STAT 2507
Dec 10 Final Exam Solutions

Part 1

1. $H_0: \mu = 50 \quad H_A: \mu \neq 50 \quad T = \frac{\bar{x} - 50}{s/\sqrt{n}}$ since $n = 9$, popn. normal

$t_{calc} = \frac{61-50}{21/\sqrt{9}} = 1.57$

p-value $= 2P(T > |1.57|)$

$t_{8,.10} = 1.397$ & $t_{8,.05} = 1.86$ $\therefore P(T > 1.57)$ is bet. .05 & .10

$\therefore$ p-value bet. $2 \times .05$ & $2 \times .10$

i.e. p-value between .1 & .2 $\therefore > .1$

$\therefore \boxed{a}$

2. $\boxed{c}$

3. $\boxed{d}$ x = r.v. with prob. of "success" .05 ("success" = interval fails to contain μ)

4. $\boxed{a}$ provided popn has a normal distn.

5. $\boxed{d}$ z score $= \frac{x - \mu}{\sigma} = \frac{4-0}{1} = 4 > 3$

& z-score > 3 $\therefore$ outlier.

6. C.I. width $= 2 z_{\alpha/2}\, 6/\sqrt{90}$

want $\frac{2 z_{\alpha/2}\, 6/\sqrt{90}}{3} = 2 z_{\alpha/2} \frac{6}{\sqrt{9 \times 90}}$ $\therefore \boxed{810}$ $\boxed{c}$

7. $\boxed{c}$

8. $\boxed{c}$

9. $\boxed{b}$ From Poisson tables $\mu = 2.5$ $P(0) = 0.082$

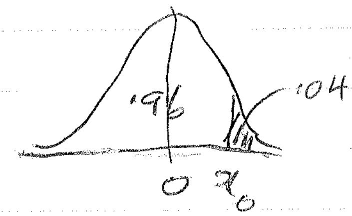
OR $P(0) = \dfrac{e^{-2.5}(2.5)^0}{0!} = 0.0821$

10. Let X = time to school $X \sim N(15, 4)$

Let x_0 = # min. she must allow for getting to school on time

Want $P(X > x_0) \le .04$ $\therefore P(Z > \frac{x_0 - 15}{2}) \le .04$

$\therefore P(Z > z_0) = .04$ $z_0 = \frac{x_0 - 15}{2}$



$\therefore P(Z < z_0) = .96$

$\therefore z_0 = 1.75$ $\therefore \frac{x - 15}{2} = 1.75$ $\boxed{x = 18.5}$

$\therefore \boxed{d}$

11. chem: $X_1 \sim N(90, 64^2)$ stats: $X_2 \sim N(70, 16^2)$

$\therefore$ Z score for chem mark: $\frac{102 - 90}{64} = 0.1875$

Z score for stats: $\frac{77 - 70}{16} = 0.4375$

$\therefore \boxed{b}$

12. X

13. $\boxed{d}$ $P(A \cup B) = .5 + .7 - .3 = 0.9$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.3}{.7} = 0.42$

14. $\boxed{d}$ $P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= 0.3 + 0.4 - P(B)P(A|B) = .3 + .4 - .4 \times .6 = 0.46$

Part II

1. Let X = height of a man $X \sim N(174, 6^2)$

(a) $P(170 < X < 179) = P\left(\frac{170-174}{6} < \frac{X-\mu}{6} < \frac{179-174}{6}\right)$

$= P(-0.67 < Z < 0.83) = P(Z < 0.83) - P(Z < -0.67)$

$= 0.7967 - 0.2514 = \boxed{0.5453}$

(b) Want x_0 such that only 5% of men are taller than this value

i.e. $P(X > x_0) = .05$ i.e. $P\left(Z > \frac{x_0 - 174}{6}\right) = .05$

i.e. $P(Z > z_0) = .05$ where $z_0 = \frac{x_0 - 174}{6}$

i.e. $P(Z < z_0) = .95$ $\therefore z_0 = 1.645$

$\therefore \frac{x_0 - 174}{6} = 1.645$ $\therefore x_0 = 174 + 6 \times 1.645$

$= 183.87$

$\therefore$ Required ceiling height is $\boxed{183.87 \text{ cm}}$

(c) $P(\bar{X} > 176) = P\left(\frac{\bar{X} - \mu}{6/\sqrt{n}} > \frac{176 - \mu}{6/\sqrt{n}}\right)$

$= P\left(Z > \frac{176-174}{6/\sqrt{49}}\right) = P(Z > 2.33)$

$= 1 - P(Z < 2.33) = 1 - .9901 = \boxed{0.0099}$

2. μ_1 = mean room rate for all Marriott hotels
μ_2 = " " " " " Radisson "

$n_1 = 50$ & $n_2 = 50$ are large enough that

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \text{ is approx } N(0,1)$$

$H_0: \mu_1 - \mu_2 = 0$ $\alpha = .01$
$H_A: \mu_1 - \mu_2 \neq 0$

test stat: $Z = \frac{\bar{X}_1 - \bar{X}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

R.R.: $Z_{calc} > Z_{.005} = 2.575$
OR $Z_{calc} < -Z_{.005} = -2.575$

Our $Z_{calc} = \frac{170-145}{\sqrt{\frac{15^2}{50} + \frac{10^2}{50}}} = 9.81 > 2.575$

∴ Reject H_0 & conclude at $\alpha = .01$ that there is a difference in the average room rates for the 2 hotel chains

(b) p-value $= 2P(Z > 9.81) = 1 - P(Z < 9.81)$
$\doteq 1 - 1 \approx 0 < \alpha = .01$

$\therefore$ same conclusion!

3. Matched pairs expt. since before & after measurements at same companies.

$d_i = X_b - X_a$ X_b = # soft drinks bottled before, X_a = # " " " after

d_i: $-10 \quad -5 \quad 0 \quad -5 \quad -3$ $\quad \Sigma d_i = -23, \quad \Sigma d_i^2 = 159$

$H_0: \mu_d = 0$ i.e. $H_0: \mu_b - \mu_a = 0$ program not effective

$H_A: \mu_d < 0$ i.e. $H_0: \mu_b - \mu_a < 0$ program effective

test stat: $T = \dfrac{\bar{d} - 0}{s_d / \sqrt{n}}$ $\quad s_d^2 = \dfrac{159 - \frac{(-23)^2}{5}}{4} = 13.3$

R.R. $t_{calc} < -t_{4, .05} = -2.13$

Our $t_{calc} = \dfrac{-23/5}{\sqrt{\frac{13.3}{5}}} = -2.825 < -2.13$

$\therefore$ Reject H_0 and conclude at $\alpha = .05$ that system was effective in increasing the average # of cases bottled / hour.

4. $n_A = 10$, $n_B = 10$ small sample sizes

$\therefore$ assuming popns of times to spoilage are normally distributed & $\sigma_1^2 = \sigma_2^2$ $\quad \dfrac{\bar{x}_A - \bar{x}_B - (\mu_1 - \mu_2)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ has a T-distn.

(a) $H_0: \mu_A = \mu_B$

$H_A: \mu_A \neq \mu_B$

N.B. $\sigma_1^2 = \sigma_2^2$ reasonable since $\frac{11.5^2}{9.5^2} = 1.47 < 3$

(b) test stat: $T = \dfrac{\bar{X}_A - \bar{X}_B - 0}{\sqrt{S_p^2\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}}$

where $S_p^2 = \dfrac{9 \times 9.5^2 + 9 \times 11.5^2}{18} = 111.25$

$t_{calc} = \dfrac{108.7 - 98.7}{\sqrt{111.25\left(\frac{1}{10} + \frac{1}{10}\right)}} = 2.11$

(c) R.R. $t_{calc} > t_{18, .025} = 2.101$

OR $t_{calc} < -2.101$

Our $t_{calc} = 2.11 > 2.101$

$\therefore$ Reject H_0 and conclude at $\alpha = .05$ that there is a difference between the average spoilage times for the 2 preservative

5. Since $n = 225$ $\therefore Z = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ is approx $N(0,1)$ for any distn of X

Provided sample random

$\therefore$ a 95% C.I. for the average online time for all users of the particular server is

$\left(\bar{x} \pm z_{.025}\, S/\sqrt{n}\right) = \left(12.5 \pm 1.96 \times \dfrac{5.4}{\sqrt{225}}\right)$

$= (12.5 \pm 0.71) = (11.79, 13.21)$ hrs

7. Let $X = \#$ incorrect reports out of 200

$X \sim bin(200, .1)$

Since $n = 200$ & $np = 20 \geq 5$ & $nq = 180 \geq 5$ ($200 \times .9$)

$\therefore$ Use normal approx to binomial with

$\mu = np = 20$ & $\sigma^2 = 200 \times .1 \times .9 = 18$

$\therefore P(X > 40) = 1 - P(X \leq 40)$

$= 1 - P\left(Z < \frac{40.5 - 20}{\sqrt{18}}\right) = 1 - P(Z < 4.83)$

$\approx 1 - 1 \approx 0$

8. (a) $\bar{y} = \frac{\Sigma y_i}{8} = \frac{640}{8} = 80$

$\Sigma x_i = 25$

$s_y^2 = \frac{\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{8}}{7}$

$= \frac{61400 - (640)^2/8}{7} = \frac{10200}{7} = 1457.1429$

$s_x^2 = \frac{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{8}}{7}$

$= \frac{93 - \frac{(25)^2}{8}}{7}$

$= \frac{14.875}{7} = 2.125$

$s_x = \sqrt{2.125}$

$s_y = \sqrt{1457.1429}$

$s_{xy} = \frac{\Sigma x_i y_i - \frac{\Sigma x_i \Sigma y_i}{8}}{7} = \frac{2380 - \frac{640 \times 25}{8}}{7}$

$= \frac{380}{7} = 54.2857$

$\therefore \boxed{\bar{y} = 80,}$ $\boxed{s_x = \sqrt{2.125} = 1.4577}$

$\boxed{s_y = \sqrt{1457.1429} = 38.1725}$ $\boxed{s_{xy} = 54.2857}$

(b) $r = \frac{s_{xy}}{s_x s_y} = \frac{54.2857}{1.4577 \times 38.1725} = \boxed{0.9756}$

The fitted regression line is $\hat{y} = a + bx$ where

$b = \frac{s_{xy}}{s_x^2} = \frac{54.2857}{2.125} = 25.5462$

$\therefore a = \bar{y} - b\bar{x} = 80 - 25.5462 \times \frac{25}{8} = 0.1681$

$\therefore \hat{y} = 0.1681 + 25.5462x$

$\therefore$ when $x = 6$ estimate they spend

$0.1681 + 25.5462 \times 6 = \$153.44$/week

on groceries