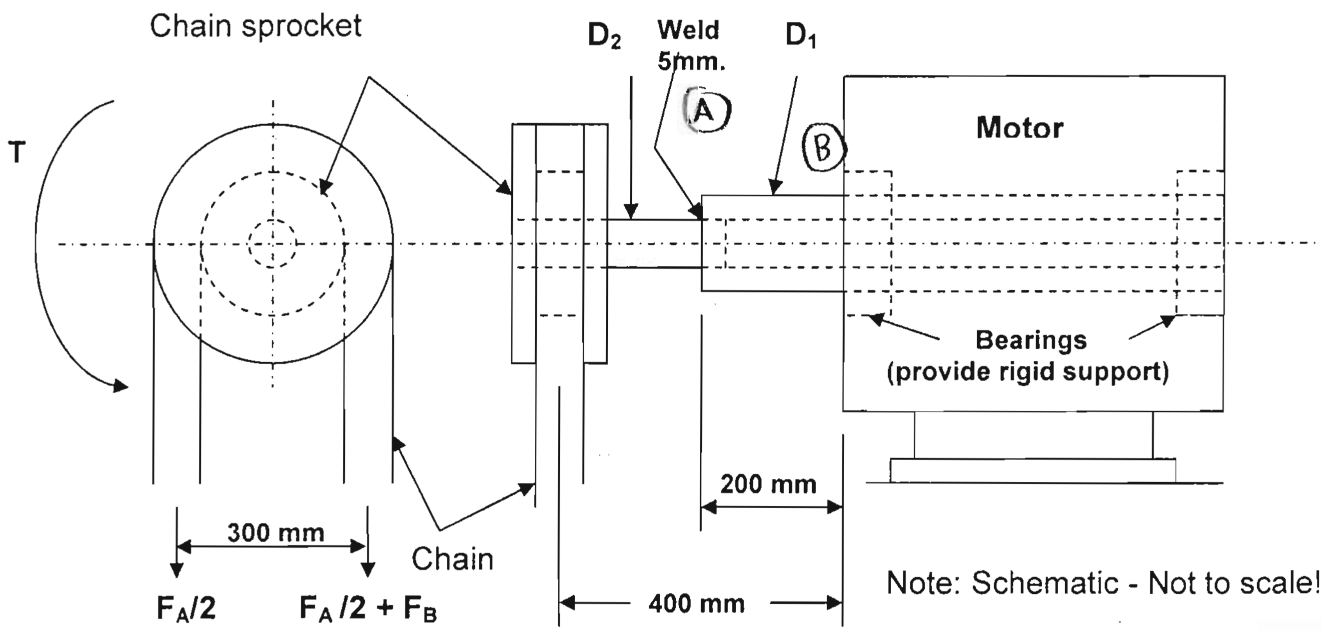
Name: Solutions Student #: ______________

Mech 326 Midterm
November 2, 2012

Instructions: Answer all questions. Indicate units where applicable and draw a box around your answers. Show all work. Open Book and Notes. Total marks = 50 Points.
No communication devices allowed.

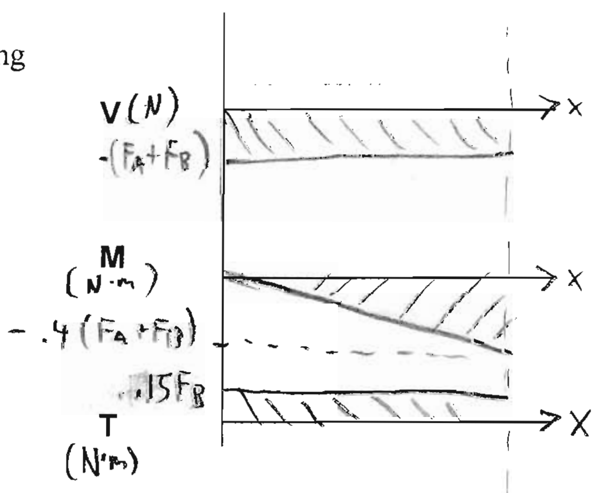
An electric motor drives a chain sprocket and chain which is used to power a wood cutting saw. The motor shaft (supported by internal bearings) is actually a tube welded to a solid shaft. The shaft is made by inserting a solid shaft (**D_2 = 50 mm**) into a hollow tube (outside dia., **D_1 = 100 mm** and inside dia., **D_2 = 50 mm**) and welding the two shafts together. A chain pretensioner is used to take the slack out of the chain which, in turn, applies a load of F_A downward on the chain as shown.
During operation, the motor generates an operating force of $\mathbf{F_B}$ (provided by the motor torque **T**) to drive the cutting blade mechanism (not shown).
The motor shaft is made from **AISI 1018 (HR)** steel and has been machined after welding leaving a **5 mm. radius** weld at the shaft reduction.

Note: *Shaft $J = 2I = 613{,}592\ mm^4$ (at solid D_2 OD) & $J = 9{,}203{,}877\ mm^4$ (at tube D_1 OD & D_2 ID))*



a) (5 pts) Use this space to show the shape of the loading diagrams (i.e. V, M, T) and (please label clearly).

(3 pts)



(2pt)

$V = -F_B + (-F_A/2 + -F_A/2)$
$= -F_B - F_A$ (N)

$M_{max} = -(F_A + F_B)(400\ mm)$
$= -0.4(F_A + F_B)$ N·m

$\circlearrowleft + \quad 0 = T + (F_A/2)(.15) - (F_A/2)(.15) - F_B(.15)$
$\therefore T = .15 F_B$

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b) (20 pts): A maximum torque of **T = 1000 N-m** can be supplied by the motor to drive the chain. If the cutting blade is jammed (no rotation) and the motor is producing its maximum torque, determine the factor of safety n_y to prevent yielding of the shaft based on the **Von Mises** (max distortion energy) criteria. Assume the pretension load $F_A = 0$.

Be sure to complete the following tasks:

5pt a) Determine the critical point where you would expect failure would first occur (be specific) and label this point as on the diagram above.

8pt b) Determine the principal stresses and sketch on Mohr Circle.

7pt c) Determine if failure has occurred and calculate factors of safety n_y for static loading.

a) Max Torque (or Stall Torque) = 1000 N-m

so $T = 0.15(F_B)$ [from Part a)]

1pt $\Rightarrow F_B = 1000/.15 = 6666.67$ N

Critical Point is either i) at weld (Point A) or ii) point where M is maximum (point B)

i) The weld joint is subject to a normal bending stress and shear stress. To be conservative, use Stress Concentration Factor for calculation

1pt Fig. A-15-8 $r/d = 5/50 = 1/10 = 0.1$ $D/d = 100/50 = 2 \Rightarrow K_{Ts} = \sim 1.45$

Fig A-15-9 $r/d = 0.1$ $D/d = 2 \Rightarrow K_T = \sim 1.7$

2pt Nominal shear stress $\tau_{xy} = \dfrac{Tr}{J} = \dfrac{(1000 N\text{-}m)(1000\ mm/m)(d/2 = 50/2\ mm)}{J = 613{,}592\ mm^4} = 40.74 MPa$

Using K_{Ts} $\tau_{max} = K_{Ts}\tau_{nom} = (1.45)(40.74) = 59.08\ MPa = \tau_A$

Nominal bending stress $(x = 200\ mm)$ $\sigma_x = \dfrac{My}{I} \Rightarrow M_{@x=200} = 200(F_A + F_B)$ $F_A = 0$

$F_B = 6666.67 N$

2pt $\sigma_x = \dfrac{200mm(6666.67N)(d/2 = 50/2\ mm)}{I = \frac{1}{2}J = \frac{1}{2}(613592\ mm^4)} = 108.65\ MPa$

Using K_T $\sigma_{max\,A} = K_T\sigma_{nom} = 1.7(108.65) = 184.7\ MPa = \sigma_A$

ii) Point B - Max. value of M. occurs at x = 400mm.

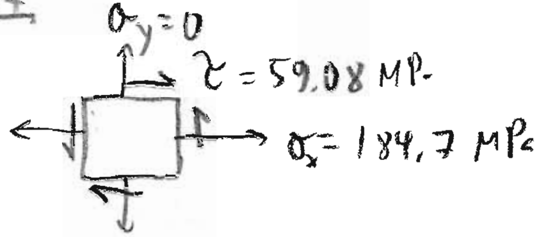
1pt $\tau_B = \tau_{xy} = \dfrac{Tr}{J} = \dfrac{(1000 N\text{-}m)(1000\ mm/m)(D/2 = 100/2\ mm)}{J = 9\,203\,877} = 5.43\ MPa$

$\sigma_B = \dfrac{My}{I} = \dfrac{(400\,mm)(6666.67\,N)(D/2 = 100/2)}{I = \frac{1}{2}J = \frac{1}{2}(9{,}203{,}877)} = 28.97\ MPa$

a) cont,

(2pt) since $\sigma_{(A)} > \sigma_{(B)}$ & $\tau_{(A)} > \tau_{(B)}$, we have the weld point (A) as the critical point

b) Principal stresses σ_1, σ_2 at (A)



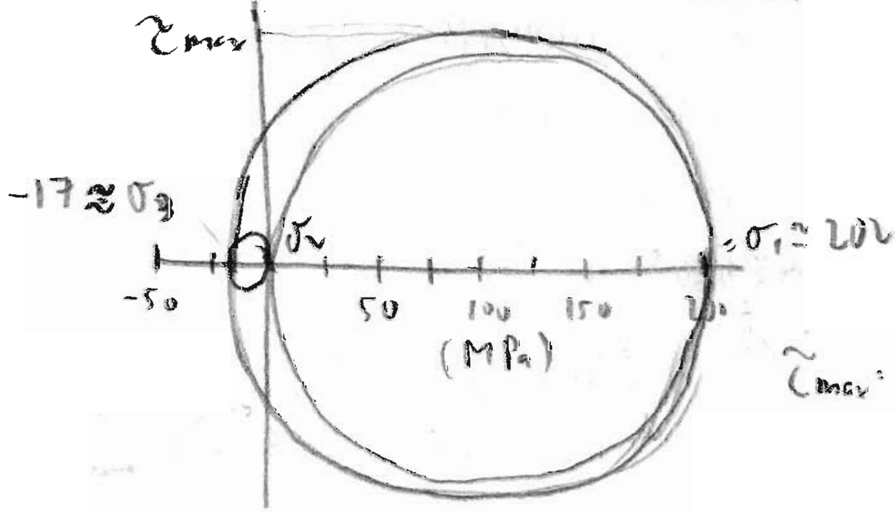
(2pt) $$\sigma_1, \sigma_3 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \frac{184.7}{2} \pm \sqrt{\left(\frac{184.7}{2}\right)^2 + (59.08)^2} = 92.35 \pm 109.67$$

So $\sigma_1 = 92.35 + 109.67 = \boxed{202.0 \text{ MPa} = \sigma_1}$

$\sigma_3 = 92.35 - 109.67 = \boxed{-17.28 \text{ MPa} = \sigma_3, \sigma_2 = 0}$

(3pt)



Note $\sigma_1 = 202.0$
$\sigma_2 = 0$
$\sigma_3 = -17.28$

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 109.64$ MPa

c) Using Von Mises Criteria and Compare with S_y

(1pt) For AISI 1018 (HR) $S_y = 220$ MPa [Table A-20]

Since we have plane stress condition

$$\sigma' = (\sigma_1^2 - \sigma_1\sigma_3 + \sigma_3^2)^{1/2} \quad \text{[Eqn 5-13]}$$

(2pt) $$\sigma' = ((202.0)^2 - (202.0)(-17.28) + (-17.28)^2)^{1/2}$$

$$\sigma' = 211.2 \text{ MPa}$$

Von Mises Criteria $\sigma' = S_y/n \Rightarrow n = S_y/\sigma'$

(2pt) $n = 220/211.2 = 1.04 \approx \boxed{n = 1.0}$

* Very close to yield point *

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Note: One might argue that AISI 1018 is ductile and so in this case we can ignore K_{Ts} & $K_{T\sigma}$

In this case we get

$\tau_{A} = 40.74$ MPa $\quad \sigma_{A} = 108.65$

so $\sigma_1, \sigma_3 = \dfrac{108.65}{2} \pm \sqrt{\left(\dfrac{108.65}{2}\right)^2 + (40.74)^2}$

$= 54.3 \pm 67.9$

$\sigma_1 = 122.2$

$\sigma_2 = 0$

$\sigma_3 = -13.6$

$\sigma' = (\sigma_1^2 - \sigma_1 \sigma_3 + \sigma_3^2)^{\frac{1}{2}} = \left(122.2^2 - (122.2)(-13.6) + (-13.6)^2\right)^{\frac{1}{2}}$

$\sigma' = 129.5$ MPa

$n = S_y/\sigma' = 220/129.5 \quad n \approx 1.7$

Note: The Shigley text discusses the fact that for ductile materials one does not apply K_t since we get local yielding at the notch tip (plastic deformation). This idea is correct but can be considered poor design practice. One should include K_t when studying elements such as welds to see if the plastic limits are reached at these points.

Note: No marks are deducted for choosing to not include K_t values.

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(25 pts) Fatigue analysis: A fatigue assessment using a Modified Goodman approach is to be performed for the shaft at the weld toe (i.e. where the weld meets the smaller diameter shaft) not for the weld itself.

During cutting operation, a constant motor torque of **T = 500 N-m** is required to cut the wood. The design parameters also required a **shaft operating temperature range of 20 °C to 50°C and a reliability level of 95%**. Assume that the chain pretension is correctly applied and thus **F_A = 0 N** for this analysis.
(See original problem statement for more information)

10 a) Determine the modified endurance limit for the shaft. Be sure to state all assumptions.

10 b) Calculate the factor of safety based on the Modified Goodman approach.

c) The fatigue assessment must be adjusted as it has been found that the chain pretensioner was improperly adjusted and adds a measured force **F_A = 500N** to the existing design loads. Determine how this will change the safety factor, **n_f**. Illustrate this analysis on the Modified Goodman diagram and highlight any change in the load line. Comment on the results.

a) Modified endurance limit $S_e = k_a k_b k_c k_d k_e k_f S_e'$

1pt i) Unmodified end. limit $S_e' = \frac{1}{2} S_{ut}$ For AISI 1018 (HR) $S_u = 400$ [Table A-20]

$S_e' = \frac{1}{2}(400)$

$S_e' = 200$ MPa

1pt ii) Surface Factor, k_a $k_a = a S_{ut}^{b}$

for machined finish: $a = 4.51$ $b = -0.265$ [Table 6-2]

$k_a = 4.51(400)^{-0.265}$

$k_a = 0.92$

1pt iii) Size factor, k_b $k_b = 1.24 d^{-0.107}$ [Eqn 6-20]

$= 1.24(50)^{-0.107}$

$k_b = 0.82$

1pt iv) Loading Factor, k_c $k_c = 1.0$ [Note: for combined loading use k_c for bending only]

1pt v) Temperature Factor, k_d @ 20°C $k_d = 1.000$ [Table 6-4]

@ 50°C $k_d = 1.010$

$\therefore$ use $k_d = 1.00$ (more conservative)

vi) Reliability factor, k_e

(2pt) For reliability of 95% we get:

$$k_e = 0.868 \quad [\text{Table 6-5}]$$

vii) Misc. Effects Factor, k_f

(1pt) Since no other information provided we assume $k_f = 1.0$

$$\text{So} \quad S_e = (0.92)(0.82)(1.0)(1.0)(0.868)(1.0)[S_e' = 200]$$

(2pt)

$$\boxed{S_e = 131.2 \text{ MPa}}$$

b) Modified Goodman Approach requires a calculation of alternating stress, σ_A, and mean stress, σ_m, using fatigue stress concentration factors, K_f due to weld fillet.

(2pt)

$$K_f = 1 + q(K_t - 1) \quad \text{from part a) } K_t = 1.7$$

for $r = 5$ mm and $S_{ut} = 400$ MPa $= 0.4$ GPa

$$q \approx 0.80 \quad [\text{Figure 6-20 with } r_{max} = 4.0 \text{ mm.}]$$

(2pt)

$$K_f = 1 + 0.80(1.7 - 1)$$

$$K_f \doteq 1.56$$

shear

$$K_{fs} = 1 + q_{shear}(K_{ts} - 1) \quad \text{from part a) } K_{ts} = 1.45$$

for $r = 5$ mm. and Brinell Hardness $= 116$ [Table A-20]

$$q_{shear} \approx 0.95$$

$$\text{so } K_{fs} = 1 + 0.95(1.45 - 1) = 1.43$$

alternating stress

$$\sigma_a' = \left\{ \left[(K_f \sigma_a)_{bend} + \cancelto{0}{\left(K_f \frac{\sigma_a}{0.85} \right)_{axial}} \right]^2 + 3 \left[K_{fs} \tau_{a_{torsion}} \right]^2 \right\}^{\frac{1}{2}}$$

midrange stress

$$\sigma_m' = \left\{ \left[(K_f \sigma_m)_{bend} + \cancelto{0}{\left(K_f \frac{\sigma_m}{0.85} \right)_{axial}} \right]^2 + 3 \left[K_{fs} \tau_{m_{torsion}} \right]^2 \right\}^{\frac{1}{2}}$$

Note: axial load is zero in this case

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b) cont.

In this case we have fully reversing bending stresses due to the belt tension (F_B) and a constant torque applied by the motor. Therefore,

i) bending: $\sigma_{x\,midrange} = 0$ $\quad \sigma_{x\,alt} = \frac{My}{I}$

(2pt)

Since $T = 500$ N·m which 50% of stall torque we can use 50% of the nominal bending stress calculated in part 1 (static case)

From part 1 $\sigma_{nom} = \frac{My}{I} = 108.65$ (do not apply K_t)

so $\sigma_a = \frac{1}{2}(108.65) = 54.3$ MPa

ii) torsion:

Constant torque, T, so $\tau_{alt} = 0$

$\tau_{midrange} = \frac{Tr}{J}$ Again since $T = 500$ N·m is 50% of

(2pt) stall torque we 50% of nominal shear stress from part 1.

From part 1, $\tau_{nom} = 40.74$ MPa

so $\tau_m = \frac{1}{2}(40.74) = 20.4$ MPa

Now

$$\sigma_a' = \left[(K_f\sigma_a)^2 + 3\left[\cancel{K_{fs}\tau_a}\right]^2\right]^{\frac{1}{2}} = \left[\left((1.56)(54.3)\right)^2\right]^{\frac{1}{2}} = 84.7 \text{ MPa}$$

$\tau_a = 0$

$$\sigma_m' = \left[(\cancel{K_f\sigma_m})^2 + 3\left[K_{fs}\tau_m\right]^2\right]^{\frac{1}{2}} = \left[3\left[(1.43)(20.4)\right]^2\right]^{\frac{1}{2}} = 50.5 \text{ MPa}$$

$\sigma_m = 0$

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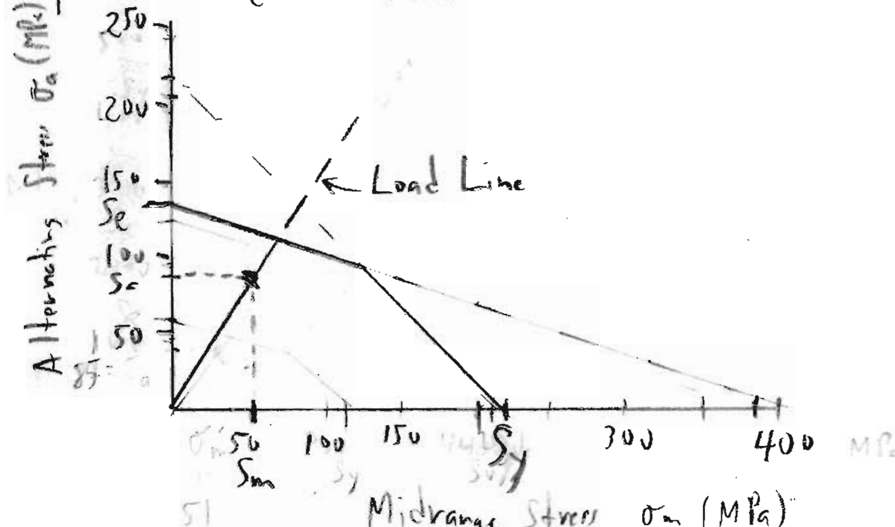
We can plot condition on Modified Goodman Diagram

AISI 1018 HR $S_y = 220 MPa$ $S_u = 400 MPa$

From part a) $S_e = 131 MPa$



For Modified Goodman

(2pt)

$$\frac{1}{n_f} = \frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{84.7}{131} + \frac{50.5}{400}$$

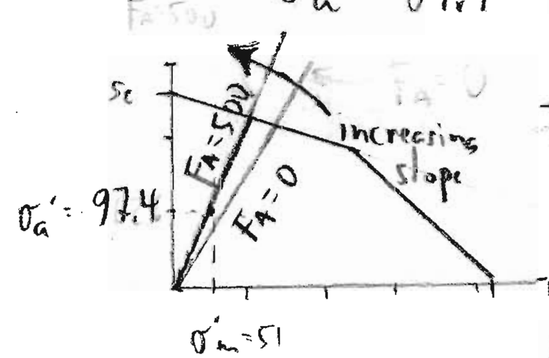
$$\boxed{\therefore\ n_f = 1.3}$$

c) For $F_A = 500 N$ we now have an additional bending stress.

Due to F_A: $\sigma_x = \sigma_a = \frac{My}{I} = \frac{(F_A)(200mm)(50/2 mm)}{I = \frac{1}{2}J = \frac{1}{2}(613592)} = 8.15 MPa$

$\therefore$ Total alternating stress $\sigma'_a = 84.7 + [\underbrace{(1.56)}_{K_f} 8.15] = 97.4 MPa$

(3pt)



(1pt)

The pre-tension cause the load line to increase in slope.

$$1/n_f = \frac{97.4}{131} + \frac{50.5}{400}$$

(1pt)

$$\boxed{n_f = 1.15}$$

Safety Factor Drops for same Torque

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