CARLETON UNIVERSITY

FINAL
EXAMINATION
December 2011

DURATION: **3** HOURS No. of Students: 90

AUTHORIZED MEMORANDA: **textbooks, course notes, calculators and laptop computers are allowed.**

Department Name & Course Number: **Mechanical & Aerospace Engineering MAAE 4102**
Instructor(s) A. Artemev

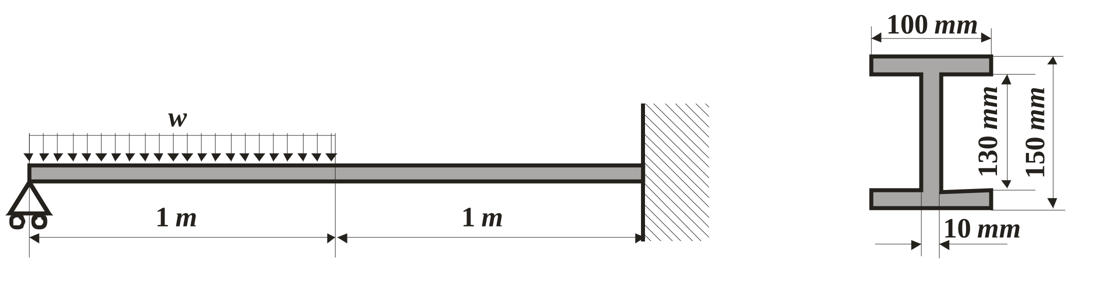
Students **MUST** count the number of pages in this examination paper **before** beginning to write, and report any discrepancy immediately to a proctor. The Examination paper has 15 pages.

In addition to this Examination paper, students require: an examination booklet yes _X_ no ___
a Scantron sheet yes ___ no _X_

This examination paper **MAY** be taken from the examination room.

All questions have the same value. Answer all questions.

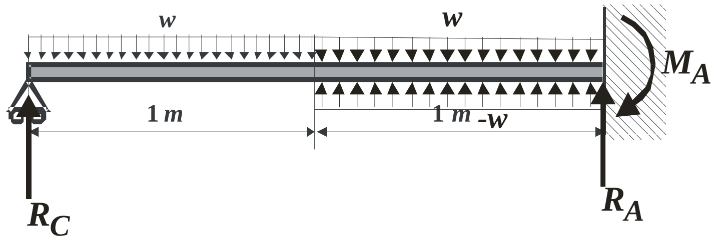
1. A 2 *m* long beam (Fig. 1 a) with the I shaped cross-section (Fig. 1 b) is made of an alloy with the yield strength $\sigma_y = 450\ MPa$, Young's modulus $E = 210\ GPa$ and Poisson's ratio $\nu = 0.31$. The beam is simply supported at one end and is rigidly built in at the other. The beam is loaded by the uniformly distributed load, with the intensity *w*, extending from the simply supported end over 1 *m* span (Fig. 1 a). Find the load intensity at the onset of yielding in the beam.



a *b*

Fig. 1

Solution:



Find the M_Y:

$$M_Y = \sigma_y \cdot \left(\frac{B \cdot D^3}{12} - \frac{b \cdot d^3}{12} \right) \cdot \frac{2}{D} = 137 \cdot 10^3 \ N \cdot m$$

Use the double integration together with step functions to resolve reactions and moment distribution:

$$EI \cdot \frac{d^2u}{dx^2} = -M(x) = -R_C \cdot x + \frac{w \cdot x^2}{2} - \frac{w}{2} \cdot [x-1]^2$$

$$EI \cdot \frac{du}{dx} = -\frac{R_C \cdot x^2}{2} + \frac{w \cdot x^3}{6} - \frac{w}{6} \cdot [x-1]^3 + A$$

$$EI \cdot u(x) = -\frac{R_C \cdot x^3}{6} + \frac{w \cdot x^4}{24} - \frac{w}{24} \cdot [x-1]^4 + A \cdot x + B$$

Boundary conditions:

$$at \ x = 0: \quad u(0) = 0 \quad \Rightarrow \quad B = 0 \qquad (1)$$

$$at \ x = 2: \quad u(2) = 0, \quad \frac{du}{dx} = 0 \qquad (2)$$

From $\frac{du}{dx}(x = 2) = 0$:

$$-\frac{R_C \cdot 4}{2} + \frac{w \cdot 8}{6} - \frac{w}{6} \cdot 1^3 + A = 0$$

$$A = \left(2 \cdot R_C - \frac{7}{6} \cdot w \right)$$

Now using $u(2)=0$:

$$\left(2\cdot R_C-\frac{7}{6}\cdot w\right)\cdot 2-\frac{R_C\cdot 8}{6}+\frac{w\cdot 16}{24}-\frac{w}{24}\cdot 1^4=0$$

$$4\cdot R_C-\frac{14}{6}\cdot w-\frac{8\cdot R_C}{6}+\frac{w\cdot 16}{24}-\frac{w}{24}\cdot 1^4=0$$

$$\left(\frac{96}{24}-\frac{32}{24}\right)\cdot R_C=\frac{56-16+1}{24}\cdot w$$

$$R_C=\frac{41}{64}\cdot w$$

Find the maximum of bending moment:

(a) Assuming the maximum location under the distributed load:

Maximum at the location with zero shear force (zero dM/dx):

$$R_C-w\cdot x=0$$

$$\frac{41}{64}\cdot w-w\cdot x=0$$

$$x=\frac{41}{64}$$

$0<x<1$ therefore maximum is located under the distributed load.

(b) Additional maximum moment value can be found at the rigidly built in end (x=2).

For $x=\frac{41}{64}$:

$$M=R_C\cdot x-\frac{w\cdot x^2}{2}=\frac{41}{64}\cdot w\cdot\frac{41}{64}-\frac{w}{2}\cdot\left(\frac{41}{64}\right)^2=\frac{1}{2}\cdot\left(\frac{41}{64}\right)^2\cdot w=0.205\cdot w$$

For $x = 2$:

$$M = R_C \cdot 2 - \frac{w \cdot 4}{2} + \frac{w}{2} \cdot 1^2 = \frac{41}{64} \cdot w \cdot 2 - \frac{w \cdot 4}{2} + \frac{w}{2}$$

$$M = w \cdot \frac{41 \cdot 2 - 4 \cdot 32 + 32}{64} = w \cdot \frac{-14}{64} = -0.219 \cdot w$$

The same value of applied load *w* produces larger bending moment (by absolute value) at *x*=2. Therefore yielding starts at *x*=2 when applied load intensity is:

$$w_y = \frac{M_Y}{0.219} = 6.26 \cdot 10^5 \ N/m$$

2. Non-linear load-deflection diagrams at constant crack areas for a double ended DCB specimen under the eccentric load were fitted by:

$$F = 350 \cdot \left(\frac{u}{A}\right) + 3250 \cdot \left(\frac{u^3}{A^2}\right)$$

where *F* is the applied load in *N*, *u* is the displacement in *mm* and *A* is the crack area in mm^2. A fracture test was performed using a specimen with the thickness of 2 *mm* and containing a crack with the length of 25 *mm*. It was observed that fracture started when the displacement of 5 *mm* was obtained under the load of approximately 200 *N*. Estimate the fracture load for the specimen with the initial crack length of 35 *mm*.

Solution:

Integrate the $F(u, A)$ relationship with respect to u at constant A to obtain elastic energy U_e:

$$U_e = \int F(u, A)_{A=const}\, du = \frac{175 \cdot u^2}{A} + \frac{1625 \cdot u^4}{2 \cdot A^2}$$

Estimate the potential energy release rate by differentiating U_e in respect to A at constant u:

$$J = -\left(\frac{\partial U_e}{\partial A}\right)_{u=const} = \frac{175 \cdot u^2}{A^2} + \frac{1625 \cdot u^4}{A^3}$$

Calculate the critical energy release rate (material toughness) by substituting u=5 mm and A=50 mm^2 into equation for J:

$$J_c\left(u = 5mm, A = 50mm^2\right) = 9.875\ (N \cdot mm)/mm^2$$

Solve toughness equation for the displacement at the onset of fracture when crack length is equal to 35 mm:

$$\frac{175 \cdot u^2}{A^2} + \frac{1625 \cdot u^4}{A^3} - J_c = 0$$

$$\frac{1625 \cdot u^4}{70^3} + \frac{175 \cdot u^2}{70^2} - 9.875 = 0$$

Solve:

$$\frac{1625 \cdot w^2}{70^3} + \frac{175 \cdot w}{70^2} - 9.875 = 0$$

where $w=u^2$ to get w=42.04 and u=6.48.

Now calculate the force at the onset of fracture for the specimen with crack length of 35 mm:

$$F = 350 \cdot \left(\frac{6.48}{70}\right) + 3250 \cdot \left(\frac{6.48^3}{70^2}\right) = 213\ N$$

3. The cantilever beam with the dimensions shown in Fig. 2 is subjected to the end load. The inspection of the beam revealed a crack located at point C shown in Fig. 2. The crack in the plane normal to the beam web was oriented at 60^o to the neutral plane. The crack size was estimated as $a \approx 4\ mm$. Estimate the maximum value of the load **W** that can be applied to the beam if the beam is made of a material with strength σ_y=880 *MPa*, Young's modulus *E*=120 *GPa*, Poisson's ratio ν=0.3 and fracture toughness K_{I_c}=85 $MPa \cdot \sqrt{m}$. Assume that stress intensity produced by the crack can be estimated as $K_I = 1.18 \cdot \sigma \cdot \sqrt{\pi \cdot a}$.

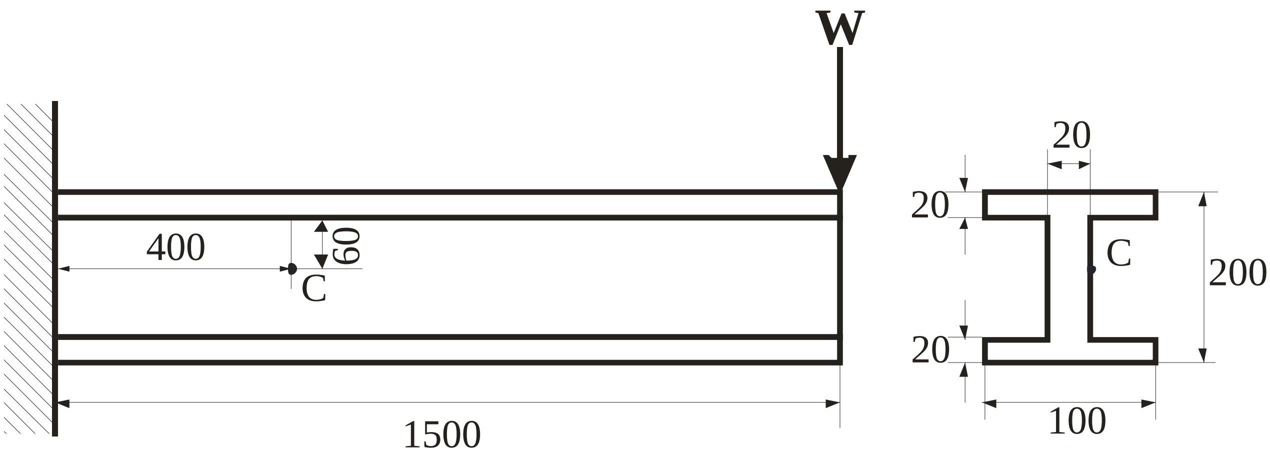


Fig. 2 Cantilever beam under the end load W. All dimensions are given in *mm*.

Solution:

Find the second moment of area for the beam cross-section:

$$I = \frac{B \cdot D^3}{12} - \frac{b \cdot d^3}{12} = \frac{0.1 \cdot 0.2^3}{12} - \frac{0.08 \cdot 0.16^3}{12} = 39.36 \cdot 10^{-6}$$

Calculate the normal and shear stress components at the crack location in the global coordinate system based on the beam geometry using unit value of applied force (W=1 N):

Bending moment: $BM(x = 0.4\,m) = W \cdot (L - 0.4\,m) = 1.1\,m$

Shear Force: Q=W=1

$\sigma_x = \dfrac{BM \cdot y}{I} = 558.9$ Pa (note that stress is tensile and therefore positive)

$$\tau_{xy} = \frac{Q}{2 \cdot I} \cdot \left[\left(\frac{d}{2} \right)^2 - y^2 \right] + \frac{Q \cdot B}{8 \cdot I \cdot t} \cdot \left(D^2 - d^2 \right) = 305\ Pa,$$

where: d=0.16 m, D=0.2 m, B=0.1 m, b=0.08 m, t=0.02 m, y=0.02 m.

Calculate the normal stress in the direction normal to the crack plane:

$$\sigma_n = \frac{\sigma_x}{2} + \frac{\sigma_x}{2} \cdot \cos\left(2 \cdot 30^o\right) + \tau_{xy} \cdot \sin\left(2 \cdot 30^o\right) = 683.2 \text{ Pa}$$

Calculate the critical stress at the onset of catastrophic crack growth:

$$\sigma_c = \frac{K_{Ic}}{1.18 \cdot \sqrt{\pi \cdot a}} = 642.6 \quad \text{MPa}$$

Calculate the critical load at the onset of catastrophic crack growth:

$$W_c = 1\,N \cdot \frac{\sigma_c}{\sigma_n} = 940 \cdot 10^3 \quad \text{N}$$

Calculate the load at the onset of yielding:

$$M_Y = \sigma_y \cdot \left(\frac{B \cdot D^3}{12} - \frac{b \cdot d^3}{12} \right) \cdot \frac{2}{D} = 346.4 \cdot 10^3 \text{ N m}$$

$$W_Y = \frac{M_Y}{L} = 230 \cdot 10^3 \quad \text{N}$$

Calculate the load at the onset of plastic collapse:

$$M_P = \sigma_y \cdot \left(\frac{B \cdot D^2}{4} - \frac{b \cdot d^2}{4} \right) = 429.4 \cdot 10^3 \text{ N m}$$

$$W_P = \frac{M_P}{L} = 286 \cdot 10^3 \text{ N}$$

Therefore failure starts by yielding at $230 \cdot 10^3$ N and complete collapse occurs at $286 \cdot 10^3$ N.

4. A through crack in a thick plate has the initial length of a_0=10 *mm*. Crack growth is approximately described by the Paris law with $A = 5.5 \cdot 10^{-12}$ m/cycle and $m = 2.5$ when ΔK is expressed in $MPa \cdot \sqrt{m}$. The plate is subjected to a cyclic load with an alternating stress range, and with the stress ratio $R = 0$ in all cycles (Fig. 3). The stress range, $\Delta\sigma$, changes with the number of cycles, N, as:

$$\Delta\sigma(N) = 160\ MPa \cdot \left| \sin\left(\frac{\pi \cdot N}{1000} \right) \right|$$

Estimate the number of cycles to failure. Material properties are: Young's Modulus, $E = 210\ GPa$, Poisson's ratio, $\nu = 0.33$, yield strength

$\sigma_{ys} = 390\ MPa$, ultimate tensile strength, $UTS = 485\ MPa$, and $K_{Ic} = 95\ MPa\sqrt{m}$. Assume that $K_I = \sigma \cdot \sqrt{\pi \cdot a / 2}$.

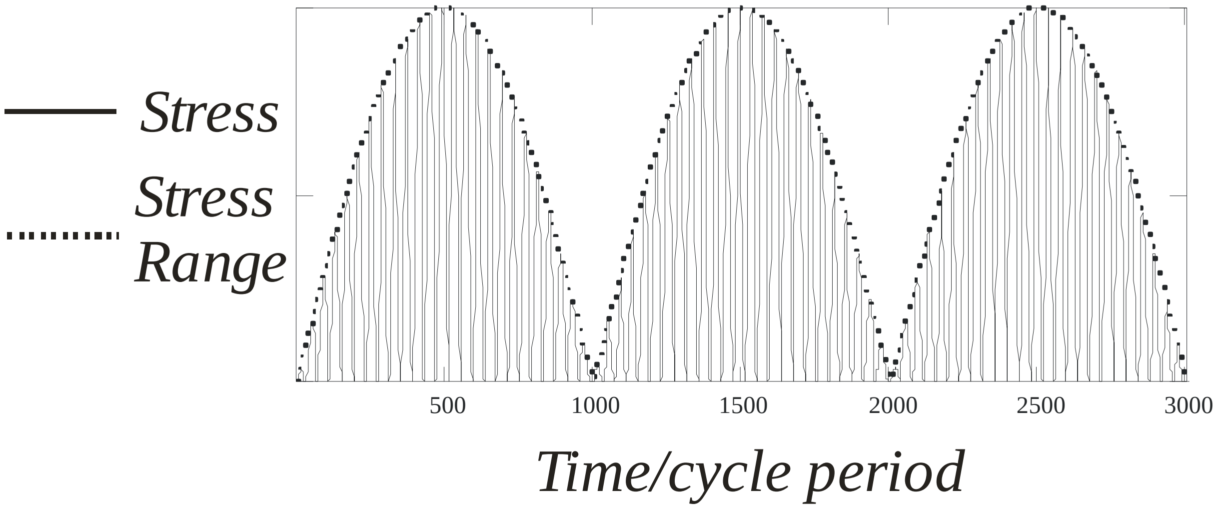


Fig. 3. The stress and stress range dependencies on time. Thin continuous line shows the stress vs. time dependency. Thick dotted line shows the stress range vs. number of cycles dependency. (The actual number of stress cycles within one stress range cycle is much larger than shown in Fig 3.)

Solution:

$l0 := 1.0 \cdot 10^{-3}$ $\quad A := 5.5 \cdot 10^{-12}$ $\quad m := 2.5$

$\sigma 0 := 160$ $\quad Nc := 1000$ $\quad \Delta\sigma max := \sigma 0$

$\sigma ys := 390$ $\quad E := 210 \cdot 10^{3}$

$UTS := 485$ $\quad KIc := 95$

$$Y := 1$$

$$\Delta KTksi := 0.04\sqrt{\frac{E}{6.895}\cdot\frac{(\sigma ys + UTS)}{2\cdot 6.895}}$$

$$\Delta KTksi = 55.606$$ ksi in$^{1/2}$

$$\Delta KT := \Delta KTksi \cdot 1.1$$

$$\Delta KT = 61.167$$ MPa m$^{1/2}$

$$Kmax := \Delta KT \qquad Kmax = 61.167$$

$$lc := \left(\frac{Kmax}{Y\cdot\sigma 0}\right)^2\cdot\frac{2}{\pi} \qquad lc = 93.041\times 10^{-3}$$

Now we need to obtain the ΔK_{rms} value by using:

$$\Delta K_{rms}^2 = \frac{1}{1000}\cdot\int_0^{1000}\left[\left(\Delta\sigma_{\max}\cdot\sqrt{\pi\cdot a}\right)\cdot\sin\left(\frac{\pi\cdot N}{1000}\right)\right]^2 dN$$

where $\Delta\sigma = 160\ MPa$**. Assuming that the crack half length does not change significantly over one cycle of the stress range we obtain:**

$$\Delta K_{rms}^2 = \frac{\left(\Delta\sigma_{\max} \cdot \sqrt{\pi \cdot a}\right)^2}{1000} \cdot \int_0^{1000} \left[\sin\left(\frac{\pi \cdot N}{1000}\right)\right]^2 dN$$

We can note, using the $x = \frac{\pi \cdot N}{1000}$ **substitution, that**

$$\int_0^{1000} \left[\sin\left(\frac{\pi \cdot N}{1000}\right)\right]^2 dN = \frac{1000}{\pi} \int_0^{\pi} [\sin(x)]^2 dx$$

From $\sin^2 x + \cos^2 x = 1$ **and by using** $\int_0^{\pi} \sin^2 x dx = \int_0^{\pi} \cos^2 x dx$ **we obtain:**

$$\int_0^{\pi} \sin^2 x dx = \frac{\pi}{2}$$

and that gives:

$$\Delta K_{rms}^2 = \frac{\left(\Delta\sigma_{\max} \cdot \sqrt{\pi \cdot a}\right)^2}{1000} \cdot \frac{1000}{\pi} \cdot \frac{\pi}{2} = \frac{\left(\Delta\sigma_{\max} \cdot \sqrt{\pi \cdot a}\right)^2}{2}$$

and

$$\Delta K_{rms} = \frac{\left(\Delta\sigma_{\max} \cdot \sqrt{\pi \cdot a}\right)}{\sqrt{2}}$$

Now we can integrate the Paris law to obtain:

$$N_f = \int_{l_0}^{l_c} \frac{da}{A \cdot \Delta K_{rms}^m} = \frac{1}{A} \cdot \frac{\left(\sqrt{2}\right)^m}{\Delta\sigma_{\max}^m \cdot \pi^{m/2}} \cdot \int_{l_0}^{l_c} \frac{da}{a^{\frac{m}{2}}}$$

$$\mathrm{Nf} := \frac{\left(\sqrt{2}\right)^{m}}{A\cdot\Delta\sigma\mathrm{max}^{m}\cdot\pi^{\frac{m}{2}}\cdot\left(1-\frac{m}{2}\right)}\cdot\left[\left(\frac{lc}{2}\right)^{1-\frac{m}{2}}-\left(\frac{l0}{2}\right)^{1-\frac{m}{2}}\right]$$

$$N_f\ 2.053\cdot 10^{6}$$

5. A hollow thin-walled shaft is subjected to the torque producing shear stress of 100 MPa and longitudinal tensile load producing normal tensile stress of 50 MPa in the axial direction. It can be assumed that stress values do not depend on time. The shaft is subjected to a corrosive environment. It can contain small surface cracks with the size, *a*, of up to 2 mm and with unknown orientation. The shaft material has Young's Modulus $E = 210\,GPa$, Poisson's ratio, $\nu = 0.33$, yield strength $\sigma_{ys} = 480\,MPa$, ultimate tensile strength, $UTS = 545\,MPa$, and $K_{Ic} = 75\,MPa\sqrt{m}$. The rate of the crack growth in the shaft material under the combined effect of applied stress and corrosive environment is described by the SCC constant $D = 20.5\cdot 10^{-4}\ yr^{-1}\cdot MPa^{-2}$. Estimate the minimum expected lifetime of the shaft. Assume that stress intensity can be approximated by $K_I = 1.24\cdot\sigma\cdot\sqrt{\pi\cdot a}$.

$a0 := 2 \cdot 10^{-3}$ $\sigma x := 50 \cdot 10^{6}$ $\tau xy := 100 \cdot 10^{6}$

$\sigma ys := 480 \cdot 10^{6}$ $KIc := 75 \cdot 10^{6}$

$D := 20.5 \cdot 10^{-4}$ $yr^{-1}\ MPa^{-2}$

$\sigma MCc := \frac{\sigma x}{2}$ $\sigma MCc = 25 \times 10^{6}$

$RMC := \sqrt{\frac{1}{4} \cdot \sigma x^{2} + \tau xy^{2}}$ $RMC = 103.078 \times 10^{6}$

$\sigma max := (\sigma MCc + RMC)$ $\sigma max = 128.078 \times 10^{6}$

$ac := \frac{1}{\pi} \cdot \left(\frac{KIc}{1.24 \cdot \sigma max} \right)^{2}$ $ac = 70.988 \times 10^{-3}$

$\sigma maxMPA := \sigma max \cdot 10^{-6}$

$tf := \frac{1}{D \cdot \sigma maxMPA^{2} \cdot \pi} \cdot (\ln(ac) - \ln(a0))$

$tf = 0.034$

$tfdays := tf \cdot 365$ $tfdays = 12.332$

6. a) A thin composite plate is attached by adhesive bonding to a wide and stiff supporting beam made of an alloy. It was found that failure in this system occurs by peeling the plate from the beam. Which of the following changes in the materials can improve the overall strength of the system?

   (a) alloy with a higher yield stress

   (b) alloy with higher toughness

   (c) alloy with a higher Young's modulus

   (d) adhesive with a higher yield stress

   (e) adhesive with higher toughness

   (f) adhesive with a higher Young's modulus

   (g) stronger composite

   (h) composite with higher toughness

   (i) composite with a higher Young's modulus.

   (e) and (i)

   b) It has been observed that one of the structure components has too short a service life as a result of a fast development of stress corrosion cracking. It is necessary to find a solution that will increase the life of the component. Two approaches have been suggested: first, to decrease by several times the stress applied to the component by increasing the component's cross section area, and by doing so reduce the stress intensity and crack growth rate; second, to improve the manufacturing process, drastically improve the surface finish and apply a thorough examination of components using non-destructive inspection methods. The resulting decrease in the size of internal defects and elimination of surface defects should result in a longer time required for the development of stress corrosion cracking. In your opinion, is one of the two suggested approaches better then the other (and so, which one is better)? Briefly substantiate your answer.

Stress reduction will not help if SCC is in stage 2 with the rate independent of stress intensity.

c) Describe briefly how the type of the dominant creep mechanism can be affected by:

- decrease in grain size: Favors the diffusional , especially Coble.
- increase in applied stress: Favors dislocation (power) creep
- increase in temperature: Favors NH over Coble