Lecture 2

MATH1119 Section B

September 12, 2012

On last class:

- Matrix
- Dimension of a matrix
- (i, j)-entry of a matrix
- Nonzero row of a matrix
- Nonzero column of a matrix
- Leading entry of a row

Echelon form matrix (or Row echelon form matrix)

An $m \times n$ matrix A is said to be an echelon form matrix if it has the following properties:

1- Every row with only zeros is below every nonzero rows.

2- Suppose rows $1, 2, \ldots, k$ (where $k \leq m$) are the nonzero rows of the matrix.
Let $c_i =$ the column number where the leading entry of the i^{th} row is.
Then we have $\quad c_1 < c_2 < \cdots < c_k$.

If one of the two properties fail then the matrix A is not an echelon form matrix.

Reduced echelon form matrix (or Row reduced echelon form matrix)

An m x n matrix is said to be a reduced echelon form matrix if it **is an echelon form matrix** plus the following 2 condiditions:

1- The leading entry in each nonzero row is 1.

2- Each leading 1 is the only nonzero entry in its column. That is, every entry above and below a leading 1 is a 0.

Three elementary row operations (Very important)

We define three operations that we can perform on the rows of a matrix so that we can obtain another matrix.

It is crucial for you to master these operations.

Interchange (swap) the i^{th} row of A and the j^{th} row of A.

(written $R_i \longleftrightarrow R_j$)

2 - Multiplication of all the entries of a row by a number k

We multiply all the entries of a particular row (say the i^{th} row) and multiply each of its entries by k.

(written $R_i \longleftrightarrow kR_i$)

Addition of a multiple of a row to a given row

Replace the i^{th} row by the sum of the i^{th} row and a number k times the j^{th} row.

(written $R_i = R_i + kR_j$, where $i \neq j$)

Linear equation of n variables

An equation of the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

where $x_1, x_2, \ldots, x_n$ are n variables and $a_1, a_2, \ldots, a_n, b$ are numbers.

Solutions of a linear system

$(b_1, b_2, \ldots, b_n)$ is a solution of a linear system

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b$$

if

$$a_1b_1 + a_2b_2 + \cdots + a_nb_n = b$$

System of linear equations

A system of linear equations is a set of m linear equations each with the same n variables. It if of the form

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\,\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m, \end{aligned}$$

where $x_1, x_2, \ldots, x_n$ are n variables and a_{ij} is a number for any integers i, j such that $1 \leq i \leq m$ and $1 \leq j \leq n$.

Solution of a System of linear equations

$(c_1, c_2, \ldots, c_n)$ is a solution of the linear system of m equations and n variables

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
&\cdot\ \cdot \\
&\cdot\ \cdot \\
&\cdot\ \cdot \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m
\end{aligned}$$

if $(c_1, c_2, \ldots, c_n)$ is a solution for <u>each</u> of the m linear equations.

Our goal

To be able to find the solution(s), if any, of any given linear system of m equations and n variables.