# CARLETON UNIVERSITY

**FINAL EXAMINATION**
**MATH 1004**
Winter 2019
**SOLUTIONS**

**DURATION: 3 HOURS**

**Department Name and Course Number:** School of Mathematics and Statistics, MATH 1004 H
**Course Instructor(s):** Dr. A. B. Mingarelli

AUTHORIZED MEMORANDA
CALCULATORS PERMITTED, NO WIRELESS DEVICES

**In addition to the EXAMINATION PAPER students will require an ADDITIONAL ANSWER BOOK, and a SCANTRON SHEET.**

1. Please verify that you are in possession of a Scantron FORM.
2. Please **fill in your COURSE CODE** (e.g., MATH 1004) and **COURSE SECTION** (e.g., A, B, C, D, E, F), **YOUR NAME** and **YOUR STUDENT NUMBER** where required on the Scantron form.
3. **The examination consists of one sheet of legal size paper with text on BOTH sides.** It is out of a total of 100 and consists of 25 multiple choice questions each worth 4 marks. **Please fill in only one answer on your Scantron sheets with a pencil** as there is only one answer to any given question. Circling two or more answers to any question invalidates that question (*i.e.,* you get 0 marks for that question).
4. **This exam may be released to the Library and may be taken away by the student.**

**Return the SCANTRON form ONLY.** .

1 Find the slope of the tangent line to the curve $y = \ln(\ln x)$ at the point where $x = e$.

**(a)** $\frac{1}{e^2}$ **(b)** $\frac{1}{e}$ **(c)** $e$ **(d)** $-\frac{1}{e}$ **(e)** None of these

**Answer**: (b)

2 Let $f(x) = \alpha\,|x| + x$. What value of $\alpha$ will make $f$ differentiable at $x = 0$?

**(a)** $\alpha = 2$ **(b)** $\alpha = -1$ **(c)** $\alpha = 1$ **(d)** $\alpha = 0$ **(e)** None of these

**Answer**: (d)

3 Given that $y(x) = f(x)^x$ and $y(e) = 1, f(e) = 1$, $f'(e) = 2$, compute $y'(e)$.

**(a)** $2e$ **(b)** $e$ **(c)** 2 **(d)** 0 **(e)** None of these

**Answer**: (a)

4 Let $f'(0)$ exist and $f(0) = 0$. If $\lim_{x\to 0} \dfrac{\sin x}{f(x)} = 2$ and L'Hospital's Rule holds here, find $f'(0)$.

**(a)** 0 **(b)** 2 **(c)** $\frac{1}{2}$ **(d)** $\frac{3}{4}$ **(e)** None of these

**Answer**: (c)

5 Let $z = \operatorname{Arctan} f(x)$ where $f(0) = -1$ and $f'(0) = 6$. Evaluate $\dfrac{dz}{dx}$ at $x = 0$.

**Answer**: (d)

**(a)** 1/2 **(b)** 2 **(c)** 4 **(d)** 3 **(e)** None of these

6 Find the derivative of the function $y = (2x)^{2x}$. **Answer**: (c)

**(a)** $2 + \ln(4x^2)$ **(b)** $(2x)^{2x}\ln(4x^2)$ **(c)** $(2x)^{2x}\left(2 + \ln(4x^2)\right)$ **(d)** $2\,(2x)^{2x}$ **(e)** None of these

7. Compute $\lim_{t\to 0} \dfrac{1 - \cos(t^5)}{t^4}$.

**(a)** 1 **(b)** 0 **(c)** 2 **(d)** $\infty$ **(e)** None of these

**Answer**: (b)

8. Evaluate $\lim_{x\to+\infty} \dfrac{\sqrt{9x^4 + 2\sin x}}{3x^2 - 5}$.

**(a)** 1 **(b)** $\dfrac{1}{3}$ **(c)** 3 **(d)** 0 **(e)** None of these or the limit does not exist

**Answer**: (a)

9 Evaluate $\lim_{x\to 0^+} \frac{d}{dx} \int_1^{\sqrt{x}} \frac{\sin t^2}{t}\, dt$.

**(a)** 0 **(b)** $\frac{5}{2}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{4}$ **(e)** None of these **Answer**: (c)

10 Find an antiderivative of the function $2(x^2+x+2)e^{-2x}$.

**(a)** $-(2x^2-8x-18)\,e^{-2x}+3$ **(b)** $-(x^2+2x+3)\,e^{-2x}-1$ **(c)** $-\frac{1}{2}(x^2-x-1)e^{-2x}$ **(d)** $(2x^2-8x-18)\,e^{-2x}-\pi$ **(e)** $(x^2-6x+2)\,e^{-2x}-\pi$ **Answer**: (b)

11 Evaluate the integral $\int \tan^{-1}(2x)\, dx$

**(a)** $\frac{8x}{4x^2+1}+C$ **(b)** $\tan^{-1}(2x)-\ln(4x^2+1)+C$ **(c)** $x\tan^{-1}(2x)-\frac{1}{2}\ln(4x^2+1)+C$ **(d)** $x\tan^{-1}(2x)-\frac{1}{4}\ln(4x^2+1)+C$ **(e)** None of these

**Answer**: (d)

12 Find all the asymptotes of the function defined by $p(x)=\frac{2x+1}{(x-1)(x-2)}$.

(a) Vertical asymptotes at $x=-1,1,2$ only. No horizontal asymptotes. (b) Vertical asymptotes at $x=1$ and $x=2$. Horizontal asymptote given by $y=0$. (c) Vertical asymptotes at $x=-1$ and $x=1$ only. Horizontal asymptote given by $y=2$. (d) Vertical asymptotes at $x=1$ and $x=2$. Horizontal asymptote at $y=-2$.

**Answer**: (b)

13 Find that antiderivative of the function $\frac{1}{t^2+2t+2}$ whose value at $t=0$ is 1.

**(a)** $\tan^{-1}(t+1)+1-\frac{\pi}{4}$ **(b)** $-t^2+2\ln|1+t|+\frac{t}{2}+1$ **(c)** $\tan^{-1}(t-1)+\frac{\pi}{4}+1$ **(d)** $\frac{1}{2}\tan^{-1}(\frac{t}{2})+1$ **(e)** None of these

**Answer**: (a)

14 Which one of the following statements is true about the definite integral $\int_{-1}^{5} |x|\, dx$

**(a)** The area under the graph of $y=|x|$, above the $x$-axis, and between the lines $x=0$ and $x=5$ is equal to 25 **(b)** The area under the graph of $y=|x|$, above the $x$-axis, and between the lines $x=-1$ and $x=0$ is equal to 1 **(c)** Its value is equal to 13 **(d)** Its value is equal to 10 **(e)** None of these

**Answer**: (c)

15 Evaluate the integral $\int (\sin^3 x-\sin^5 x)\, dx$

**(a)** $\frac{1}{6}\sin^6 x+C$ **(b)** $\sin x-\frac{1}{3}\sin^3 x+\frac{1}{5}\sin^5 x+C$ **(c)** $\frac{1}{5}\sin^5 x+\frac{1}{3}\sin^3 x+C$ **(d)** $\frac{1}{5}\cos^5 x-\frac{1}{3}\cos^3 x+C$ **(e)** None of these

**Answer**: (d)

16 Evaluate the integral $\int \tan^3 x\, dx$

**(a)** $\frac{1}{4}\sec^4 x+C$ **(b)** $\frac{1}{2}\sec^2 x-\ln|\sec x|+C$ **(c)** $\frac{1}{2}\tan^2 x-\ln|\sec x|+C$ **(d)** $\frac{1}{4}\tan^4 x+C$ **(e)** $\frac{1}{4}\tan^2 x+\ln|\sec x|+C$

**Answer**: (c)

17 Evaluate the integral $\displaystyle\int_{-1}^{0} \frac{x^{-1/3}}{1+x^{2/3}}\,dx$

**(a)** $-\dfrac{3}{2}\ln 2$ **(b)** $-\ln 2$ **(c)** $\dfrac{2}{3}\ln 2$ **(d)** $-\infty$ **(e)** None of these

**Answer**: (a)

18 Find an expression for the area between the curves defined by $y=\sqrt{x}$ and $y=x^2$.

(a) $\displaystyle\int_0^2 (x^2-1)\,dx$ (b) $\displaystyle\int_0^1 \sqrt{x}\,dx$ (c) $\displaystyle\int_0^1 (\sqrt{x}-x^2)\,dx$ (d) $\displaystyle\int_1^0 \left(x-\sqrt{x}\right)\,dx$ (e) None of these

**Answer**: ( c)

19 Compute the volume of the solid of revolution obtained by rotating the region above the $x$-axis and bounded by $y=x^2$, for $x$ between 0 and 1, about the $y$-axis.

**(a)** $\pi/2$ **(b)** $3\pi/2$ **(c)** $2\pi$ **(d)** $\pi$

**Answer**: (a)

20 A function $f$ has an antiderivative $\mathcal{F}$ such that $\mathcal{F}(x)=\dfrac{(\ln x)^3}{3}-2$. Find $f(x)$.

**(a)** $\dfrac{(\ln x)^4}{2x}$ **(b)** $\dfrac{(\ln x)^4}{12}$ **(c)** $\dfrac{(\ln x)^2}{x}$ **(d)** $\dfrac{(\ln x)^2}{3x}$ **(e)** None of these

**Answer**: (c)

21 Let $f(x)=\sin|x|$. Calculate $\displaystyle\lim_{h\to 0}\frac{f(h)-f(0)}{h}$.

**(a)** 1 **(b)** $-1$ **(c)** 0 **(d)** 2 **(e)** None of these or the limit does not exist

**Answer**: (e)

22 A function $f$, with a differentiable inverse function $F$, is such that $f'(1)=1/3$, $f(1)=2$ and $f(2)=0$. Find $F'(2)$.

**(a)** 2 **(b)** 1 **(c)** $\dfrac{1}{3}$ **(d)** 3 **(e)** None of these

**Answer**: (d)

23 Evaluate $\displaystyle\int_0^1 xe^{-x}\,dx$

**(a)** 1 **(b)** $1-\dfrac{2}{e}$ **(c)** $e-1$ **(d)** $e$ **(e)** None of these

**Answer**: (b)

24 Evaluate the improper integral $\displaystyle\int_0^\infty x^5\,e^{-x}\,dx$.

**(a)** 120 **(b)** 24 **(c)** 6 **(d)** 2 **(e)** None of these

**Answer**: (a)

25 Let $u$ be given implicitly as a differentiable function of $v$ by $e^{uv}+u^2v=1$.

Calculate the value of the derivative $\dfrac{du}{dv}$ at the point $(u,v)$ where $u=0$, $v=1$:

**(a)** 1 **(b)** 2 **(c)** 0 **(d)** $-1$ **(e)** None of these

**Answer**: (c)

[Total: 100 marks]

END OF THE EXAMINATION.