CVG 3116 Tutoria 4 - Solutions

Q1.

$Q = 30\,L/s = 0.03\ m^3/s$

$H_s = 20\ m$

$L = 100\ m$, $D = 15\ cm = 0.15\ m$

$k_s = 0.12\ mm = 0.12 \times 10^{-3}\ m$

Local losses: Ball check valve ($k_L = 70$), entrance ($k_L = 0.5$) and exit ($k_L = 1.0$)

$\nu = 1.0 \times 10^{-6}\ m^2/s$

$Q, H_p, \eta = ?$

Solution:

To deliver the required discharge, each pump must provide a total energy head of:

$$H_p = H_s + \left(\frac{\lambda L}{D} + \Sigma k_L\right)\frac{V^2}{2g}$$

$$\frac{k_s}{D} = \frac{0.12 \times 10^{-3}}{0.15} = 0.0008$$

$$V = \frac{Q}{A} = \frac{0.03}{\left(\frac{\pi \times 0.15^2}{4}\right)} = 1.698\ m/s$$

$$Re = \frac{DV}{\nu} = \frac{0.15 \times 1.698}{1.0 \times 10^{-6}} = 2.55 \times 10^5$$

From Moody diagram: $\lambda = 0.0204$

$$\Sigma k_L = 70 + 0.5 + 1.0 = 71.5$$

$$H_p = 20 + \left(\frac{0.0204 \times 100}{0.15} + 71.5\right) \times \frac{1.698^2}{2 \times 9.81} = \underline{32.50\ m}$$

From Figure 5.23, with $Q = 30\,L/s$ and $H_p = 32.50\ m$ try Pump I or Pump II.

Obtain more points on the system curve (Q vs H_{SH}) by assuming different values of Q and calculati H_{SH}. Plot Q vs H_{SH} (system curve) on Figure 5.24, Pumps I and II.

The two selected pumps will have the following operating conditions:

Pump I: $n = 3850$ rpm, $Q = 30.5$ L/s, $H_p = 33$ m, $\eta = 41\%$
$P_{in} = 37$ Hp

Pump II: $n = 3550$ rpm, $Q = 30$ L/s, $H_p = 34$ m, $\eta = 50\%$
$P_{in} = 28$ Hp.

Pump II would be the better choice as it can supply the required head & discharge at a greater efficiency and lower power input requirements.

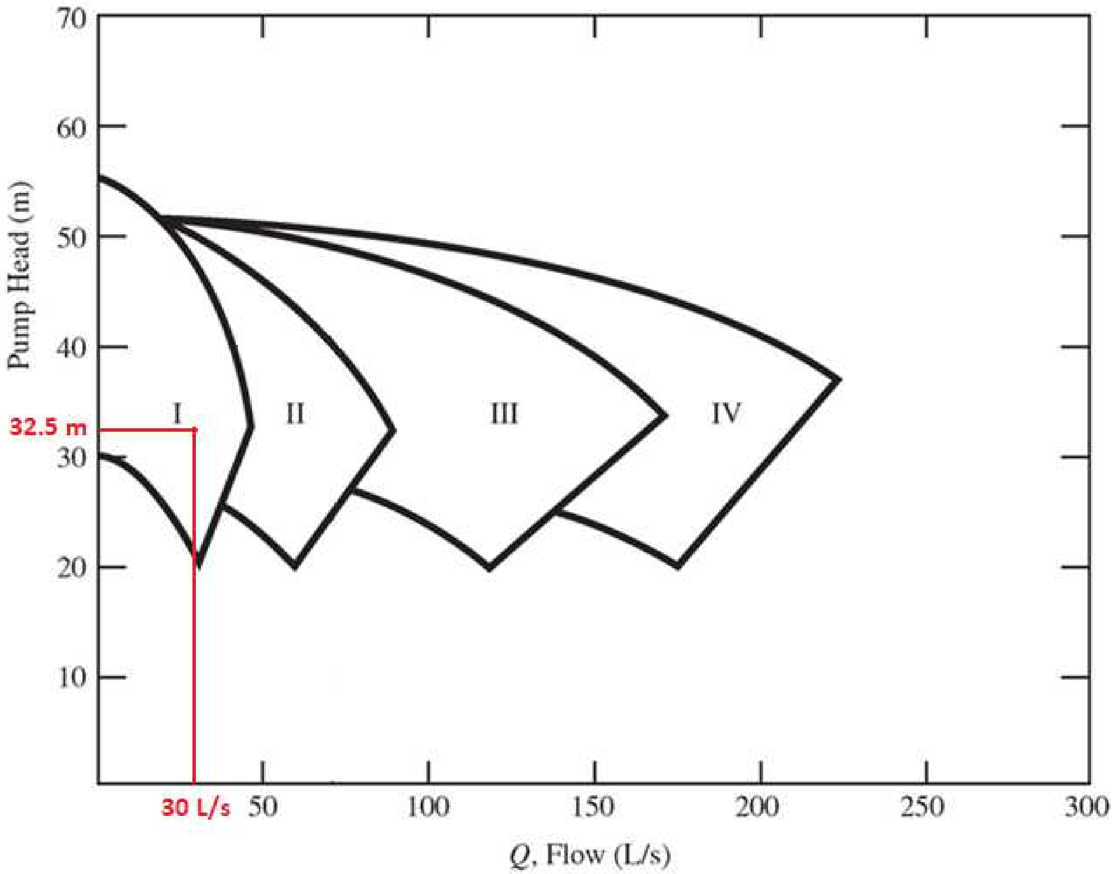
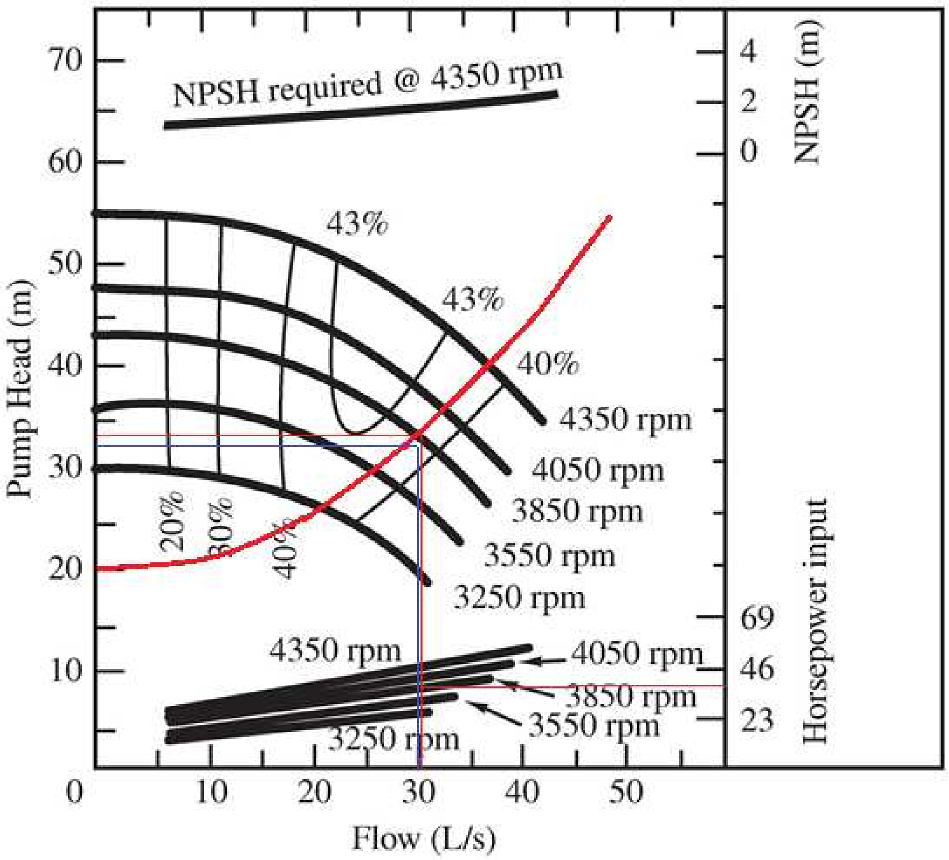
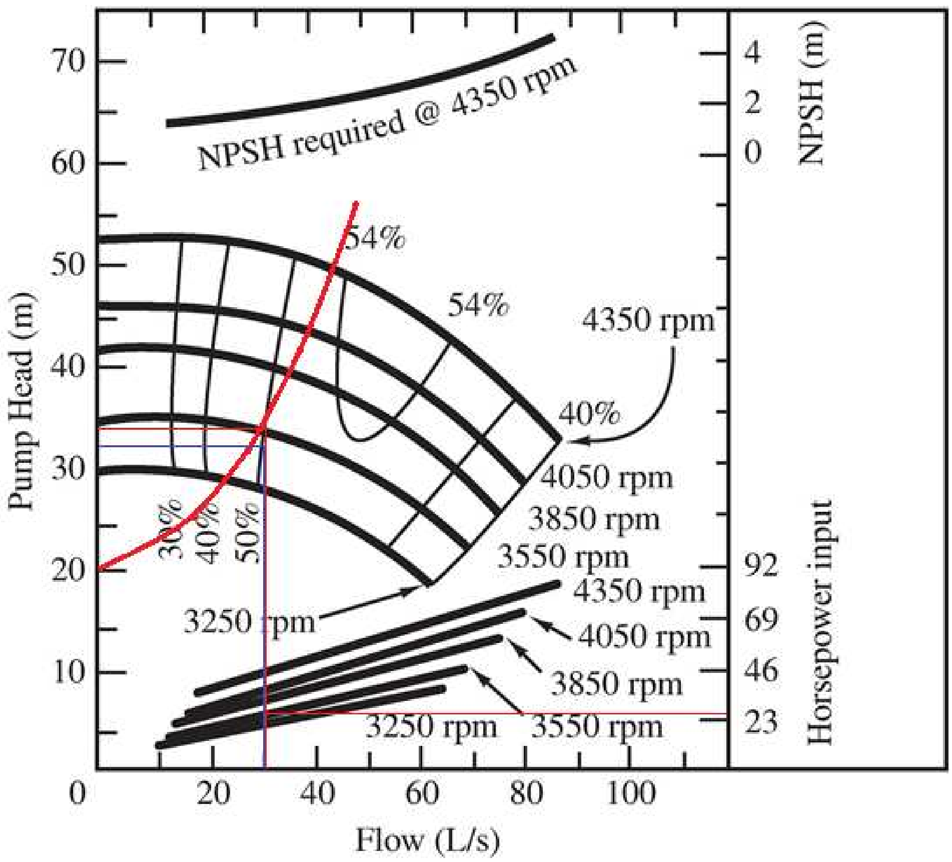


Figure 5.23 Pump model selection chart (Houghtalen et al., 2010)

Pump, I



Pump, II



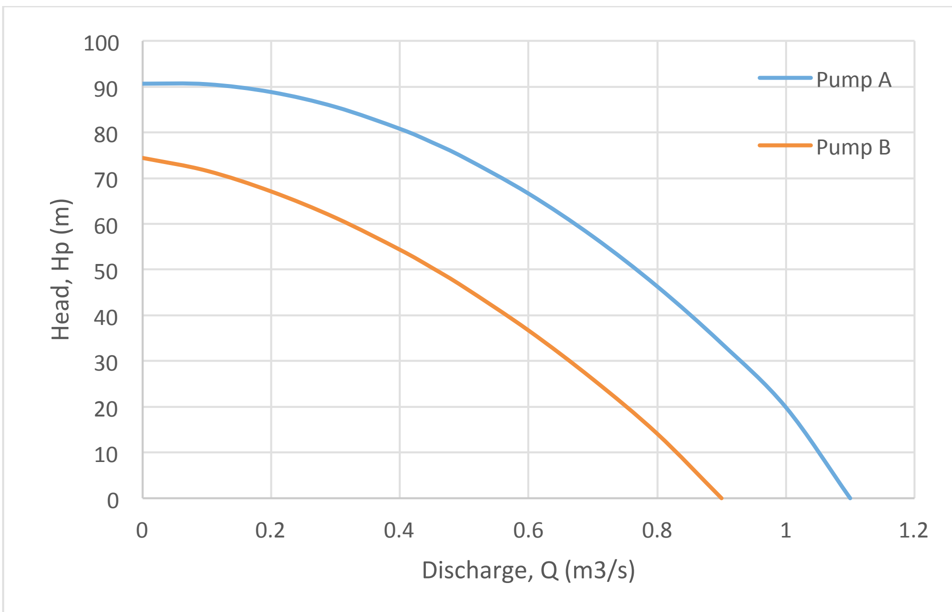
Question 2

Given:

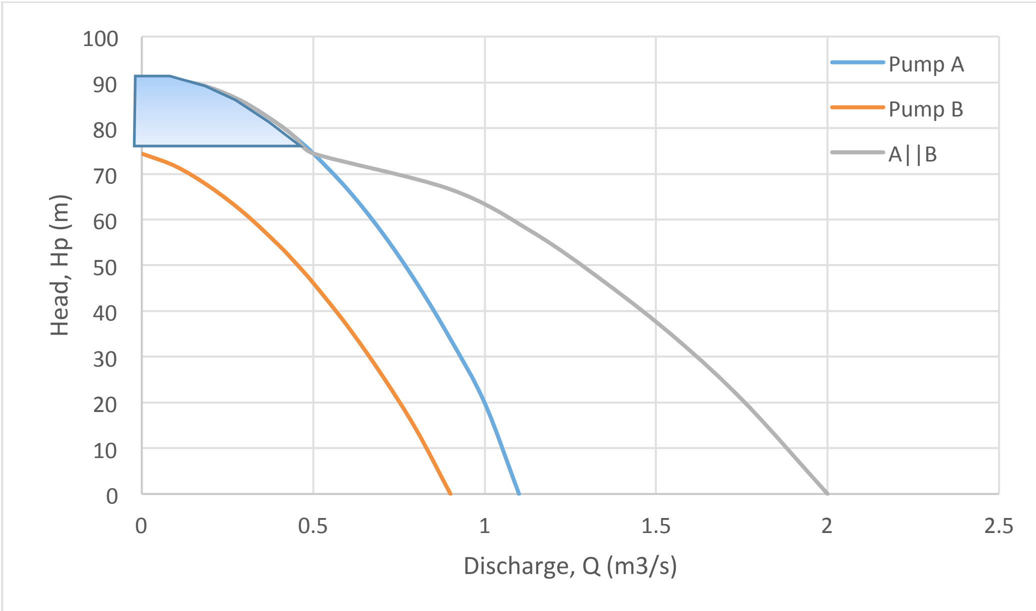
$D = 0.4\ m$
$L = 1000\ m$
$h_L = 10\frac{v^2}{2g}$
$\lambda = 0.02$
$\Delta z = 60\ m\ \equiv\ z_{R2} - z_{R1}$

a) Plot the combined pump curve ($H_p vs\ Q$)

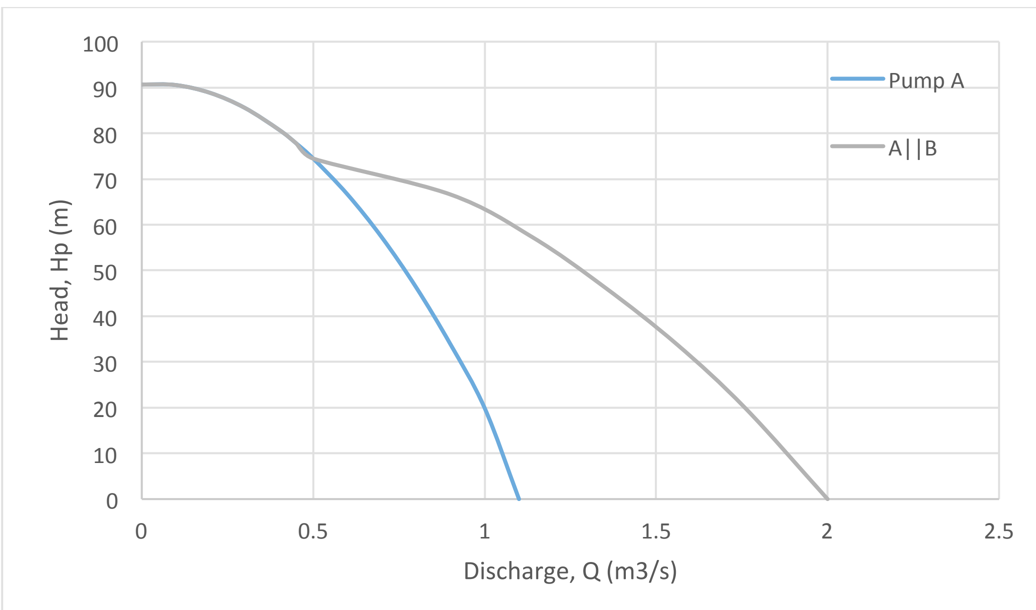
The individual pump performance curves can be generated using the pumps` data given in the tables.



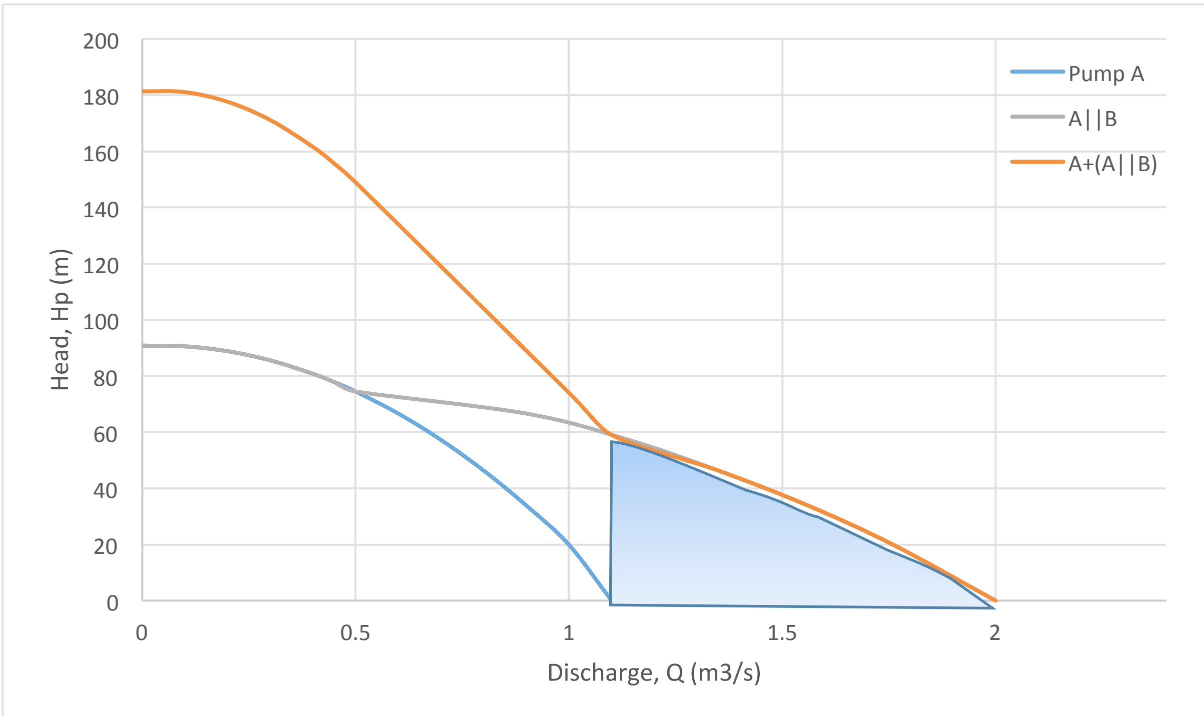
To generate a higher discharge, two of the three pumps are operated in parallel. Thus, the discharges of the individual pumps are summed in order to produce the performance curve for Pumps A and B in parallel; however, in the shaded region, pump B does not work, as its head is lower. Therefore, in this region, the performance curve for pumps A and B working in parallel is the same as for pump A. Afterwards, both discharges are considered.



Performance curve for pumps A and B combined in parallel

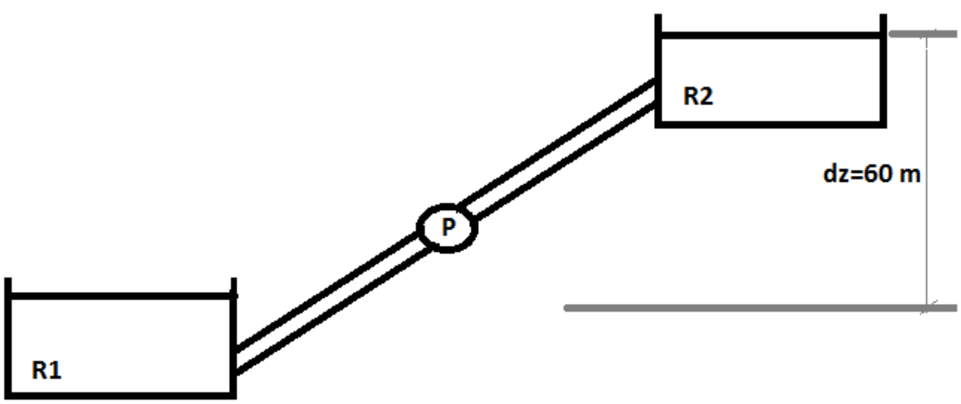


To generate higher head, pump A is connected in series to both pumps A and B. In this case, the heads of the pumps in series must be summed. However, in the shaded region, pump A does not work as its discharge is lower.



b) Plot the system curve ($H_p vs\ Q$)

First, write the energy equation between the two reservoirs and calculate Q for different H_p values.



Bernoulli Eq. from R_1 to R_2:

$$\frac{P_1}{\rho g} + z_1 + \frac{{V_1}^2}{2g} + H_p = \frac{P_2}{\rho g} + z_2 + \frac{{V_2}^2}{2g} + h_L + h_f$$

$$\Delta z + H_p = h_L + h_f = 10\frac{V^2}{2g} + \frac{\lambda L V^2}{2gD}$$

$$-60 + H_p = 10\frac{V^2}{2g} + \frac{0.02\times1000}{0.4}\frac{V^2}{2g} = 60\frac{V^2}{2g} = 3.1\,V^2$$

$$V = \frac{Q}{A} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{Q}{\frac{\pi(0.4)^2}{4}} \rightarrow V = 7.96\,Q$$

Therefore,

$$-60 + H_p = 3.1\,(7.96\,Q)^2$$

$$-60 + H_p = 196.5\,Q^2$$

$$H_p = 196.5\,Q^2 + 60$$

$$H_p = 60 \rightarrow Q = 0\,m^3/s$$

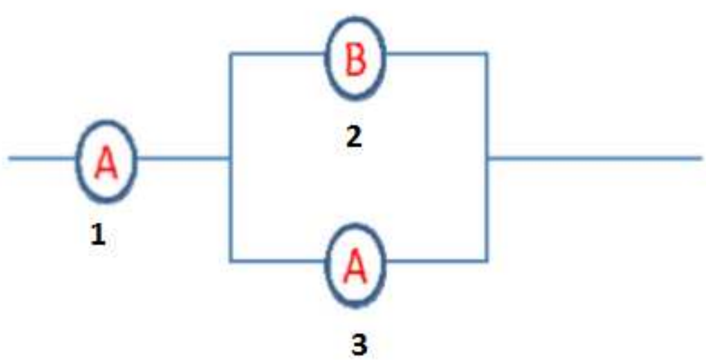
$$H_p = 80 \rightarrow Q = 0.32\,m^3/s$$

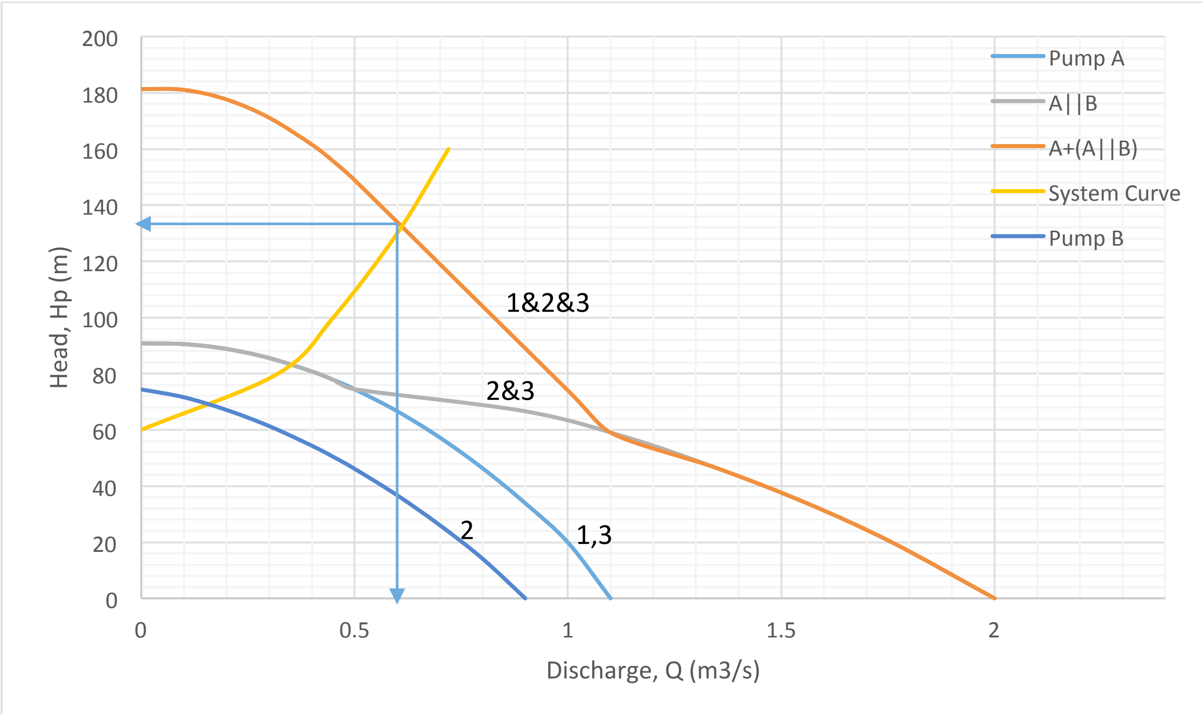
$$H_p = 100 \rightarrow Q = 0.45\,m^3/s$$

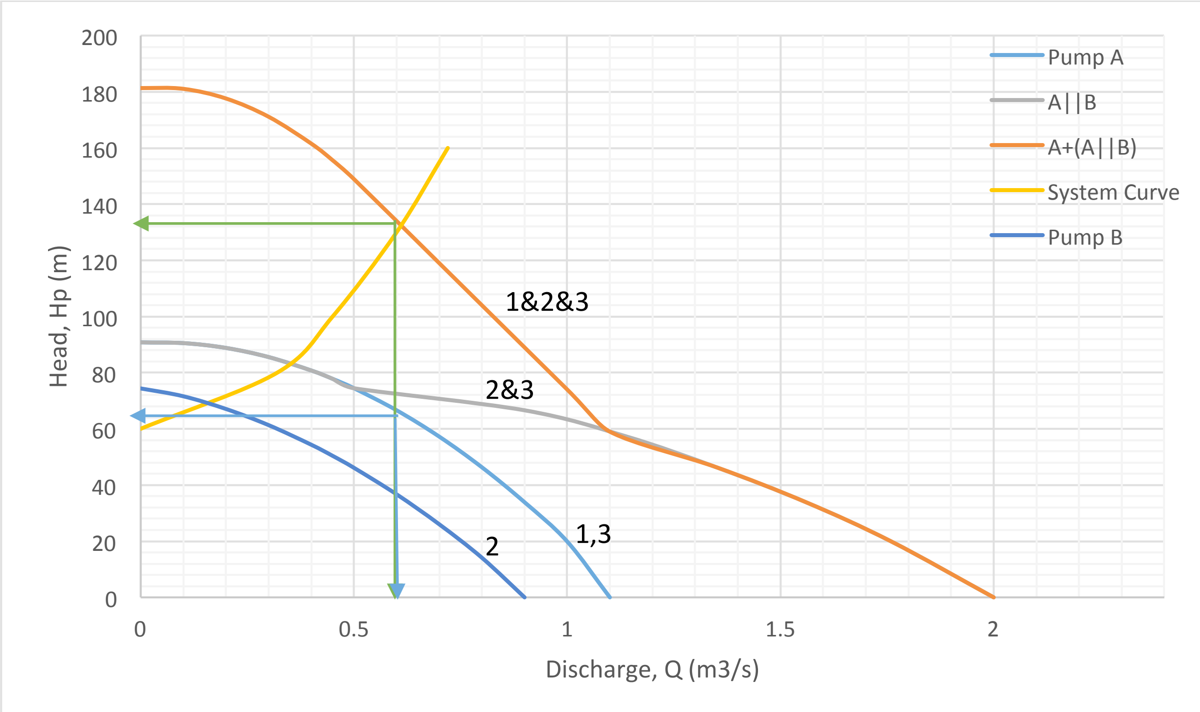
$$H_p = 130 \rightarrow Q = 0.60\,m^3/s$$

$$H_p = 160 \rightarrow Q = 0.72\,m^3/s$$

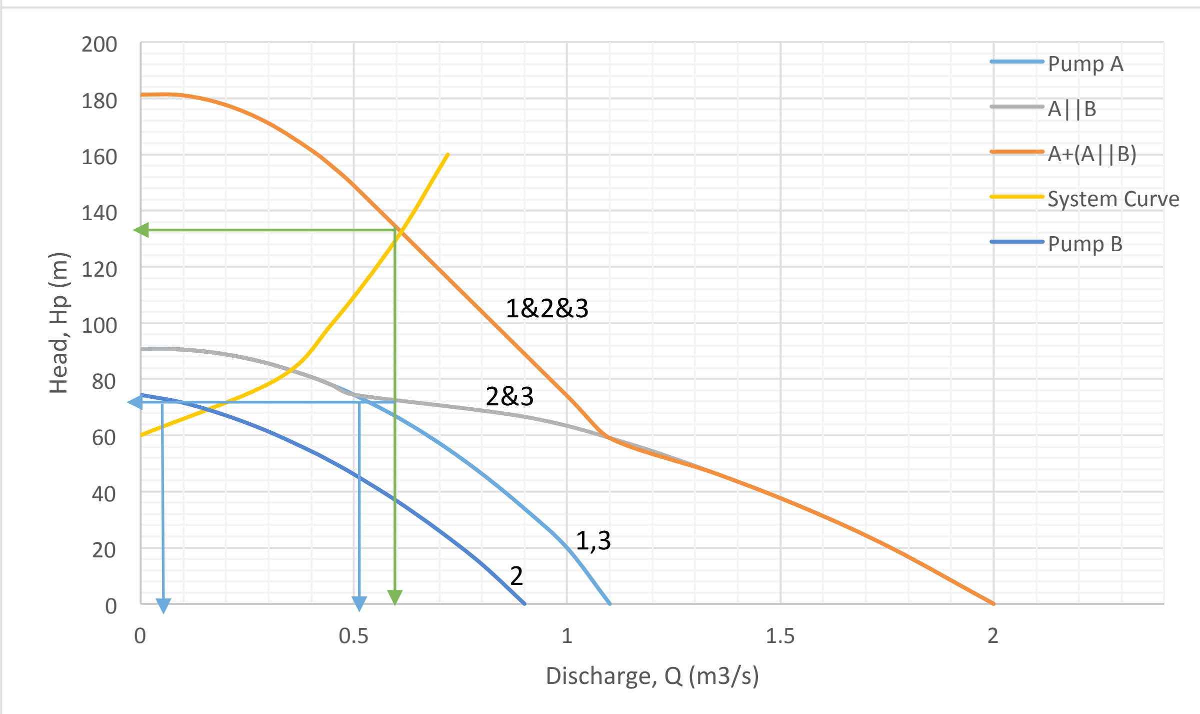
c) Find the discharge and the head in the system and define the operating conditions for each of the three pumps.




200
180
160
140
120
100
80
60
40
20
0
Head, Hp (m)
0
0.5
1
1.5
2
Discharge, Q (m3/s)
Pump A
A||B
A+(A||B)
System Curve
Pump B
1&2&3
2&3
2
1,3



200
180
160
140
120
100
80
60
40
20
0
Head, Hp (m)
0
0.5
1
1.5
2
Discharge, Q (m3/s)
Pump A
A||B
A+(A||B)
System Curve
Pump B
1&2&3
2&3
1,3
2



200
180
160
140
120
100
80
60
40
20
0
Head, Hp (m)
0
0.5
1
1.5
2
Discharge, Q (m3/s)
Pump A
A||B
A+(A||B)
System Curve
Pump B
1&2&3
2&3
1,3
2

$$Duty\ point\ of\ the\ system\ \begin{cases} H_{pt} = 134\ m \\ Q_t = 0.61\ m^3/s \end{cases}$$

$$Pump\ 1\ \begin{cases} H_{p1} = 64\ m \\ Q_1 = 0.61\ m^3/s \\ \eta_1 = 69\ \% \end{cases}$$

$$Pump\ 2\ \begin{cases} H_{p2} = 70\ m \\ Q_{t2} = 0.08\ m^3/s \\ \eta_2 = 15\ \% \end{cases}$$

$$Pump\ 3\ \begin{cases} H_{p3} = 70\ m \\ Q_{t3} = 0.53\ m^3/s \\ \eta_3 = 66\ \% \end{cases}$$

Note:

$Q_1 = Q_2 + Q_3 = Q_t = 0.61\ m^3/s$

$H_{p2} = H_{p3}$

$H_{pt} = H_{p1} + H_{p2} = H_{p1} + H_{p3} \cong 134\ m$