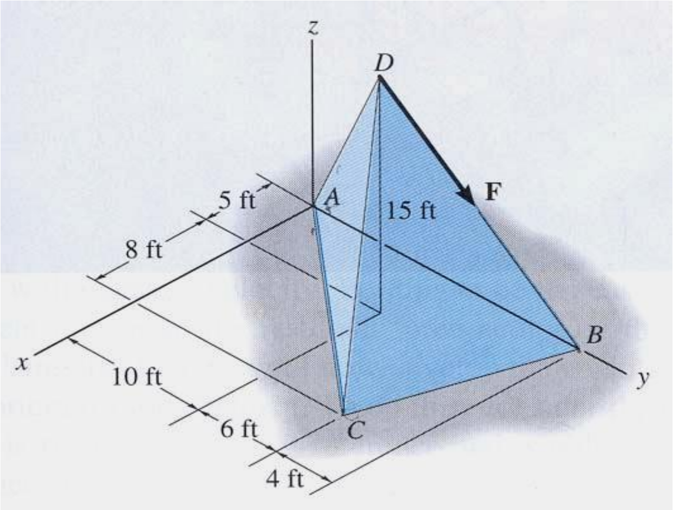
1) We need to use $M_{AC} = \boldsymbol{u_{AC}} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F_{DB}})$

2) Find $\boldsymbol{u_{AC}} = \boldsymbol{r_{AC}} / r_{AC}$

3) Find $\boldsymbol{F_{DB}} = 80 \text{ lb } \boldsymbol{u_{DB}} = 80 \text{ lb } (\boldsymbol{r_{DB}} / r_{DB})$

4) Complete the triple scalar product.

SOLUTION



$\boldsymbol{r}_{AB} = \{\ 20\,\boldsymbol{j}\ \}$ ft

$\boldsymbol{r}_{AC} = \{\ 13\,\boldsymbol{i}\ + 16\,\boldsymbol{j}\ \}$ ft

$\boldsymbol{r}_{DB} = \{\ -5\,\boldsymbol{i}\ + 10\,\boldsymbol{j} - 15\,\boldsymbol{k}\ \}$ ft

$\boldsymbol{u}_{AC} = (\ 13\,\boldsymbol{i}\ + 16\,\boldsymbol{j}\ )\ \text{ft} / (13^{2} + 16^{2})^{½}\ \text{ft}$

$= \ 0.6306\,\boldsymbol{i}\ + \ 0.7761\,\boldsymbol{j}$

$\boldsymbol{F}_{DB} = 80\ \{\boldsymbol{r}_{DB} / (5^{2} + 10^{2} + 15^{2})^{½}\ \}\ \text{lb}$

$= \{-21.38\,\boldsymbol{i}\ + \ 42.76\ \boldsymbol{j}\ - \ 64.14\,\boldsymbol{k}\ \}\ \text{lb}$

Solution (continued)

Now find the triple product, $M_{AC} = \boldsymbol{u_{AC}} \bullet (\boldsymbol{r_{AB}} \times \boldsymbol{F_{DB}})$

$$M_{AC} = \begin{vmatrix} 0.6306 & 0.7706 & 0 \\ 0 & 20 & 0 \\ -21.38 & 42.76 & -64.14 \end{vmatrix} \begin{matrix} \text{ft} \\ \text{lb} \end{matrix}$$

$$M_{AC} = 0.6306 \{20 (-64.14) - 0 - 0.7706 (0 - 0)\} \text{ lb}\cdot\text{ft}$$

$$= -809 \text{ lb}\cdot\text{ft}$$

The negative sign indicates that the sense of $\boldsymbol{M_{AC}}$ is opposite to that of $\boldsymbol{u_{AC}}$

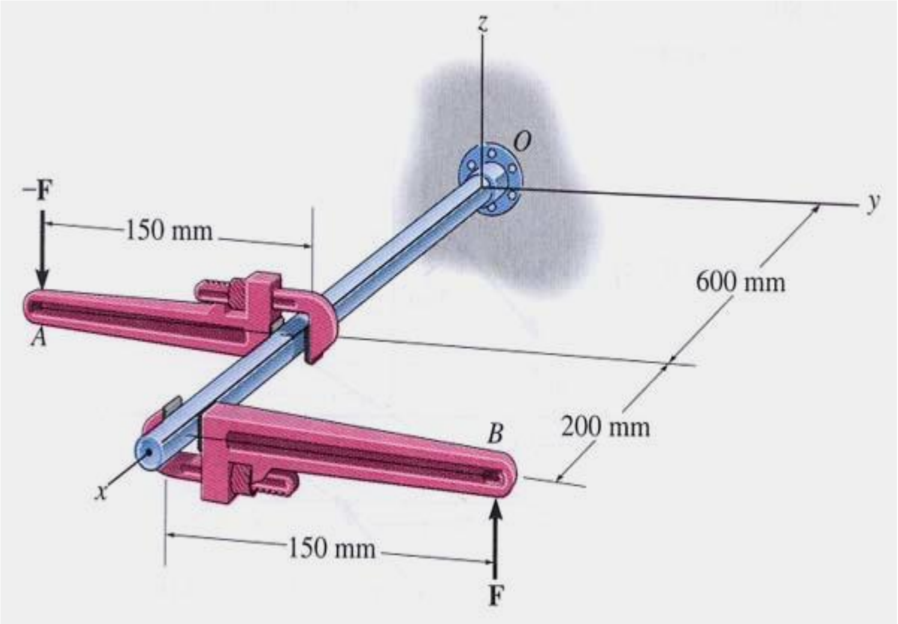
MOMENT OF A COUPLE

Objectives:

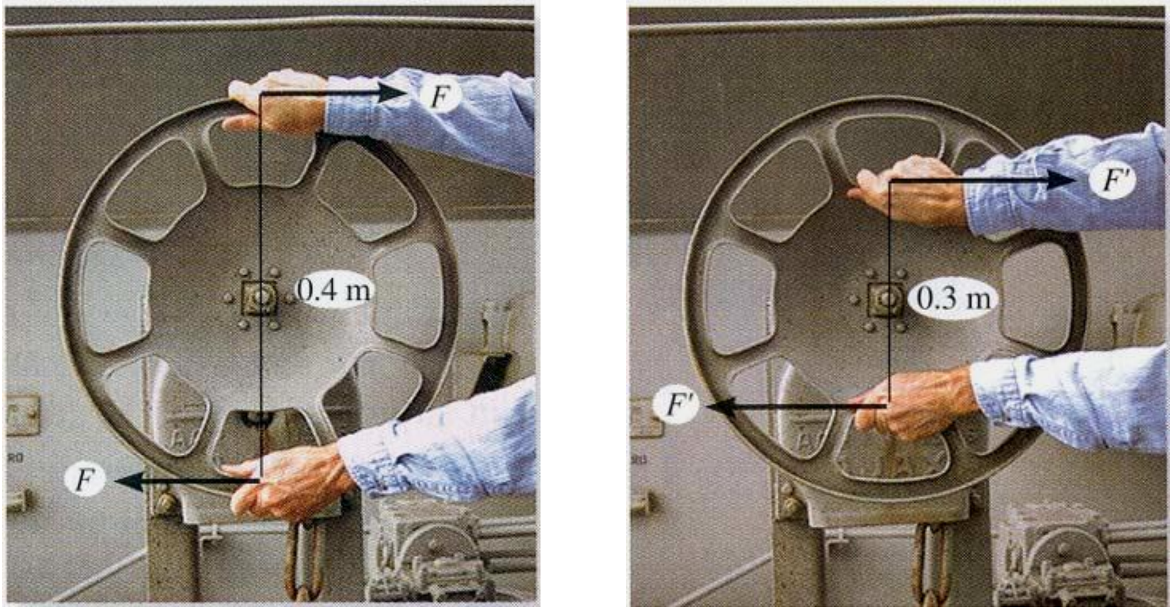
Students will be able to

a) define a couple, and,

b) determine the moment of a couple.

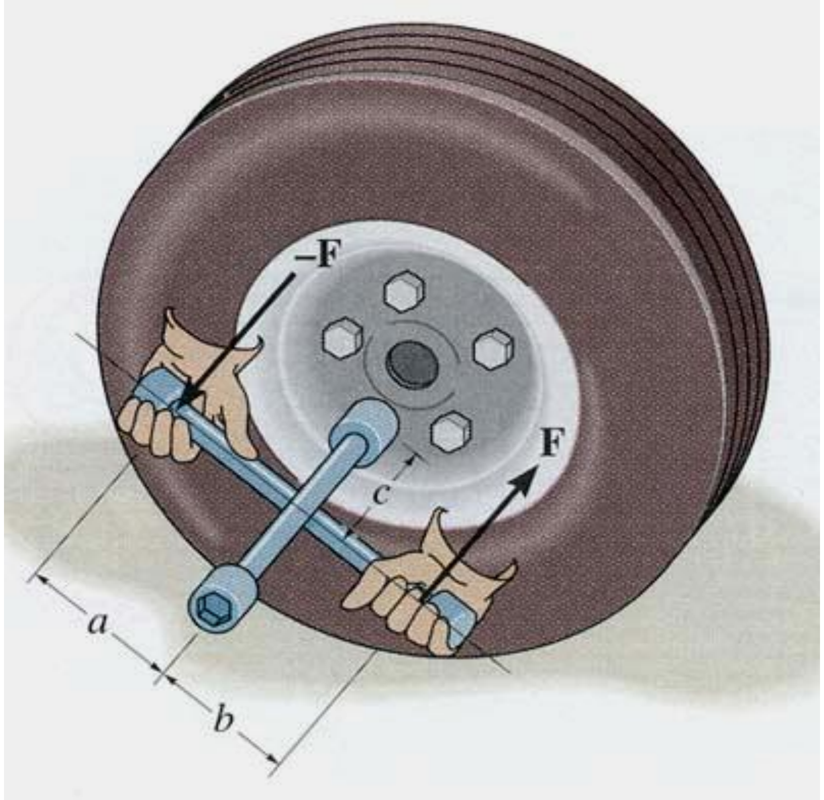


APPLICATIONS



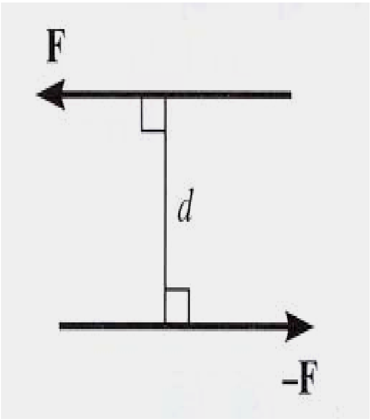
A torque or moment of 12 N · m is required to rotate the wheel. Which one of the two grips of the wheel above will require less force to rotate the wheel?

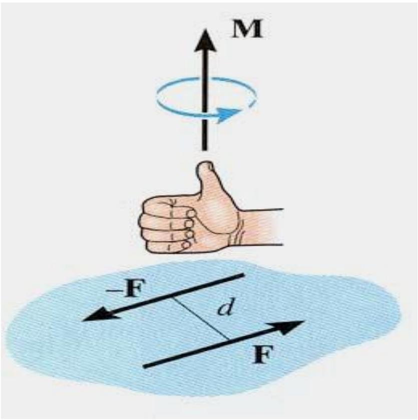
APPLICATIONS
(continued)



The crossbar lug wrench is being used to loosen a lug net. What is the effect of changing dimensions a, b, or c on the force that must be applied?

MOMENT OF A COUPLE





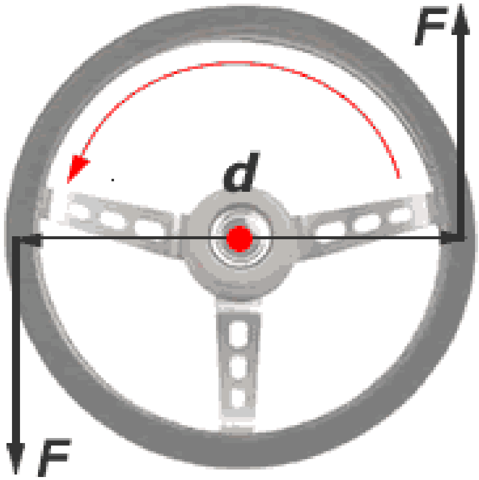
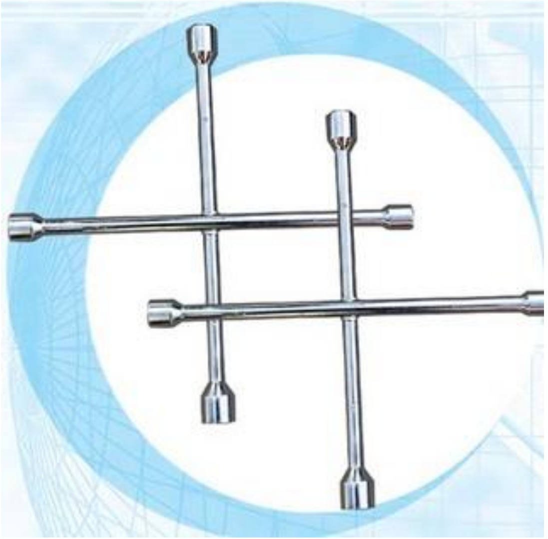
A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

The moment of a couple is defined as

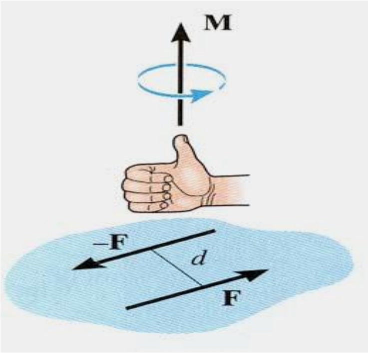
M_O = F d (using a scalar analysis) or as

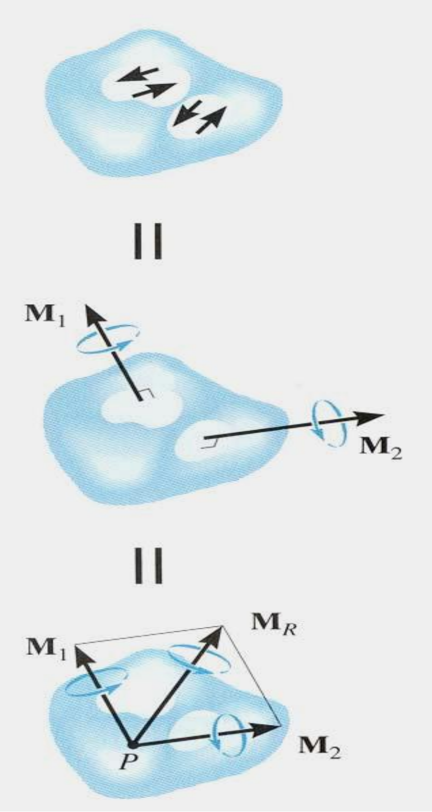
$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis).

Here ***r*** is any position vector from the line of action of ***–F*** to the line of action of ***F***.


F
d
F



MOMENT OF A COUPLE
(continued)





The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals F d

Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a free vector. It can be moved anywhere on the body and have the same external effect on the body.

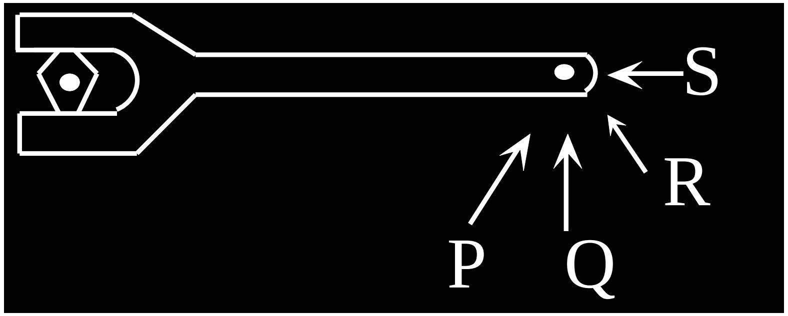
Moments due to couples can be added using the same rules as adding any vectors.

QUIZ

1. **If a force of magnitude F can be applied in 4 different 2-D configurations (P,Q,R, & S), select the cases resulting in the maximum and minimum torque values on the nut. (Max, Min).**

A) (Q, P) **B) (R, S)**

C) (P, R) **D) (Q, S)**



2. **If $M = r \times F$, then what will be the value of $M \bullet r$?**

A) 0 **B) 1**

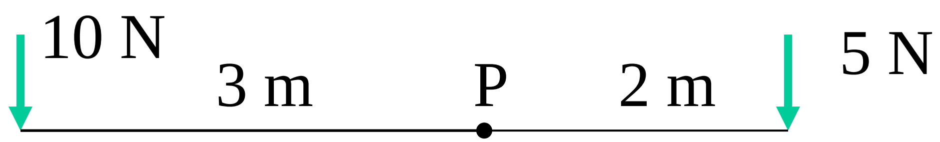
C) r^2F **D) None of the above.**

QUIZ

3. Using the CCW direction as positive, the net moment of the two forces about point P is

A) 10 N ·m B) 20 N ·m C) - 20 N ·m

D) 40 N ·m E) - 40 N ·m



4. If *r* = { 5 *j* } m and *F* = { 10 *k* } N, the moment *r* x *F* equals { ________ } N·m.

A) 50 *i* B) 50 *j* C) –50 *i*

D) – 50 *j* E) 0

QUIZ

5. When determining the moment of a force about a specified axis, the axis must be along ______________.

A) the x axis B) the y axis C) the z axis

D) any line in 3-D space E) any line in the x-y plane

6. The triple scalar product $\boldsymbol{u} \bullet (\boldsymbol{r} \times \boldsymbol{F})$ results in

A) a scalar quantity (+ or -)

B) a vector quantity.

C) zero.

D) a unit vector.

E) an imaginary number.

QUIZ

7. The vector operation ($P \times Q$) • R equals

   A) $P \times (Q \cdot R)$.

   B) $R \cdot (P \times Q)$.

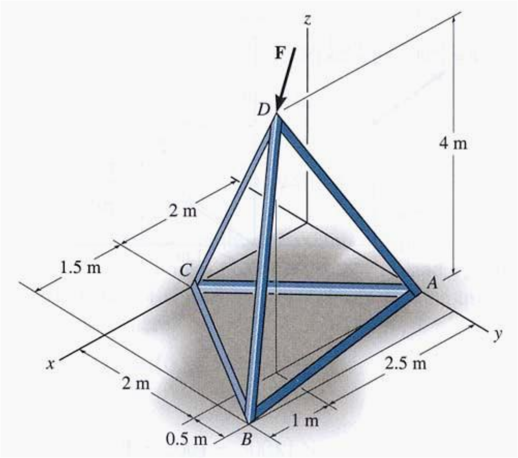
   C) $(P \cdot R) \times (Q \cdot R)$.

   D) $(P \times R) \cdot (Q \times R)$.

8. The force F is acting along DC. Using the triple product to determine the moment of F about the bar BA, you could use any of the following position vectors except

A) r_{BC} B) r_{AD} C) r_{AC}

D) r_{DB} E) r_{BD}

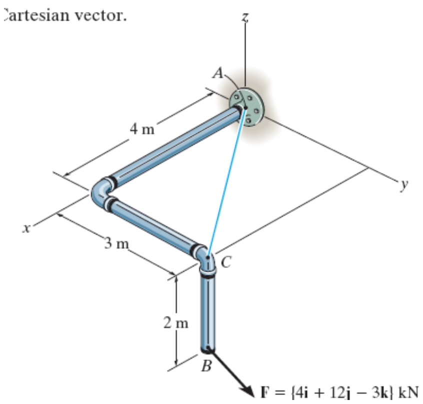


QUIZ

9. For finding the moment of the force *F* about the x-axis, the position vector in the triple scalar product should be ___ .

A) r_{AC}　　B) r_{BA}

C) r_{AB}　　D) r_{BC}



10. If $r = \{1\,i + 2\,j\}$ m and $F = \{10\,i + 20\,j + 30\,k\}$ N, then the moment of *F* about the y-axis is ____ N·m.

A) 10　　B) -30

C) -40　　D) None of the above.

QUIZ

11. **In statics, a couple is defined as ___________ separated by a perpendicular distance.**

A) two forces in the same direction

B) two forces of equal magnitude

C) two forces of equal magnitude acting in the same direction

D) two forces of equal magnitude acting in opposite directions

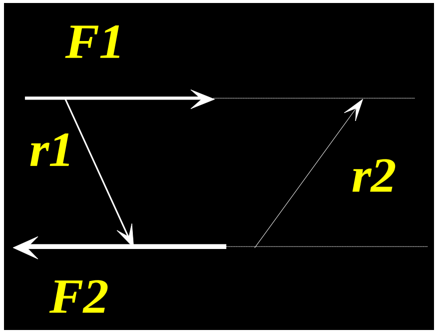
12. **The moment of a couple is called a _____ vector.**

A) Free **B) Spin**

C) Romantic **D) Sliding**

QUIZ

13. ***F1*** **and** ***F2*** **form a couple. The moment of the couple is given by _____ .**



A) $r1 \times F1$ **B)** $r2 \times F1$

C) $F2 \times r1$ **D)** $r2 \times F2$

14. If three couples act on a body, the overall result is that

A) The net force is not equal to 0.

B) The net force and net moment are equal to 0.

C) The net moment equals 0 but the net force is not necessarily equal to 0.

D) The net force equals 0 but the net moment is not necessarily equal to 0 .

QUIZ

15. A <u>general system</u> of forces and couple moments acting on a rigid body can be reduced to a ___ .

   A) single force

   B) single moment

   C) single force and two moments

   D) single force and a single moment

16. The original force and couple system and an equivalent force-couple system have the same _____ effect on a body.

   A) internal

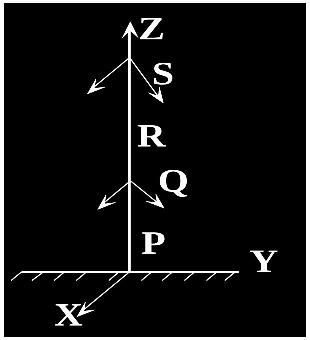
   B) external

   C) internal and external

   D) microscopic

QUIZ

17. The forces on the pole can be reduced to a single force and a single moment at point _____ .



A) P B) Q C) R

D) S E) Any of these points.

18. Consider <u>two couples</u> acting on a body. The simplest possible equivalent system at any arbitrary point on the body will have

A) One force and one couple moment.

B) One force.

C) One couple moment.

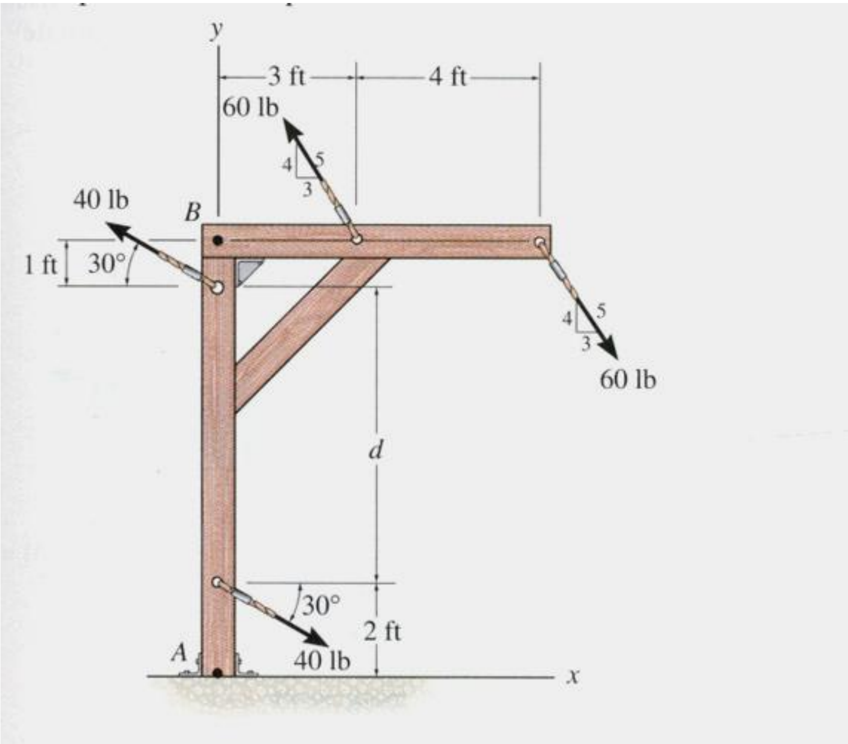
D) Two couple moments.

QUIZ

19. Consider three couples acting on a body. Equivalent systems will be _______ at different points on the body.

   A) Different when located
   B) The same even when located
   C) Zero when located
   D) None of the above.

EXAMPLE - SCALAR APPROACH



Given: Two couples act on the beam and d equals 8 ft.

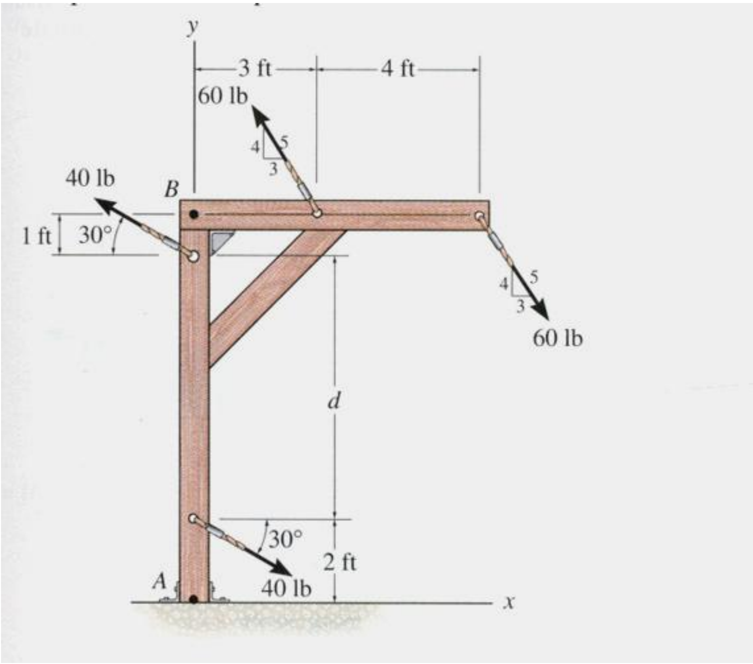
Find: The resultant couple

<u>Plan</u>:

1) Resolve the forces in x and y directions so they can be treated as couples.

2) Determine the net moment due to the two couples.

EXAMPLE - SCALAR APPROACH



The x and y components of the top 60 lb force are:

(4/5)(60 lb) = 48 lb vertically up

(3/5)(60 lb) = 36 lb to the left

Similarly for the top 40 lb force:

(40 lb) (sin 30°) up

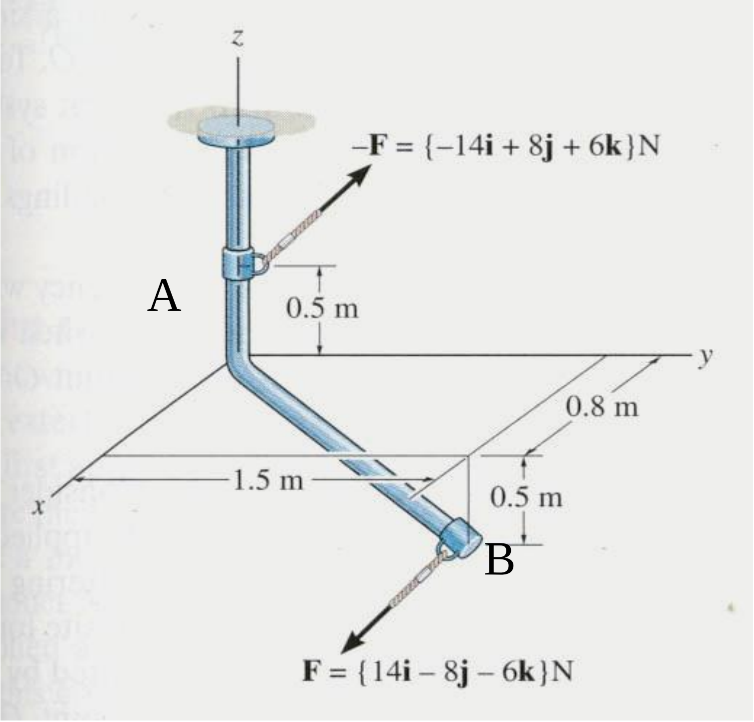
(40 lb) (cos 30°) to the left

The net moment equals to

$$+\circlearrowright \Sigma M = -(48\text{ lb})(4\text{ ft}) + (40\text{ lb})(\cos 30^{o})(8\text{ft})$$

$$= -192.0 + 277.1 = 85.1\text{ ft}\cdot\text{lb}$$

EXAMPLE – VECTOR APPROACH



Given: A force couple acting on the rod.

Find: The couple moment acting on the rod in Cartesian vector notation.

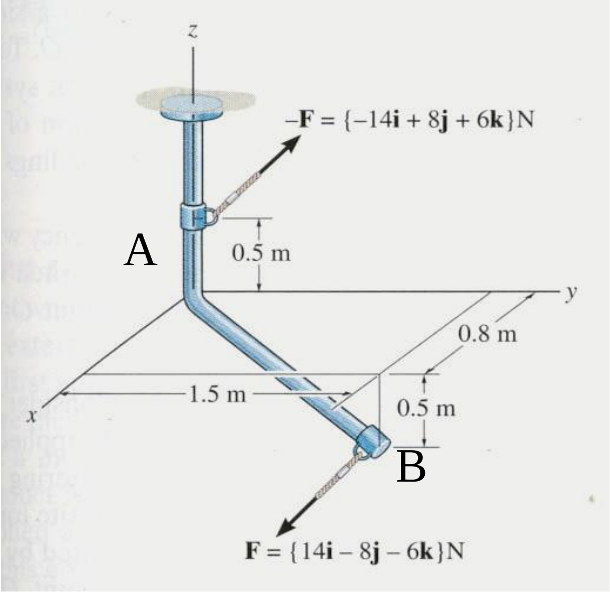
Plan:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{14\,\boldsymbol{i} - 8\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

EXAMPLE – VECTOR APPROACH



$\boldsymbol{r}_{AB} = \{0.8\,\boldsymbol{i} + 1.5\,\boldsymbol{j} - 1\,\boldsymbol{k}\}$ m

$\boldsymbol{F} = \{14\,\boldsymbol{i} - 8\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ N

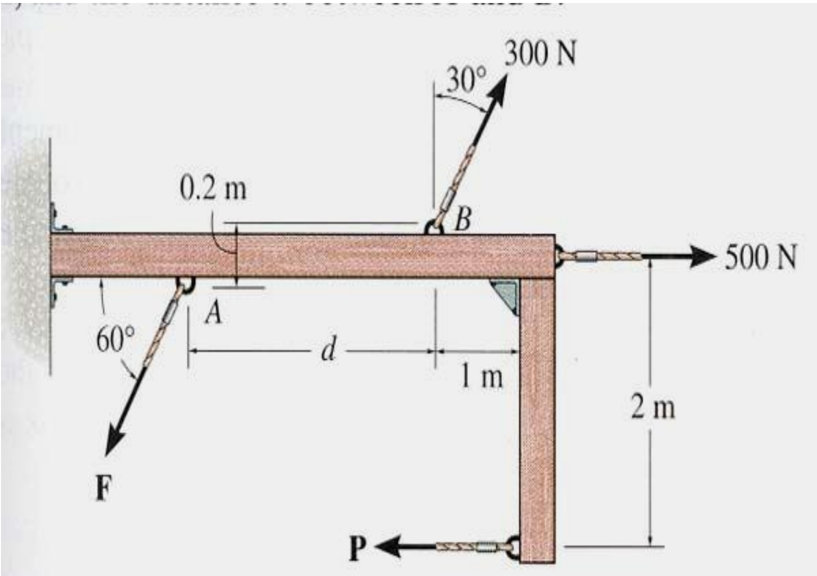
$$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.8 & 1.5 & -1 \\ 14 & -8 & -6 \end{vmatrix} \text{ N·m}$$

$$= \{\boldsymbol{i}\,(-9 - (8)) - \boldsymbol{j}\,(-4.8 - (-14)) + \boldsymbol{k}\,(-6.4 - 21)\} \text{ N·m}$$

$$= \{-17\,\boldsymbol{i} - 9.2\,\boldsymbol{j} - 27.4\,\boldsymbol{k}\} \text{ N·m}$$

GROUP PROBLEM SOLVING – SCALAR APPROACH



Given: Two couples act on the beam. The resultant couple is zero.

Find: The distance d.

PLAN:

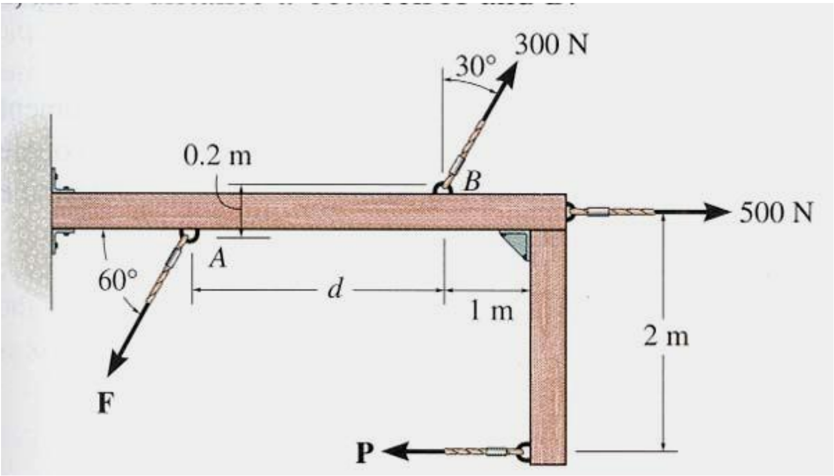
1) Use definition of a couple to find P and F.

2) Resolve the 300 N force in x and y directions.

3) Determine the net moment.

4) Equate the net moment to zero to find d.

GROUP PROBLEM SOLVING – SCALAR APPROACH



From the definition of a couple:

P = 500 N and

F = 300 N.

Resolve the 300 N force into vertical and horizontal components. The vertical component is (300 cos 30º) N and the horizontal component is (300 sin 30º) N.

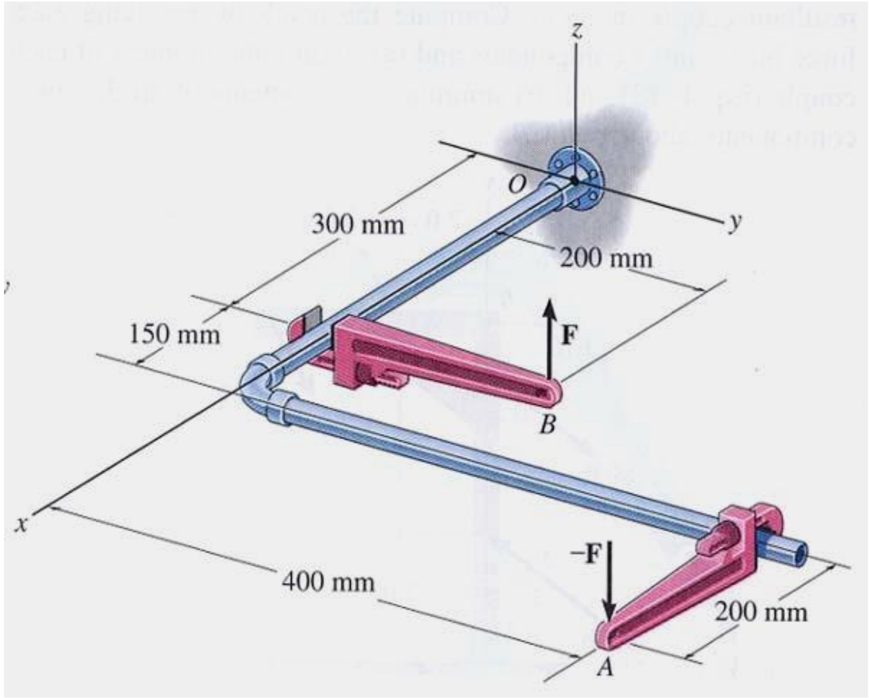
It was given that the net moment equals zero. So

$+\circlearrowright \Sigma M = -(500)(2) + (300 \cos 30^\circ)(d) - (300 \sin 30^\circ)(0.2) = 0$

Now solve this equation for d.

$d = (1000 + 60 \sin 30^\circ) / (300 \cos 30^\circ) = 3.96\ m$

GROUP PROBLEM SOLVING – VECTOR APPROACH



Given: $\boldsymbol{F} = \{25\ \boldsymbol{k}\}$ N and
$-\boldsymbol{F} = \{-\ 25\ \boldsymbol{k}\}$ N

Find: The couple moment acting on the pipe assembly using Cartesian vector notation.

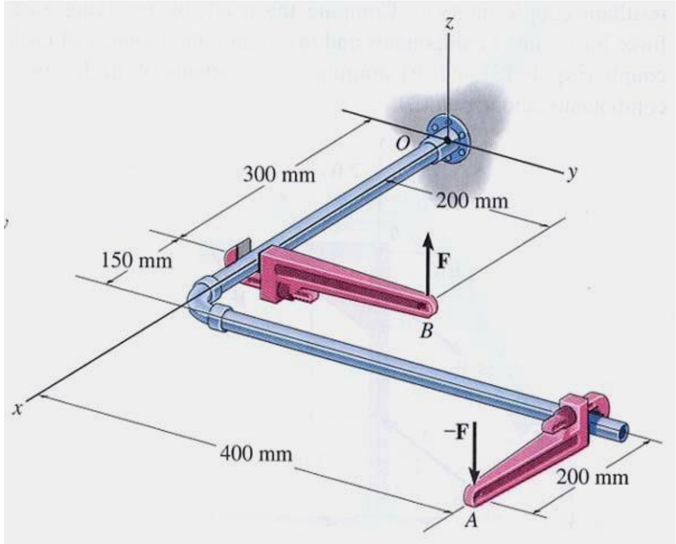
<u>PLAN</u>:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{25\ \boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

GROUP PROBLEM SOLVING – VECTOR APPROACH



$$\boldsymbol{r}_{AB} = \{ -350\,\boldsymbol{i} - 200\,\boldsymbol{j} \}\ \text{mm}$$

$$= \{ -0.35\,\boldsymbol{i} - 0.2\,\boldsymbol{j} \}\ \text{m}$$

$$\boldsymbol{F} = \{25\,\boldsymbol{k}\}\ \text{N}$$

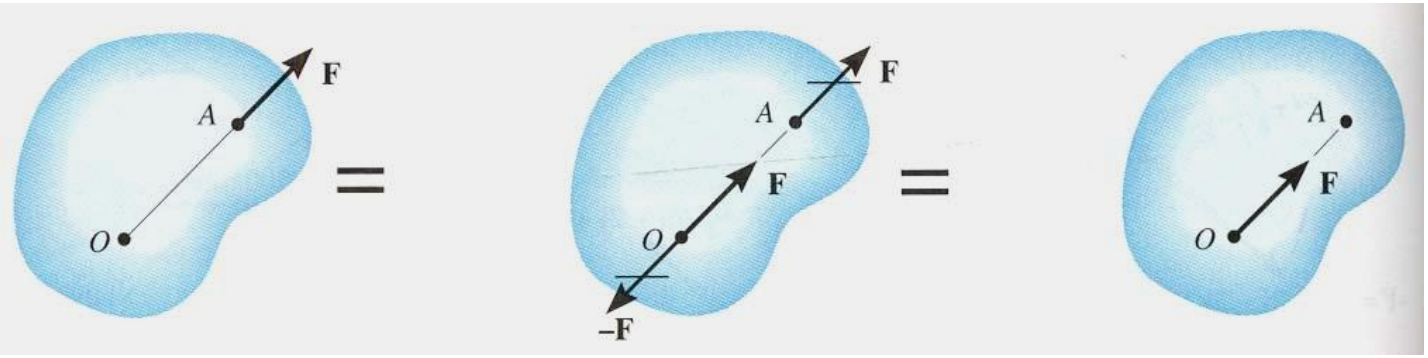
$$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix}\ \text{N} \cdot \text{m}$$

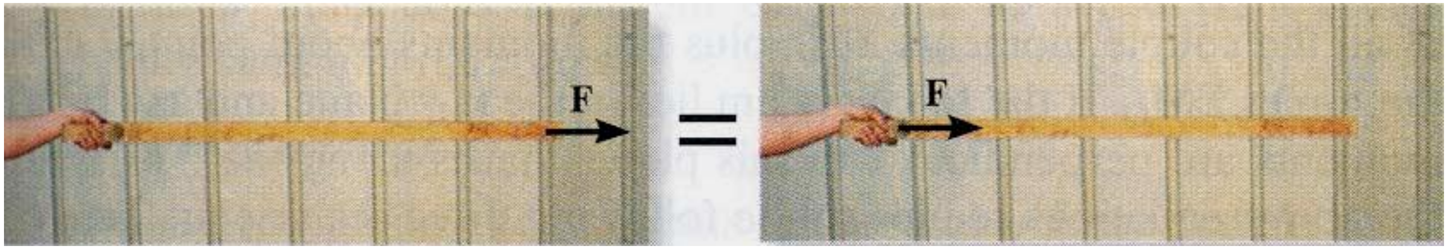
$$= \{ \boldsymbol{i}\,(-5 - 0) - \boldsymbol{j}\,(-8.75 - 0) + \boldsymbol{k}\,(0) \}\ \text{N} \cdot \text{m}$$

$$= \{ -5\,\boldsymbol{i} + 8.75\,\boldsymbol{j} \}\ \text{N} \cdot \text{m}$$

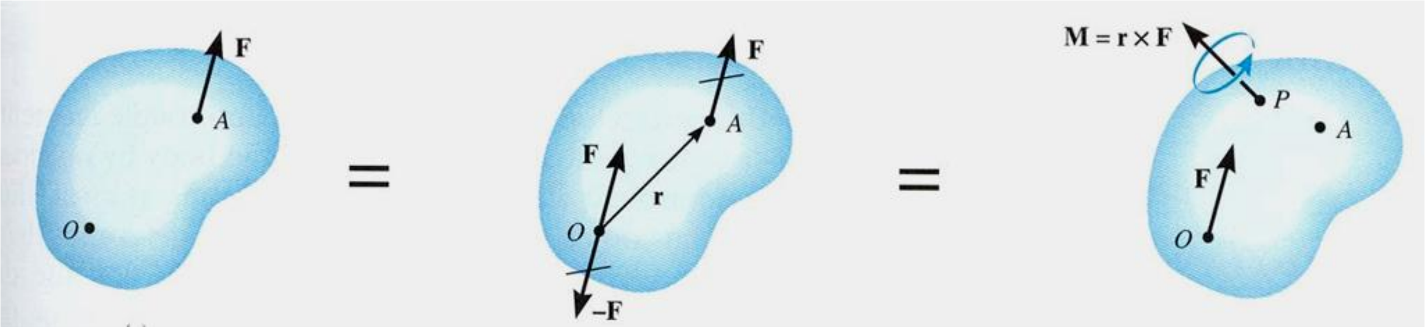
MOVING A FORCE ON ITS LINE OF ACTION



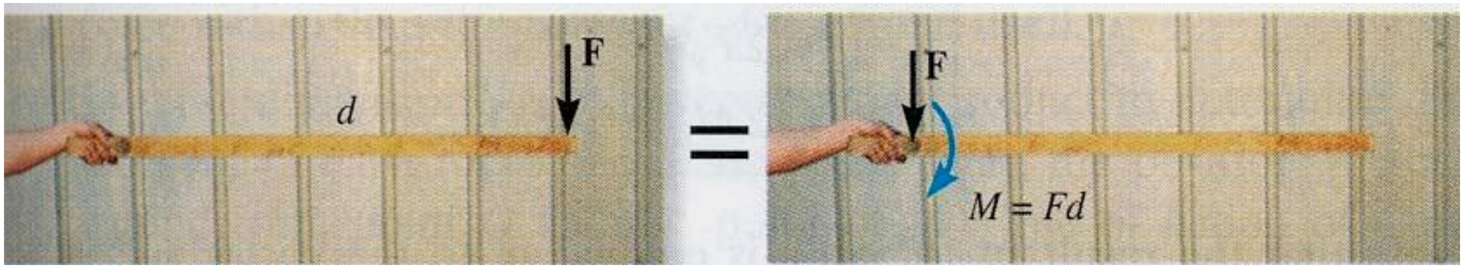
Moving a force from A to O, when both points are on the vectors' line of action, does not change the external effect. Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).

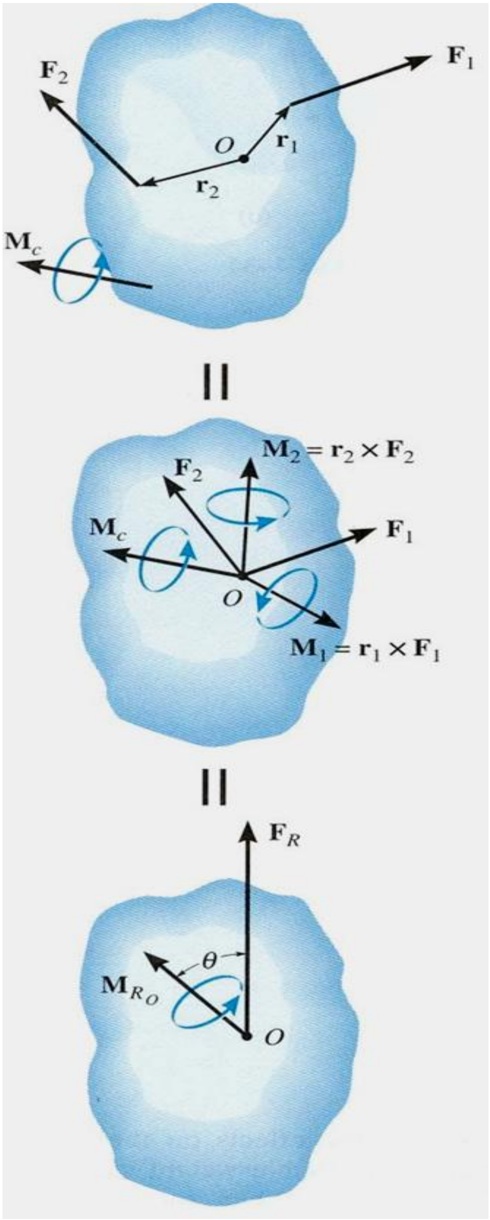


MOVING A FORCE OFF OF ITS LINE OF ACTION



Moving a force from point A to O (as shown above) requires creating an additional couple moment. Since this new couple moment is a "free" vector, it can be applied at any point P on the body.





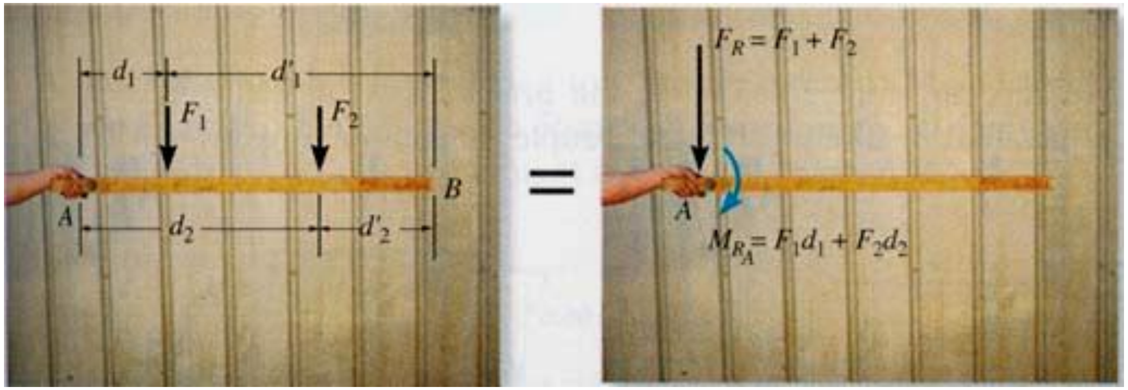
RESULTANTS OF A FORCE AND COUPLE SYSTEM
(Section 4.8)

When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

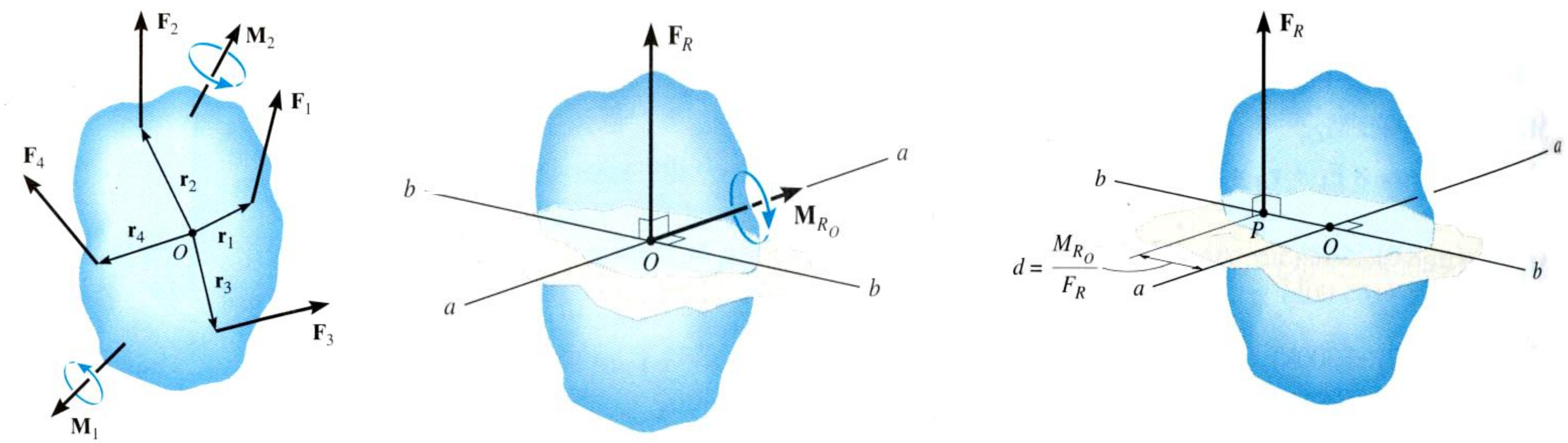
RESULTANT OF A FORCE AND COUPLE SYSTEM
(continued)



If the force system lies in the x-y plane (the 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

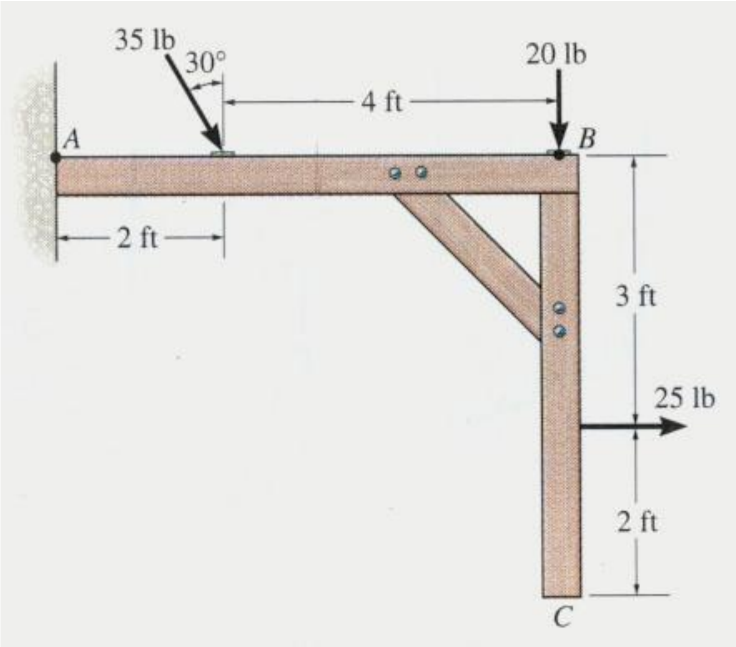
FURTHER REDUCTION OF A FORCE AND COUPLE SYSTEM
(Section 4.9)



If $\boldsymbol{F_R}$ and $\boldsymbol{M_{RO}}$ are perpendicular to each other, then the system can be further reduced to a single force, $\boldsymbol{F_R}$, by simply moving $\boldsymbol{F_R}$ from O to P.

In three special cases, <u>concurrent, coplanar, and parallel</u> systems of forces, the system can always be reduced to a single force.

EXAMPLE #1



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location along the beam AB.

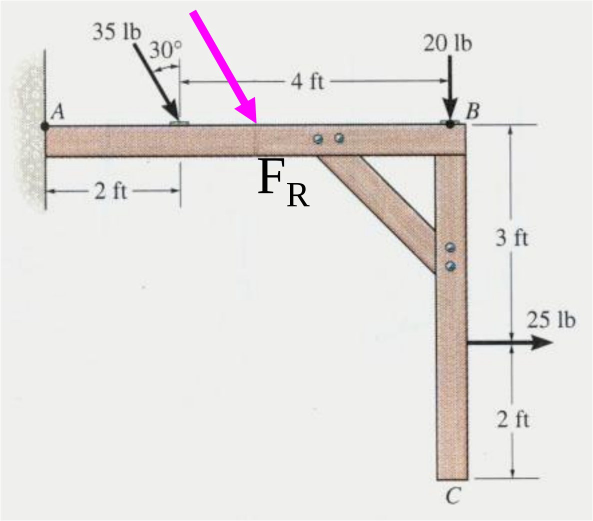
<u>Plan</u>:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A.

3) Shift the F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE #1
(continued)



$$+\rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30° = 42.5 \text{ lb}$$

$$-\downarrow \Sigma F_{Ry} = -20 - 35 \cos 30° = -50.31 \text{ lb}$$

$$+\circlearrowleft M_{RA} = -35 \cos 30° (2) - 20(6) + 25(3)$$

$$= -105.6 \text{ lb}\cdot\text{ft}$$

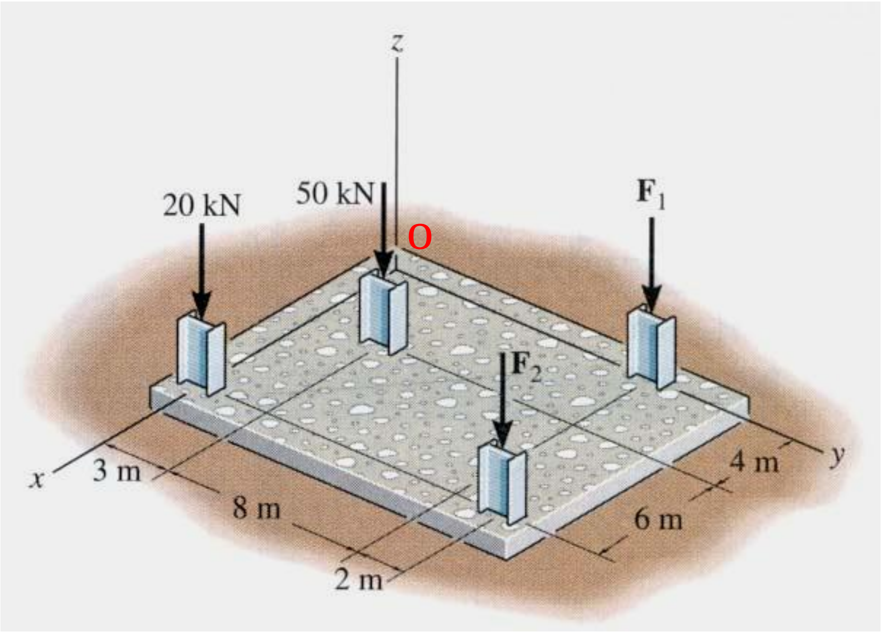
$$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9 \text{ lb}$$

$$\searrow \theta = \tan^{-1} (50.31/42.5) = 49.8°$$

The equivalent single force F_R can be located on the beam AB at a distance d measured from A.

$$d = M_{RA}/F_{Ry} = -105.6/-50.31 = 2.10 \text{ ft.}$$

EXAMPLE #2



Given: The building slab has four columns. F_1 and $F_2 = 0$.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x,y) of the single equivalent resultant force.

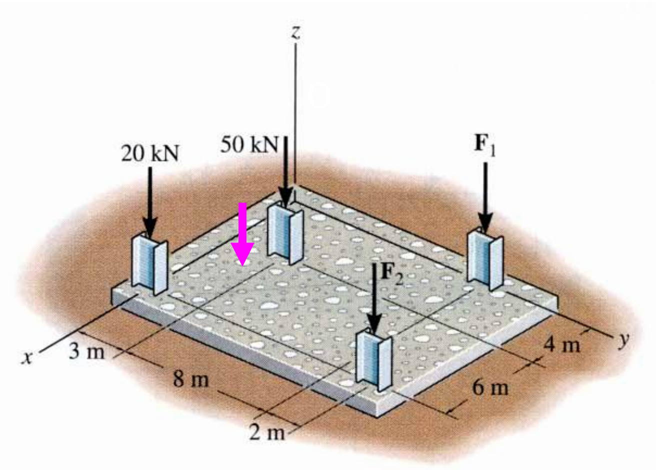
Plan:

1) Find $\boldsymbol{F_{RO}} = \sum \boldsymbol{F_i} = F_{Rzo}\, \boldsymbol{k}$

2) Find $\boldsymbol{M_{RO}} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i}) = M_{RxO}\, \boldsymbol{i} + M_{RyO}\, \boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO}/F_{RzO}$ and $y = M_{RxO}/F_{RzO}$

EXAMPLE #2
(continued)



$\boldsymbol{F}_{RO} = \{-50\,\boldsymbol{k} - 20\,\boldsymbol{k}\} = \{-70\,\boldsymbol{k}\}$ kN

$\boldsymbol{M}_{RO} = (10\,\boldsymbol{i}) \times (-20\,\boldsymbol{k}) + (4\,\boldsymbol{i} + 3\,\boldsymbol{j})\text{x}(-50\,\boldsymbol{k})$

$= \{200\,\boldsymbol{j} + 200\,\boldsymbol{j} - 150\,\boldsymbol{i}\}$ kN·m

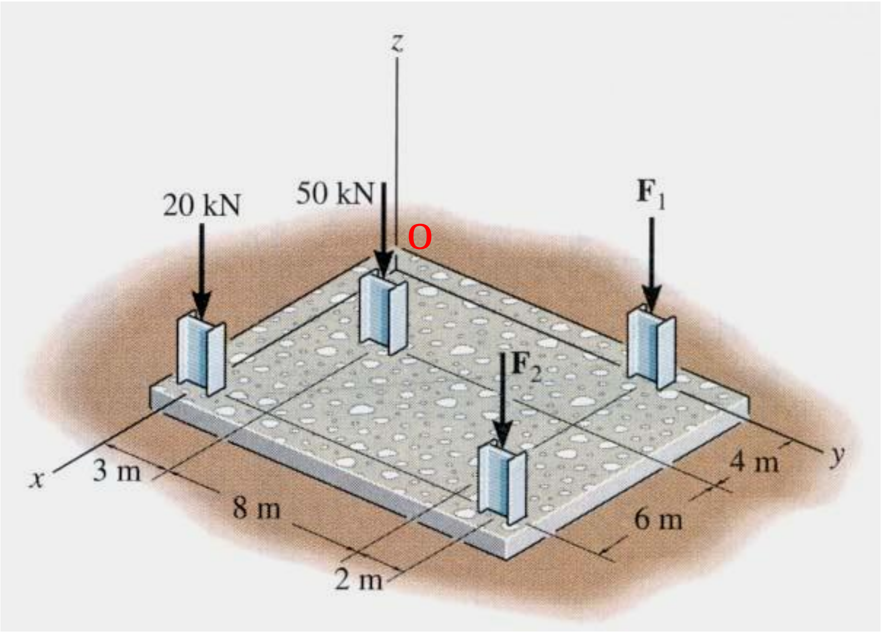
$= \{-150\,\boldsymbol{i} + 400\,\boldsymbol{j}\}$ kN·m

The location of the single equivalent resultant force is given as,

$x = M_{Ryo}/F_{Rzo} = 400/(70) = 5.71$ m

$y = M_{Rxo}/F_{Rzo} = (-150)/(-70) = 2.14$ m

EXAMPLE #3



Given: The building slab has four columns. F_1 and F_2 = 50 KN.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x,y) of the single equivalent resultant force.

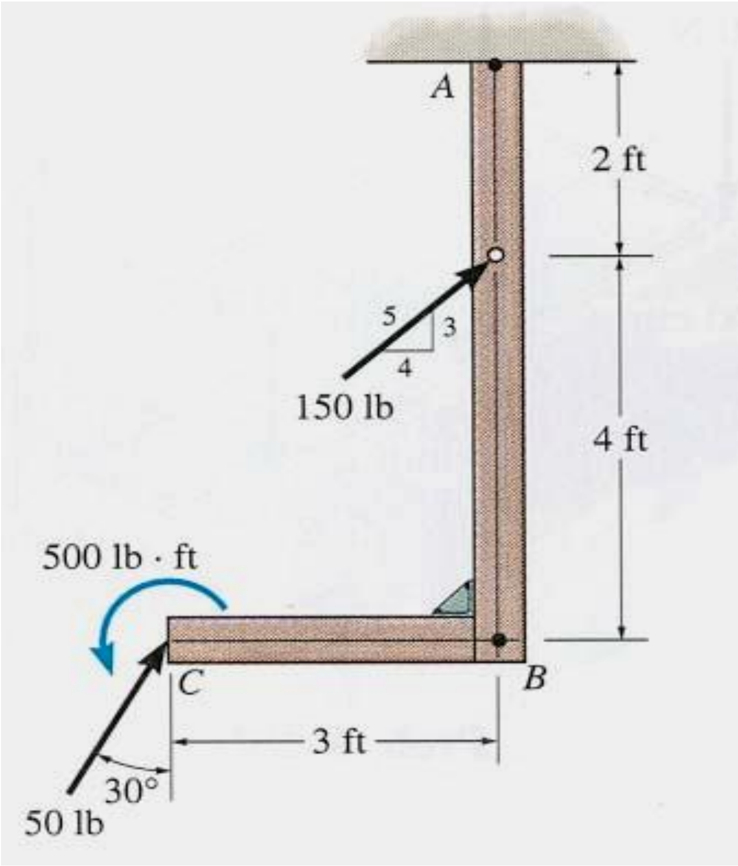
Plan:

1) Find $\boldsymbol{F}_{RO} = \sum \boldsymbol{F}_i = F_{Rzo}\,\boldsymbol{k}$

2) Find $\boldsymbol{M}_{RO} = \sum (\boldsymbol{r}_i \times \boldsymbol{F}_i) = M_{RxO}\,\boldsymbol{i} + M_{RyO}\,\boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO}/F_{RzO}$ and $y = M_{RxO}/F_{RzO}$

GROUP PROBLEM SOLVING



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A. Also, find the position of the resultant force on AB creating an equivalent moment.

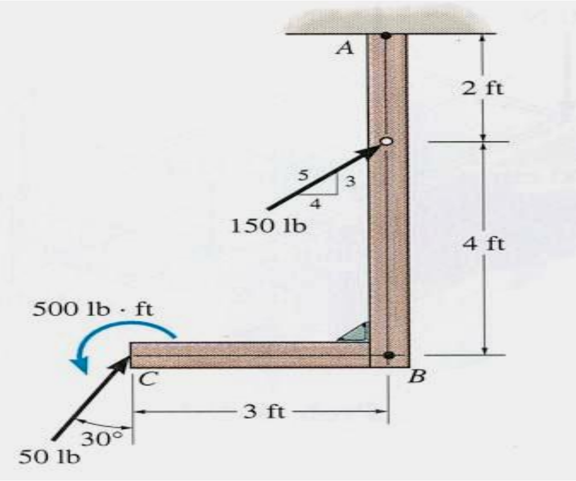
Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A and add them to the 500 lb - ft free moment to find the resultant M_{RA} .

GROUP PROBLEM SOLVING (continued)

Summing the force components:



$+ \rightarrow \Sigma F_x = (4/5)\ 150\ \text{lb} + 50\ \text{lb} \sin 30° = 145\ \text{lb}$

$+ \uparrow \Sigma F_y = (3/5)\ 150\ \text{lb} + 50\ \text{lb} \cos 30° = 133.3\ \text{lb}$

Now find the magnitude and direction of the resultant.

$F_{RA} = (145^2 + 133.3^2)^{1/2} = 197\ \text{lb}$ and $\theta = \tan^{-1}(133.3/145) = 42.6°$ ∠

$+ \circlearrowleft M_{RA} = \{(4/5)(150)(2) - 50 \cos 30° (3) + 50 \sin 30° (6) + 500\}$

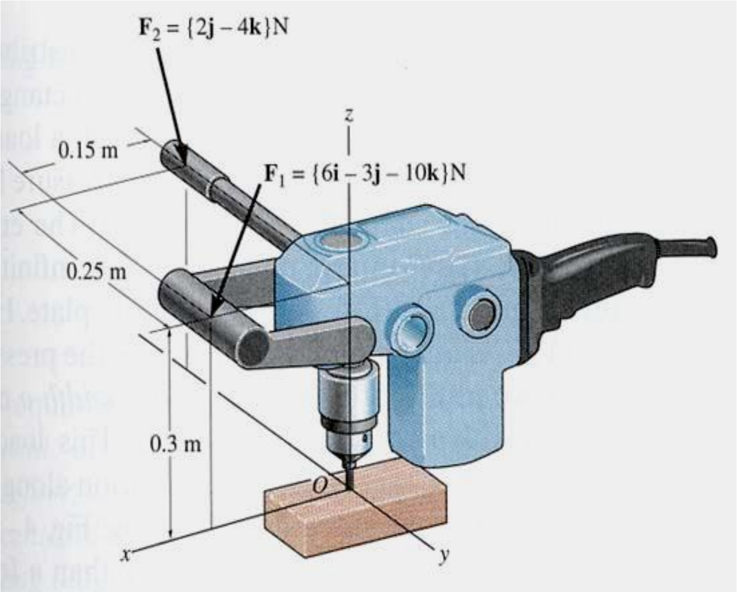
$= 760\ \text{lb} \cdot \text{ft}$

Position of the resultant force which creates the same magnitude Of moment is:

$F_x.d = M_{RA}$

$d = 760/145 = 5.24$ ft

GROUP PROBLEM SOLVING (continued)



Given: Handle forces F_1 and F_2 are applied to the electric drill.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $\boldsymbol{F_{RO}} = \Sigma\, \boldsymbol{F_i}$

b) Find $\boldsymbol{M_{RO}} = \Sigma\, \boldsymbol{M_C} + \Sigma\, (\, \boldsymbol{r_i} \times \boldsymbol{F_i}\,)$

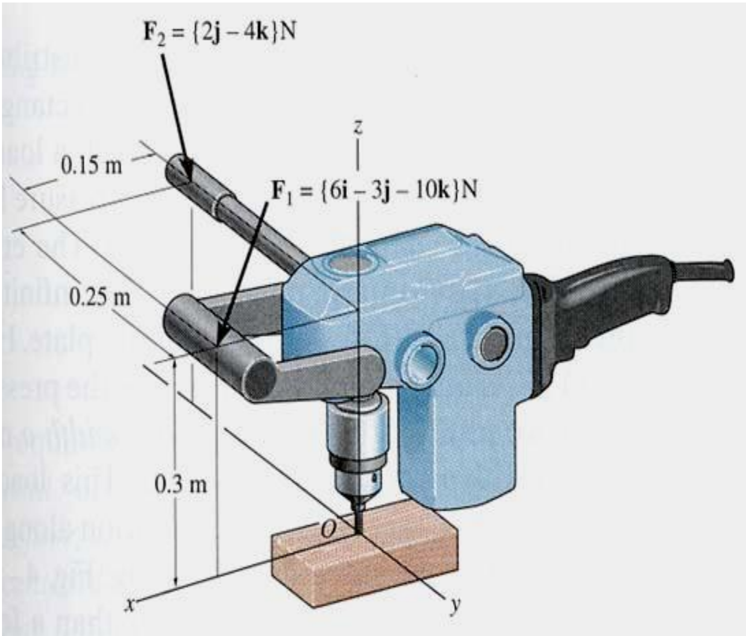
Where,

$\boldsymbol{F_i}$ are the individual forces in Cartesian vector notation (CVN).

$\boldsymbol{M_C}$ are any free couple moments in CVN (none in this example).

$\boldsymbol{R_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

SOLUTION



$$\boldsymbol{F_1} = \{6\,\boldsymbol{i} - 3\,\boldsymbol{j} - 10\,\boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F_2} = \{0\,\boldsymbol{i} + 2\,\boldsymbol{j} - 4\,\boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F_{RO}} = \{6\,\boldsymbol{i} - 1\,\boldsymbol{j} - 14\,\boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{r_1} = \{0.15\,\boldsymbol{i} + 0.3\,\boldsymbol{k}\}\ \text{m}$$

$$\boldsymbol{r_2} = \{-0.25\,\boldsymbol{j} + 0.3\,\boldsymbol{k}\}\ \text{m}$$

$$\boldsymbol{M_{RO}} = \boldsymbol{r_1} \times \boldsymbol{F_1} + \boldsymbol{r_2} \times \boldsymbol{F_2}$$

$$\boldsymbol{M_{RO}} = \left\{ \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.15 & 0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix} \right\}\ \text{N·m}$$

$$= \{0.9\,\boldsymbol{i} + 3.3\,\boldsymbol{j} - 0.45\,\boldsymbol{k} + 0.4\,\boldsymbol{i} + 0\,\boldsymbol{j} + 0\,\boldsymbol{k}\}\ \text{N·m}$$

$$= \{1.3\,\boldsymbol{i} + 3.3\,\boldsymbol{j} - 0.45\,\boldsymbol{k}\}\ \text{N·m}$$

DEMO:

$\vec{a} \times \vec{b} \times \vec{c}$ has no meaning!

$$\vec{V}_1 = \vec{a} \times (\vec{b} \times \vec{c}) = \vec{a} \times \vec{d}$$

$$\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \hat{i}(b_y c_z - b_z c_y) + \hat{j}(b_z c_x - b_x c_z) + \hat{k}(b_x c_y - b_y c_x$$

$$\vec{V}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ d_x & d_y & d_z \end{vmatrix} = \hat{i}\left(a_y(b_x c_y - b_y c_x) - a_z(b_z c_x - b_x c_z)\right)$$

$$+ \hat{j}\left(a_z(b_y c_z - b_z c_y) - a_x(b_x c_y - b_y c_x)\right)$$

$$+ \hat{k}\left(a_x(b_z c_x - b_x c_z) - a_y(b_y c_z - b_z c_y)\right)$$

$$= \hat{i}(a_y b_x c_y + a_z b_x c_z - a_y b_y c_x - a_z b_z c_x)$$

$$+ \hat{j}(a_z b_y c_z + a_x b_y c_x - a_z b_z c_y - a_x b_x c_y)$$

$$+ \hat{k}(a_x b_z c_x + a_y b_z c_y - a_x b_x c_z - a_y b_y c_z).$$

$$\vec{V}_2 = (\vec{a} \times \vec{b}) \times \vec{c} = \vec{f} \times \vec{c}$$

$$\vec{V}_2 = (\vec{a} \times \vec{b}) \times \vec{c} = \vec{f} \times \vec{c}$$

$$\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i}(a_y b_z - a_z b_y) + \hat{j}(a_z b_x - a_x b_z) + \hat{k}(a_x b_y - a_y b_x)$$

$$\vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ f_x & f_y & f_z \\ c_x & c_y & c_z \end{vmatrix} = \hat{i}\left[c_z(a_z b_x - a_x b_z) - c_y(a_x b_y - a_y b_x)\right]$$

$$+ \hat{j}\left[c_x(a_x b_y - a_y b_x) - c_z(a_y b_z - a_z b_y)\right]$$

$$+ \hat{k}\left[c_y(a_y b_z - a_z b_y) - c_x(a_z b_x - a_x b_z)\right]$$

$$= \hat{i}(a_z b_x c_z + a_y b_x c_y - a_x b_z c_z - a_x b_y c_y)$$

$$+ \hat{j}(a_x b_y c_x + a_z b_y c_z - a_y b_x c_x - a_y b_z c_z)$$

$$+ \hat{k}(a_y b_z c_y + a_x b_z c_x - a_z b_y c_y - a_z b_x c_x)$$

$$\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \hat{i}(b_y c_z - b_z c_y) + \hat{j}(b_z c_x - b_x c_z) + \hat{k}(b_x c_y - b_y c_x)$$

$$\vec{V}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ d_x & d_y & d_z \end{vmatrix} = \hat{i}\left(a_y(b_x c_y - b_y c_x) - a_z(b_z c_x - b_x c_z)\right)$$
$$+ \hat{j}\left(a_z(b_y c_z - b_z c_y) - a_x(b_x c_y - b_y c_x)\right)$$
$$+ \hat{k}\left(a_x(b_z c_x - b_x c_z) - a_y(b_y c_z - b_z c_y)\right)$$
$$= \hat{i}(a_y b_x c_y + a_z b_x c_z - a_y b_y c_x - a_z b_z c_x)$$
$$+ \hat{j}(a_z b_y c_z + a_x b_y c_x - a_z b_z c_y - a_x b_x c_y)$$
$$+ \hat{k}(a_x b_z c_x + a_y b_z c_y - a_x b_x c_z - a_y b_y c_z).$$

$$\vec{V}_2 = (\vec{a} \times \vec{b}) \times \vec{c} = \vec{f} \times \vec{c}$$

$$\vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i}(a_y b_z - a_z b_y) + \hat{j}(a_z b_x - a_x b_z) + \hat{k}(a_x b_y - a_y b_x)$$

$$\vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ f_x & f_y & f_z \\ c_x & c_y & c_z \end{vmatrix} = \hat{i}\left[c_z(a_z b_x - a_x b_z) - c_y(a_x b_y - a_y b_x)\right]$$
$$+ \hat{j}\left[c_x(a_x b_y - a_y b_x) - c_z(a_y b_z - a_z b_y)\right]$$
$$+ \hat{k}\left[c_y(a_y b_z - a_z b_y) - c_x(a_z b_x - a_x b_z)\right]$$
$$= \hat{i}(a_z b_x c_z + a_y b_x c_y - a_x b_z c_z - a_x b_y c_y)$$
$$+ \hat{j}(a_x b_y c_x + a_z b_y c_z - a_y b_x c_x - a_y b_z c_z)$$
$$+ \hat{k}(a_y b_z c_y + a_x b_z c_x - a_z b_y c_y - a_z b_x c_x)$$

NB:

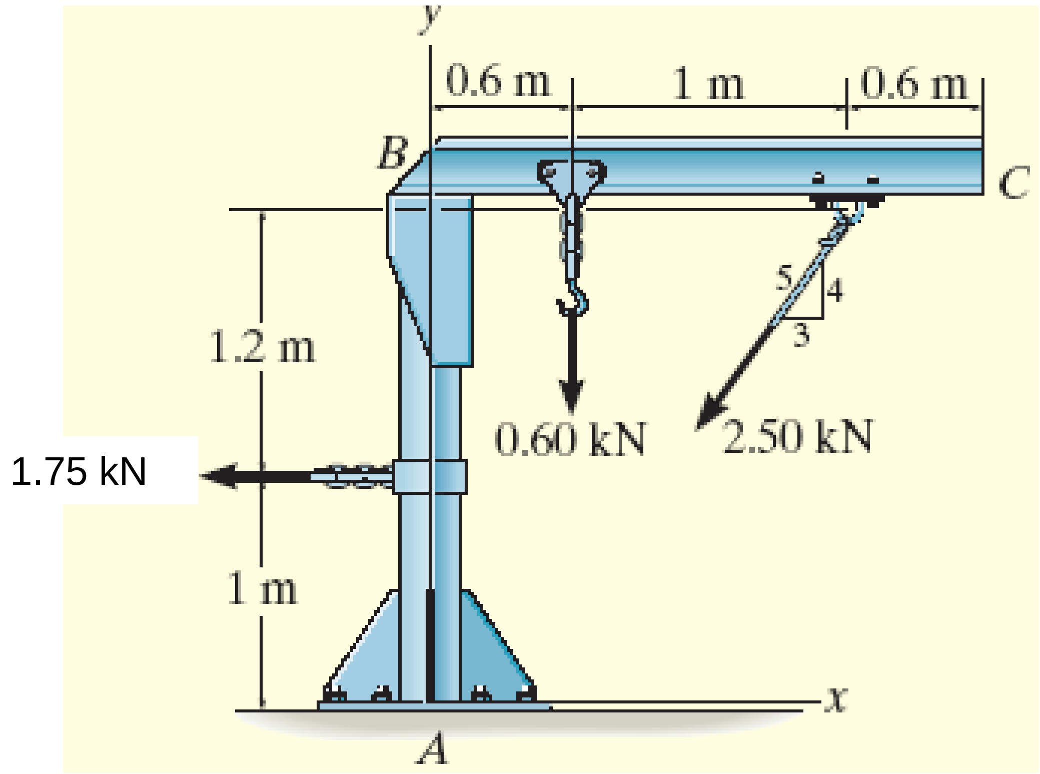
$$\vec{V}_1 \neq \vec{V}_2$$

also:

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}\,!$$

Example

The jib crane is subjected to three coplanar forces. Replace this loading by an equivalent resultant force and specify where the resultant's line of action intersects the column AB and boom BC.



Solution

Force Summation

$$+\rightarrow F_{Rx} = \Sigma F_x;$$

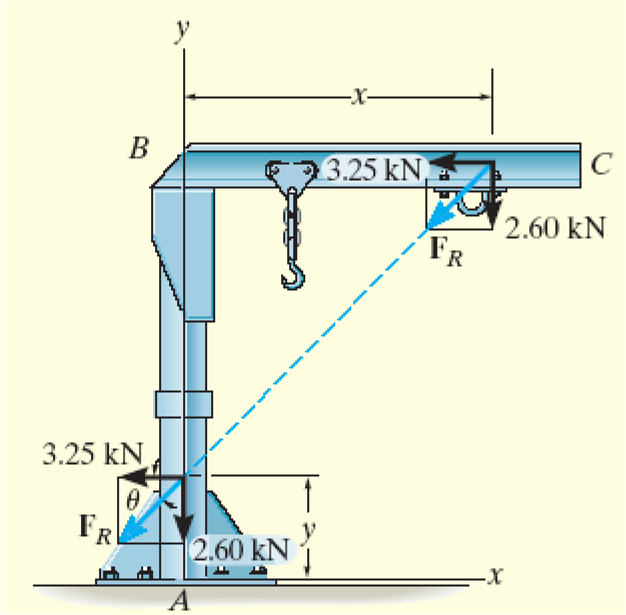
$$F_{Rx} = -2.5kN\left(\frac{3}{5}\right) - 1.75kN$$

$$= -3.25kN = 3.25kN \leftarrow$$

$$+\rightarrow F_{Ry} = \Sigma F_y;$$

$$F_{Ry} = -2.5N\left(\frac{4}{5}\right) - 0.6kN$$

$$= -2.60kN = 2.60N \downarrow$$



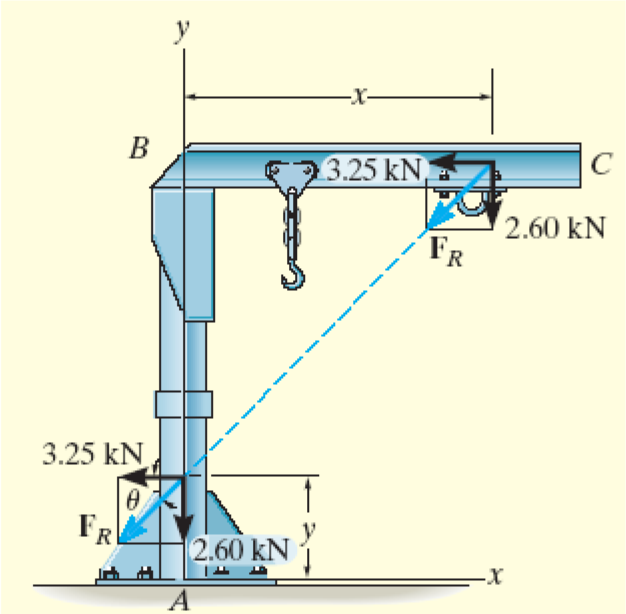
Solution

For magnitude of resultant force,

$$F_R = \sqrt{(F_{Rx})^2 + (F_{Ry})^2} = \sqrt{(3.25)^2 + (2.60)^2}$$
$$= 4.16kN$$

For direction of resultant force,

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{2.60}{3.25}\right)$$
$$= 38.7^\circ$$



Solution

Moment Summation

➔ Summation of moments about point A,

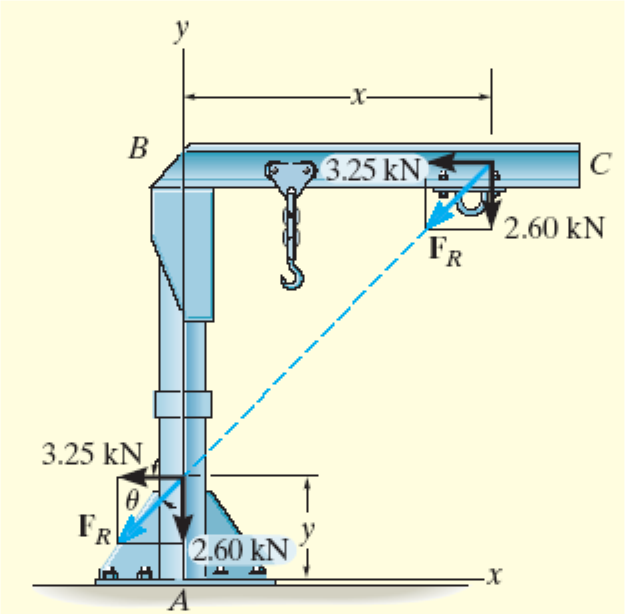
$$M_{RA} = \Sigma M_A;$$

$$3.25kN(y) + 2.60kN(0)$$

$$= 1.75kn(1m) - 0.6kN(0.6m)$$

$$+ 2.50kN\left(\frac{3}{5}\right)(2.2m) - 2.50kN\left(\frac{4}{5}\right)(1.6m)$$

$$y = 0.458m$$



Solution

Moment Summation

➔ Principle of Transmissibility

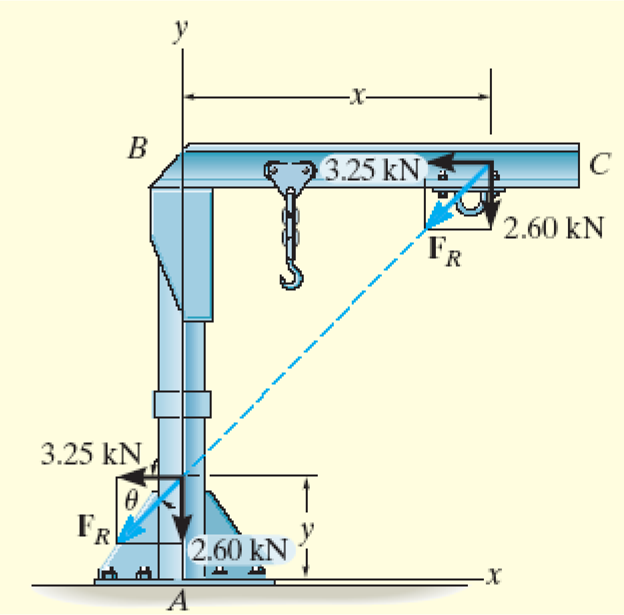
$$M_{RA} = \Sigma M_A;$$

$$3.25kN(2.2m) - 2.60kN(x)$$

$$= 1.75kn(1m) - 0.6kN(0.6m)$$

$$+ 2.50kN\left(\frac{3}{5}\right)(2.2m) - 2.50kN\left(\frac{4}{5}\right)(1.6m)$$

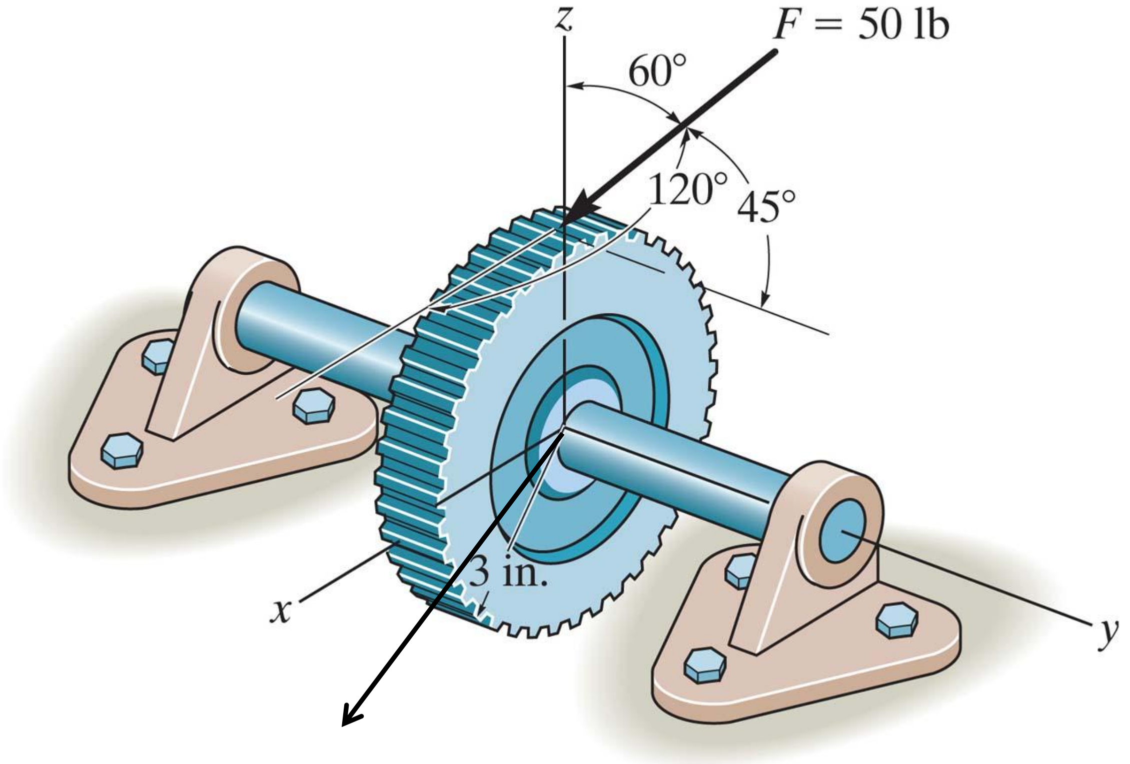
$$x = 2.177m$$



Tutorial Problem

The force F acts on the gear in the direction shown. Determine the moment of this force about the *y* axis.

$$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ a \end{pmatrix} \quad \mathbf{F_v} = F\begin{pmatrix} -\cos(\theta_3) \\ -\cos(\theta_2) \\ -\cos(\theta_1) \end{pmatrix} \quad M_y = (\mathbf{r} \times \mathbf{F_v})\cdot\mathbf{j} \qquad M_y = 75\,\text{lb}\cdot\text{in}$$



Tutorial Problem

Determine the moment of each force acting on the handle of the wrench about the a axis.

$$\mathbf{F_1} = \begin{pmatrix} -2 \\ 4 \\ -8 \end{pmatrix} \text{lb} \qquad \mathbf{F_2} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \text{lb}$$

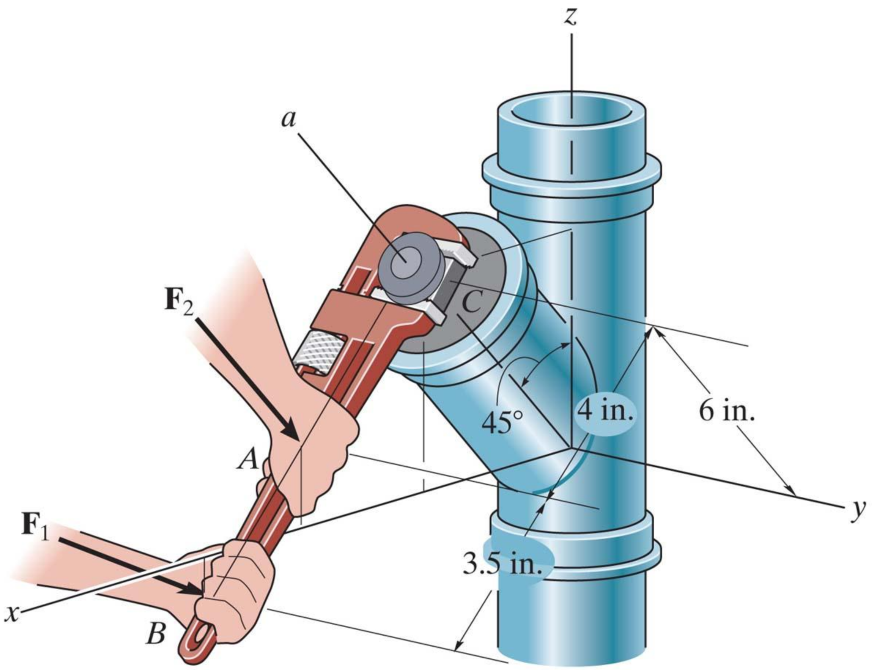
$b = 6$ in

$c = 4$ in

$d = 3.5$ in

$\theta = 45$ deg

$$\mathbf{u_a} = \begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix}$$



$$\mathbf{r_1} = b\begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix} + (c+d)\begin{pmatrix} \sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix} \qquad \mathbf{r_2} = b\begin{pmatrix} \cos(\theta) \\ 0 \\ \sin(\theta) \end{pmatrix} + c\begin{pmatrix} \sin(\theta) \\ 0 \\ -\cos(\theta) \end{pmatrix}$$

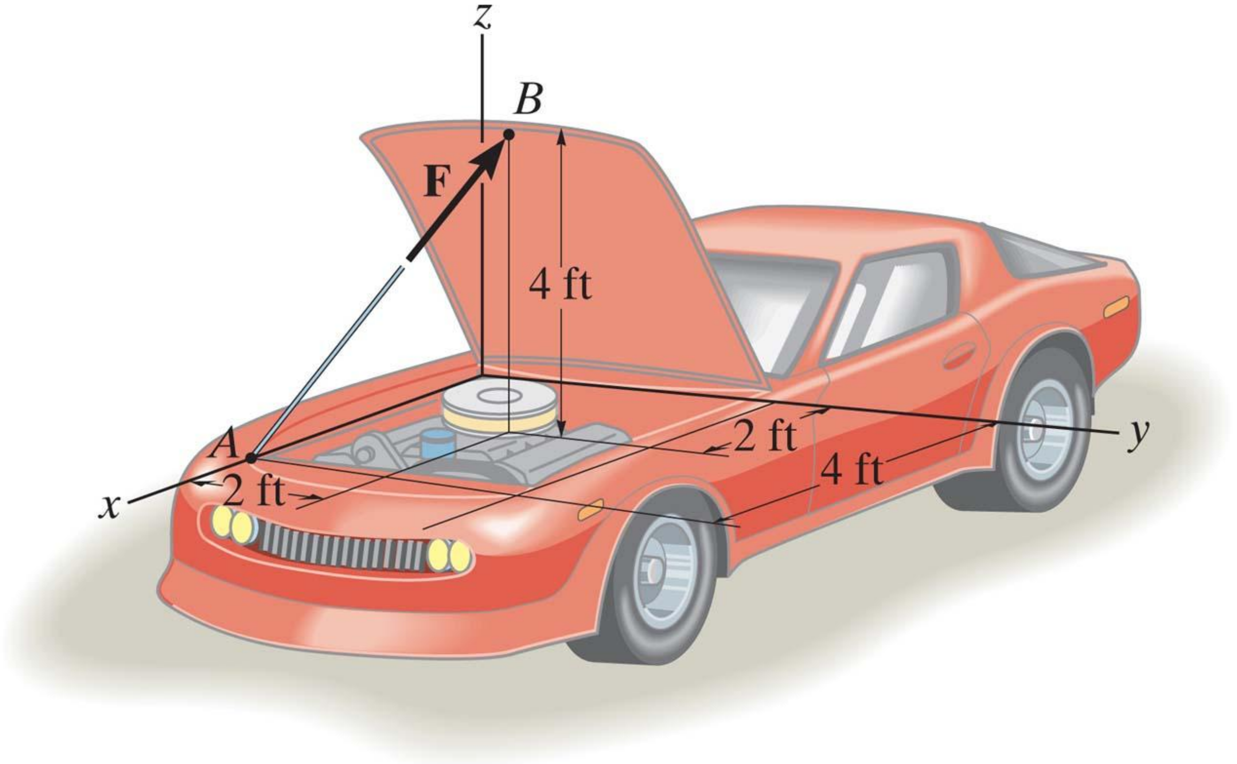
$$M_{1a} = (\mathbf{r_1} \times \mathbf{F_1}) \cdot \mathbf{u_a} \qquad M_{1a} = 30\,\text{lb}\cdot\text{in}$$

Tutorial Problem

Problem 4-58

The hood of the automobile is supported by the strut AB, which exerts a force $\mathbf{F}$ on the hood. Determine the moment of this force about the hinged axis y.

$F = 24$ lb



$$\mathbf{r_A} = \begin{pmatrix} b \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{r_{AB}} = \begin{pmatrix} -b+c \\ a \\ d \end{pmatrix} \qquad \mathbf{F_v} = F\frac{\mathbf{r_{AB}}}{\left|\mathbf{r_{AB}}\right|} \qquad \mathbf{F_v} = \begin{pmatrix} -9.798 \\ 9.798 \\ 19.596 \end{pmatrix} \text{lb}$$

$$\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad M_y = \left(\mathbf{r_A} \times \mathbf{F_v}\right)\cdot \mathbf{j} \qquad M_y = -78.384 \text{ lb}\cdot\text{ft}$$

Tutorial Problem

Problem 4-78

Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is *M* counterclockwise. Where on the beam does the resultant couple moment act?

$M = 450$ lb·ft

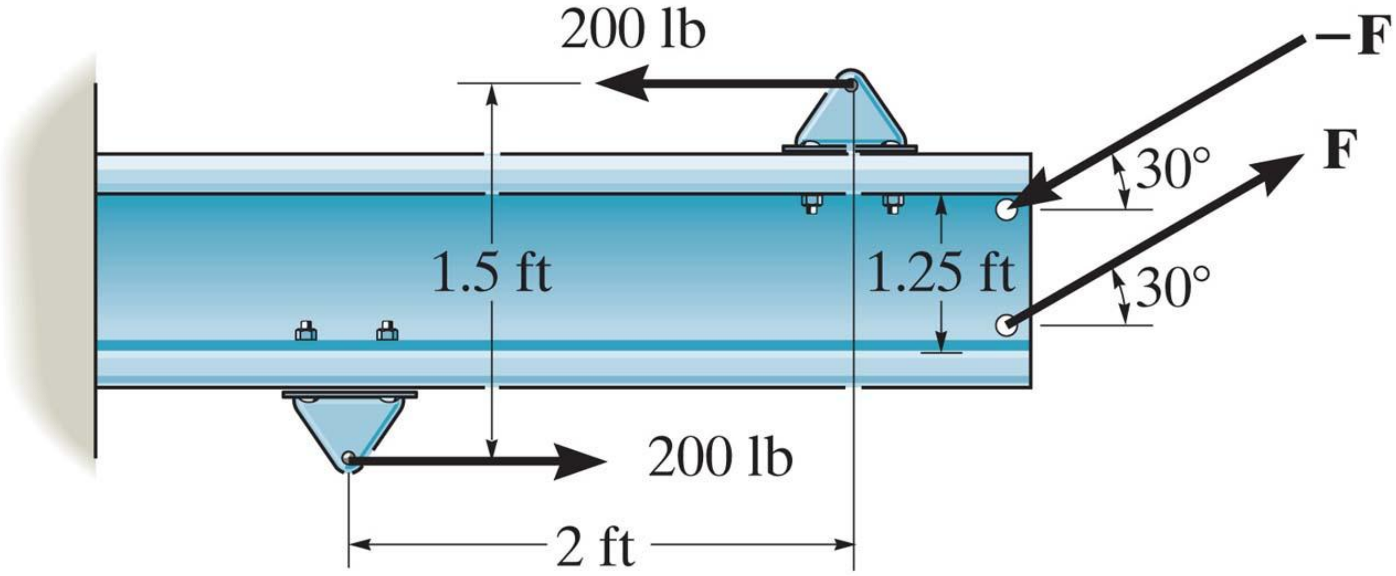
$P = 200$ lb

$a = 1.5$ ft

$b = 1.25$ ft

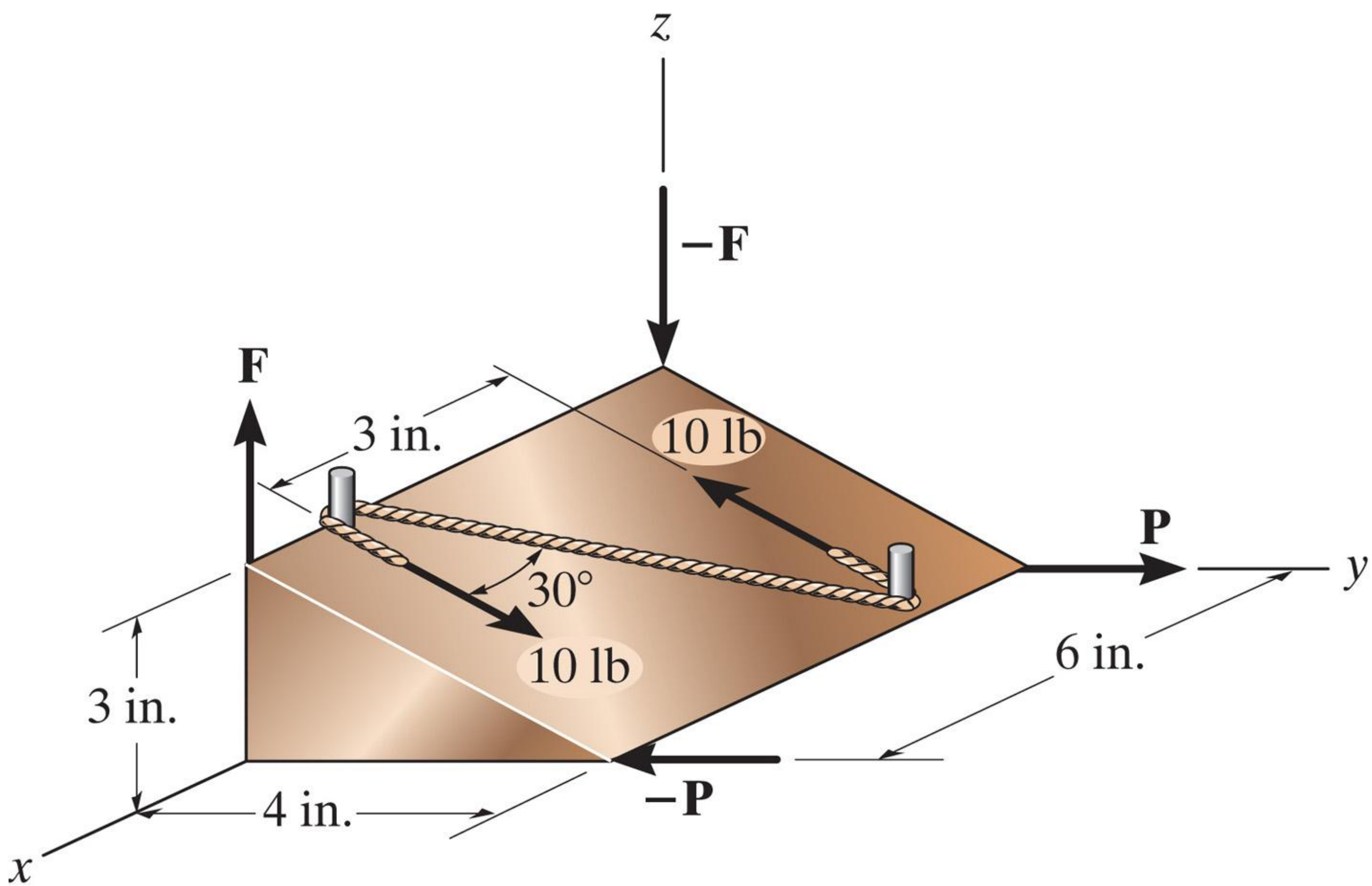
$c = 2$ ft

$\theta = 30$ deg

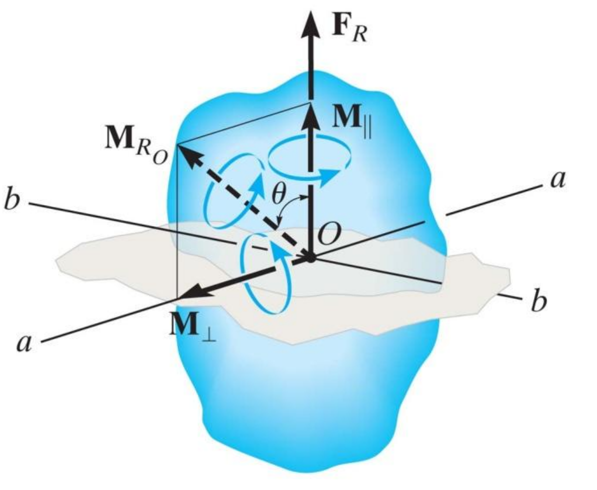


$$\circlearrowright + \quad M_R = \Sigma M \qquad M = F\,b\cos(\theta) + P\,a \qquad F = \frac{M - P\,a}{b\cos(\theta)} \qquad F = 139\,\text{lb}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

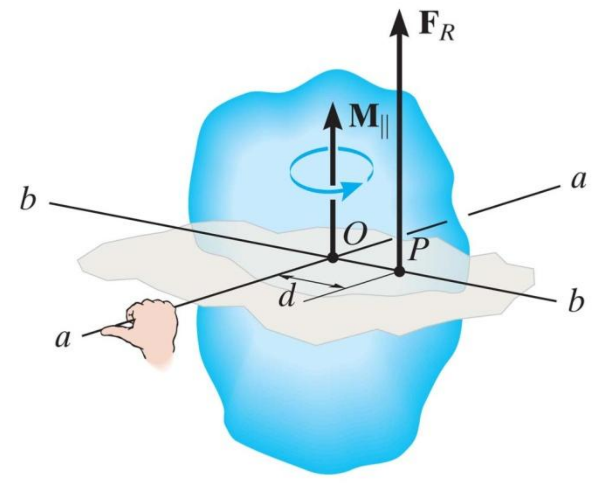

z
−F
F
3 in.
10 lb
P
y
30°
10 lb
6 in.
3 in.
4 in.
−P
x

Reduce to a Wrench



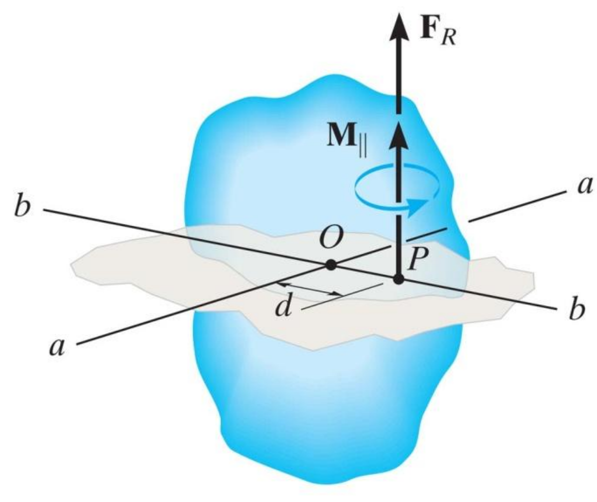
(a)

=

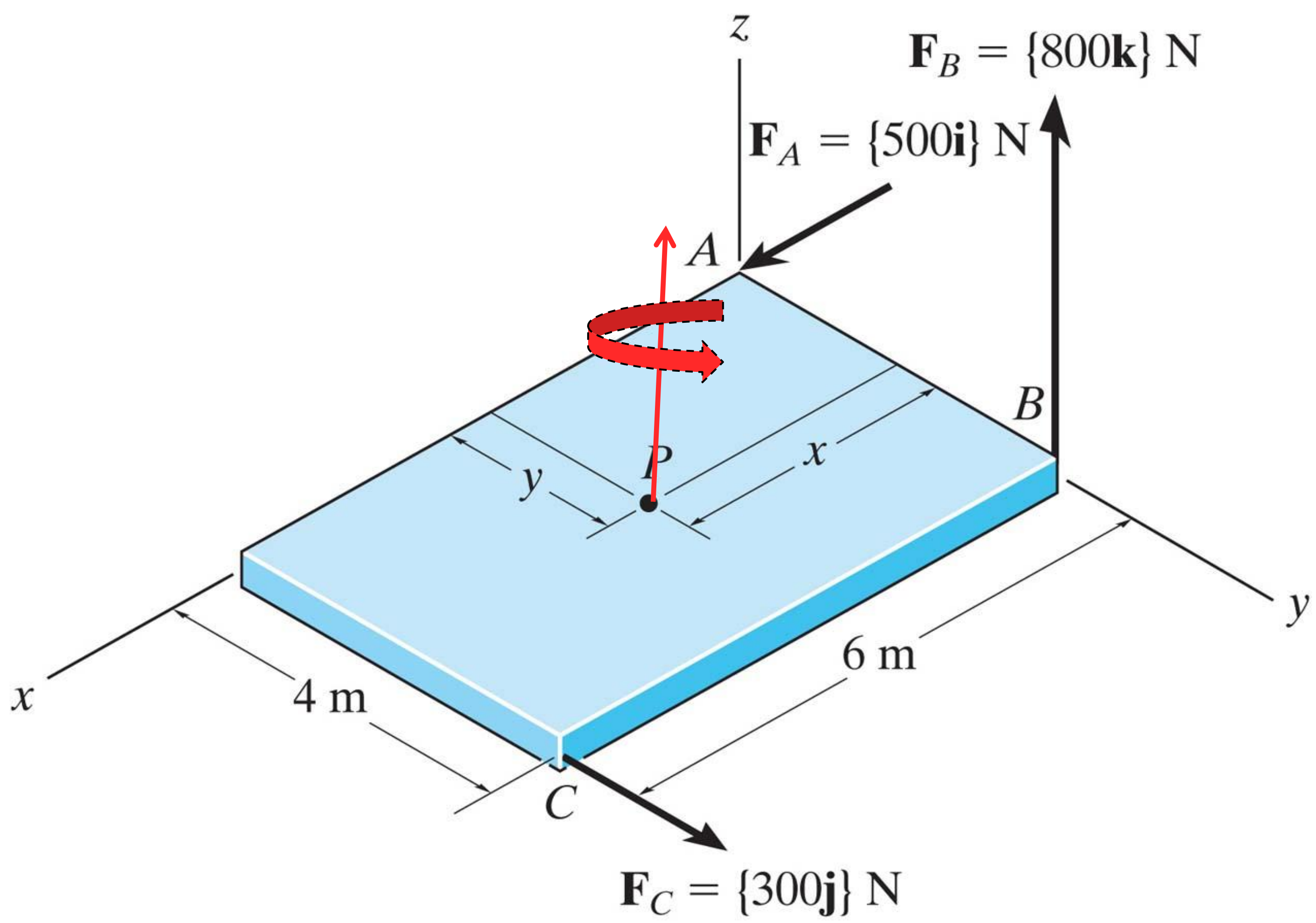


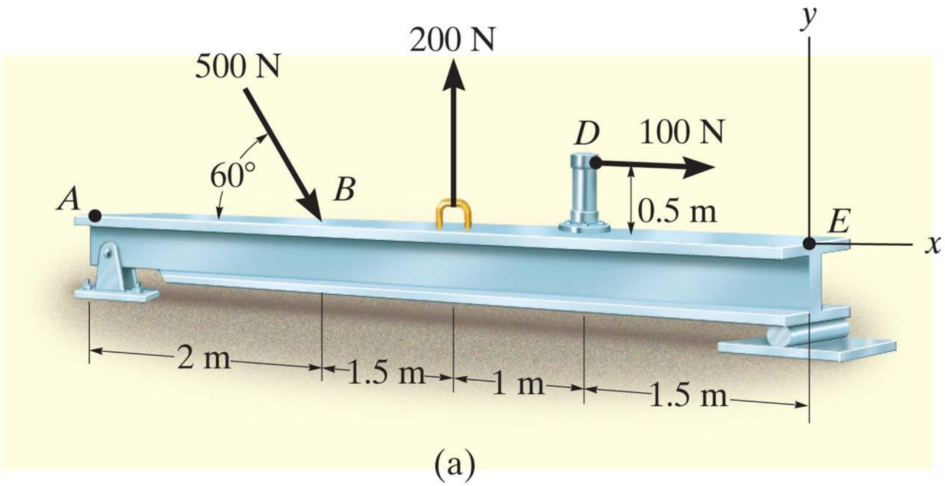
(b)

=



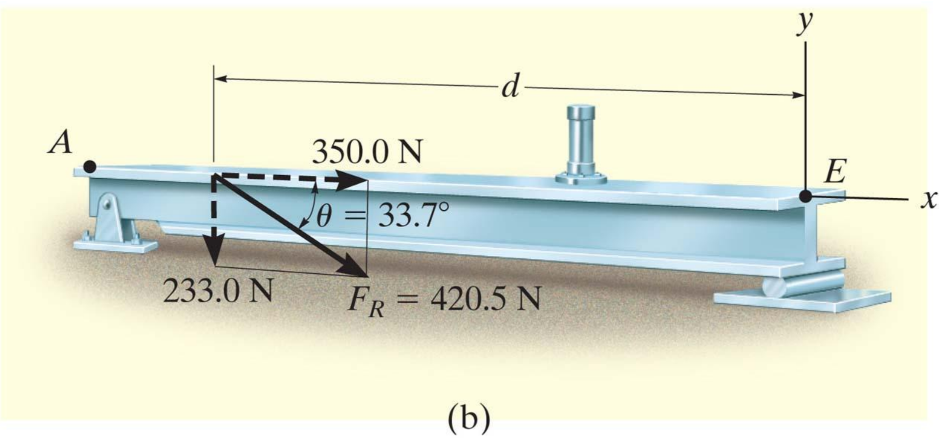
(c)


z
$F_B = \{800k\}$ N
$F_A = \{500i\}$ N
A
B
P
x
y
x
y
4 m
6 m
C
$F_C = \{300j\}$ N



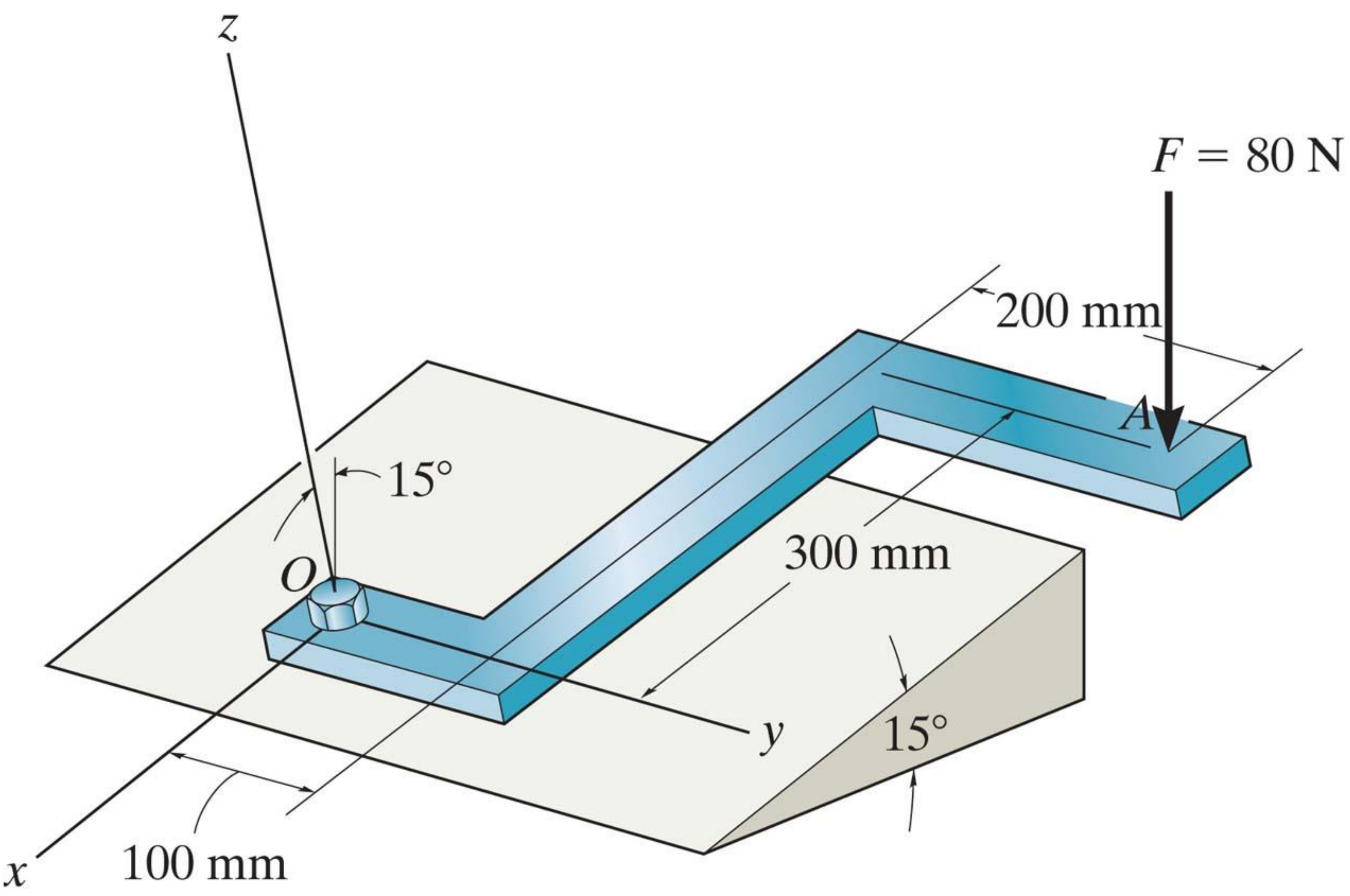
500 N
200 N
D
100 N
60°
A
B
0.5 m
E
y
x
2 m
1.5 m
1 m
1.5 m

(a)



y
d
A
350.0 N
E
x
θ = 33.7°
233.0 N
F_R = 420.5 N

(b)



z
F = 80 N
200 mm
A
15°
O
300 mm
y
15°
x
100 mm

Tutorial Problem

Problem 4-146

The beam supports the distributed load caused by the sandbags. Determine the resultant force on the beam and specify its location measured from point *A*.

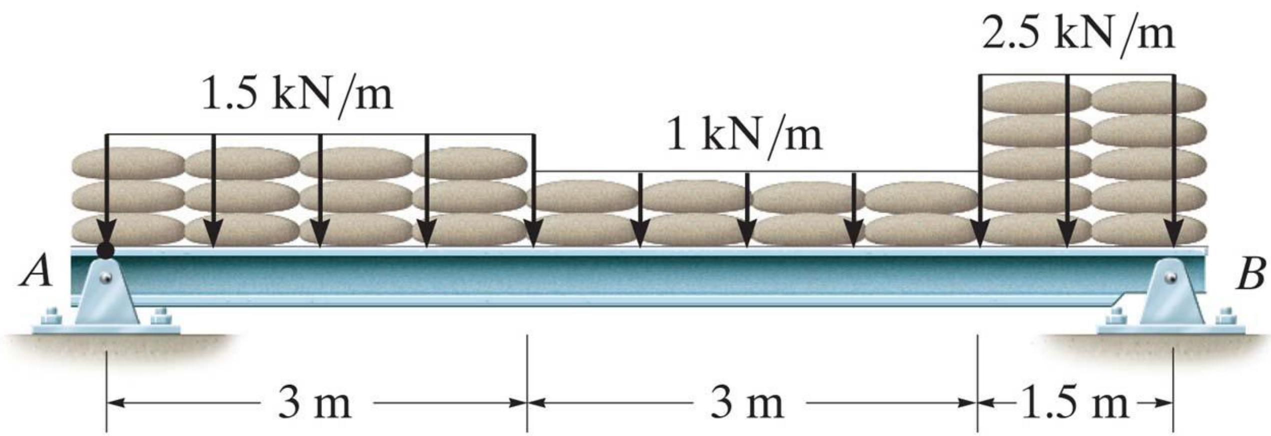
Units Used: $kN = 10^3\ N$

Given:

$w_1 = 1.5 \frac{kN}{m}$ $\quad a = 3\ m$

$w_2 = 1 \frac{kN}{m}$ $\quad b = 3\ m$

$w_3 = 2.5 \frac{kN}{m}$ $\quad c = 1.5\ m$



Solution:

$$F_R = w_1\, a + w_2\, b + w_3\, c$$

$$F_R = 11.25\ \text{kN}$$

$$M_A = w_1\, a\frac{a}{2} + w_2\, b\left(a + \frac{b}{2}\right) + w_3\, c\left(a + b + \frac{c}{2}\right)$$

$$M_A = 45.563\ \text{kN}\cdot\text{m}$$

$$d = \frac{M_A}{F_R}$$

$$d = 4.05\ \text{m}$$