MATH 1104A   Test 1   January 23, 2007

This test paper has two Parts.
Part I has 6 multiple choice questions. Part II has 2 long answer questions.
It cannot be taken from the examination room.
Only nonprogrammable calculators are allowed. Duration: 50 Minutes.

NAME :   **STUDENT NO :**

PART I: Multiple Choice Questions. Circle the correct answer. Three marks each.

1) Consider the following augmented matrix of a system of linear equations:

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 1 \\ 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right]$$

The system has

a) infinitely many solutions with one free variable

b) infinitely many solutions with two free variables

c) unique solution

d) no solutions

2) Let $A = \left[\begin{array}{ccc|c} 1 & 2 & 0 & 10 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & k^2-16 & k+4 \end{array}\right]$ be the augmented matrix of a given system.

For which value(s) of k will the system has infinitely many solutions?

a) $k = 4$ **b)** $k = -4$ **c)** $k \neq \pm 4$ **d)** $k \neq -4$

3) Let $A = \begin{bmatrix} 1 & -3 & -4 & -3 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -3 & -4 \end{bmatrix}$.

Reduced row echolon form (RREF) of A is

a) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ **b)** $\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$, **c)** $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$, **d)** $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$,

4) Let $u = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $v = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ -6 \\ h \end{bmatrix}$.

For what value(s) of h, the vector b is in the span of the vectors u and v?

a) for $h = -4$

b) for $h = 7$

c) for all values of h

d) there is no value of h such that b is in the span of u and v.

5) For what value(s) of h is the set of vectors $\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} h \\ -3 \end{bmatrix} \right\}$ linearly <u>independent</u>?

a) Vectors are linearly <u>independent</u> for $h = 3$

b) Vectors are linearly <u>independent</u> for all $h \neq 3$

c) Vectors are linearly <u>independent</u> for all h

d) Vectors are linearly <u>dependent</u> for all h

6) What is the general solution of the following system of linear equations?

$$\begin{aligned} x + y - 3z &= 0 \\ -x + 3y - z &= 0 \\ x - y - z &= 0. \end{aligned}$$

a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ t \\ 2t \end{bmatrix}$, **b)** $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ 3t \\ t \end{bmatrix}$, **c)** $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2t \\ t \\ t \end{bmatrix}$, **d)** $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2t \\ -t \\ t \end{bmatrix}$,

where t is a parameter

PART II: Long answer questions. Show all your work.

[10] 1) Determine whether each the following sets of vectors is linearly independent. State your reason clearly.

$$S_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}, \quad S_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$$

[12] **2)** Solve the following system of linear equations.
Write the solution in vector form.

$$x_1 - 2x_2 + x_3 - x_4 = 4$$
$$-3x_1 + 5x_2 - 3x_3 + 4x_4 = -3$$
$$-2x_1 + 3x_2 - 2x_3 + 3x_4 = 1$$
$$x_1 - x_2 + x_3 - 2x_4 = -5$$