GENERAL KNOWLEDGE TEST 1 MATH 1004 E, Fall, 2019

Last name: First name: Student no.:

This test has two parts, the first part has 5 multiple choice questions (3 marks each) and the second part has 3 long answer questions each 5. **Calculator is NOT allowed**

A1) The $\lim_{x\to+\infty}\left(\frac{\sqrt{x^2+x-1}}{8x^2-3}\right)^{\frac{1}{3}}$

(a) $\frac{1}{8}$

(b) $\frac{1}{2}$

(c) $\sqrt{2}$

(d) ∞

(e) None.

Ans (e)

A2) The $\lim_{x\to0^+}\frac{\tan 3x}{\sin 8x}$ is:

(a) $\frac{8}{3}$ (b) $\frac{3}{8}$ (c) 0 (d) $+\infty$ (e) None.

Ans (b)

A3) The $\lim_{x\to3^+}\frac{[x]-3}{x-3}$ where $[x]$ is the bracket (floor) function.

(a) 0 (b) ∞ (c) $+\infty$ (d) 3 (e) None.

Ans (a)

A4) The $\lim_{x\to3^-}\frac{|x-3|[x]}{3-x}$, (where $[x]$ is the bracket (floor) function) is:

(a) ∞

(b) 0

(c) 2

(d) -2

(e) Doesn't exist.

Ans (c)

A5) The $\lim_{x\to4^-}\frac{x-4}{x^2-16}$ is:

(a) 0 (b) $-\infty$ (c) $+\infty$ (d) $\frac{1}{8}$ (e) None.

Ans (d)

Part 2: long answer questions

B1)Evaluate the following limits, justify your answers:

B1) $\lim_{x\to+\infty}(\sqrt{x^2+3x+3}-x)$

$$\lim_{x\to+\infty}(\sqrt{x^2+3x+3}-x)=\lim_{x\to+\infty}(\sqrt{x^2+3x+3}-x)\times\frac{\sqrt{x^2+3x+3}+x}{\sqrt{x^2+3x+3}+x}=\lim_{x\to+\infty}\frac{x^2+3x+3-x^2}{\sqrt{x^2+3x+3}+x}=$$
$$\lim_{x\to+\infty}\frac{3x+3}{\sqrt{x^2+3x+3}+x}=\lim_{x\to+\infty}\frac{3x+3}{|x|\sqrt{1+\frac{3}{x}+\frac{3}{x^2}}+x}=\lim_{x\to+\infty}\frac{x(3+\frac{3}{x})}{x\sqrt{1+\frac{3}{x}+\frac{3}{x^2}}+x}=\lim_{x\to+\infty}\frac{(3+\frac{3}{x})}{\sqrt{1+\frac{3}{x}+\frac{3}{x^2}}+1}=$$
$$\frac{(3+0)}{\sqrt{1+0+0}+1}=\frac{3}{2}.$$

B2) $\lim_{x\to0}3x^2\sin\left(\frac{5}{x}\right)$

Since $-1\le\sin\left(\frac{5}{x}\right)\le1$, we have $\lim_{x\to0}3x^2\sin\left(\frac{5}{x}\right)=0$.

B3) Find value(s) of the constant c that makes the function $f(x)$ to be continuous everywhere:

$$f(x)=\begin{cases}\frac{\sin x}{x}+2c\ , & x<0\\ 3x-c-2\ , & 0\le x.\end{cases}$$

$\lim_{x\to0^-}f(x)=\lim_{x\to0^+}f(x)$.

$\lim_{x\to0}(\frac{\sin x}{x}+2c)=\lim_{x\to0}(3x-c-2)\Rightarrow1+2c=0-c-2\Rightarrow c=-1$