Test 2 MATH 1004D, Fall, 2019

Last name: First name: Student no.:

This test has two parts, the first part has 4 multiple choices questions (3 marks each) and the second part has 4 long answers (which you show your works). Calculator is NOT allowed

A1: Let $f(x) = \dfrac{x^5 + x}{x^3 + 2x}$. What is $f'(1)$?

(a) $\dfrac{-30}{9}$ (b) $\dfrac{7}{3}$ (c) $\dfrac{3}{5}$ (d) $\dfrac{20}{9}$ (e) None.

Ans. $\dfrac{8}{3}$

A2: Given $f(5) = 3$, $f'(5) = 7$, $f'(3) = 6$. What is $(f^{-1})'(3)$?

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{5}$ (c) $\dfrac{1}{6}$ (d) $\dfrac{1}{7}$ (e) None.

$f^{-1}(3) = x \iff f(x) = 3$
$\Downarrow$
$x = 5$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(5)} = \frac{1}{7}$$

A3: What is $y' = \frac{dy}{dx}$, if $y = \sec x + \csc x$:

(a) $\tan x + \cot x$

(b) $\csc x \cot x + \csc x \cot x$

(c) $\csc x \cot x + \csc^2 x$

(d) $\sec x \tan x - \csc x \cot x$

(e) None.

A4: Let $f(x) = \tan^{-1}(\sqrt[3]{x})$, find $f'(8)$.

(a) $\dfrac{1}{4}$ (b) $\dfrac{4}{17}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{4}{5}$ (e) None.

$\boxed{\dfrac{1}{60}}$

B1-[4 marks]: Let $f(x) = x^2$. Use the definition of derivative, ONLY, to find $f'(3)$.

$$f'(2) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h \to 0} \frac{(3+h)^2 + 3^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{9} + 6h + h^2 - \cancel{9}}{h} = \lim_{h \to 0} (6 + h) = 6$$

OR: $$f'(3) = \lim_{x \to 3} \frac{f(x) - f(3)}{x - 3} = \lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \lim_{x \to 3} (x + 3) = 6$$

B2-[3+3 marks]: Find the derivative of: $A:\ y = \sin(\cos x)$, $B: y = \cos^{-1}(\sqrt{1-x^2}),\ x \le 0$

A: $y' = u' \cos u$, $u = \cos x \implies y' = -\sin x \cos(\cos x)$

B: $y' = -\dfrac{u'}{\sqrt{1-u^2}} = -\dfrac{\dfrac{-2x}{2\sqrt{1-x^2}}}{\sqrt{1-(\sqrt{1-x^2})^2}} = \dfrac{\dfrac{-x}{\sqrt{1-x^2}}}{\sqrt{x^2}}$, $\sqrt{x^2} = |x|$

$$= \frac{-x}{|x|\sqrt{1-x^2}}$$

B3-[4 marks]: Find $\lim\limits_{x\to 0} \dfrac{\sin^{-1}(2x)}{\tan^{-1} x}$

$$= \lim_{x\to 0} \frac{(\sin^{-1} 2x)'}{(\tan^{-1} x)'} = \lim_{x\to 0} \frac{\dfrac{2}{\sqrt{1-x^2}}}{\dfrac{1}{1+x^2}}$$

$$= \frac{2}{1} = 2$$

B4-[4 marks]: Let $\sin x + \cos y = 2x$. What is $\dfrac{dy}{dx}$ at $(0, \frac{\pi}{6})$? (Hint: use implicit differentiation). (Hint: $\cos(\pi/6) = \sqrt{3}/2$, $\sin(\pi/6) = 1/2$)

$\dfrac{d}{dx}$: $\cos x - y' \sin y = 2 \implies y' = \dfrac{\cos x - 2}{\sin y}\Big|_{(0, \frac{\pi}{6})} = \dfrac{1-2}{\frac{1}{2}} = -2$

OR:

$F = \sin x + \cos y - 2x$

$\dfrac{dy}{dx} = -\dfrac{F_x}{F_y} = -\dfrac{\cos x - 2}{-\sin y} = \dfrac{\cos x - 2}{\sin y}\Big|_{(0, \frac{\pi}{6})} = \dfrac{1-2}{\frac{1}{2}} = -2$

Scanned by CamScanner