Student name:--

Student ID:--

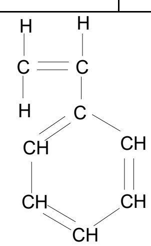
Manufacturing of composites

Mid term exam

October 17, 2019

The information below can be used for the exam.

	Diameter (µm)	Strength (MPa)	Strain limit (%)	Density (g/cm^3)	Modulus (GPa)
S glass fibers	10	4544	4	2.5	70
Carbon fibers	7	3280	1.4	1.8	234
Kevlar fibers	12	2900	2.8	1.44	131
Epoxy		130		1.20	4.5



Styrene

Bond	Energy (kJ/mole)
C-C	370
C=C	680
C-O	360
C=O	535

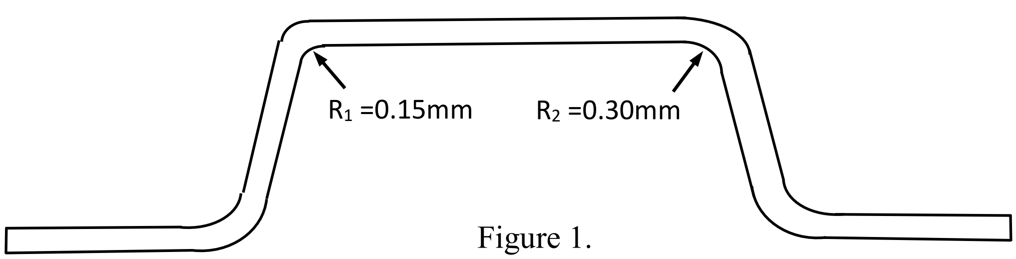


Figure 1.

$$|\varepsilon_{\max}| = \frac{d}{2\rho} \qquad \frac{l}{d} = \frac{\sigma_f}{2\tau} \qquad E_c = E_f v_f + E_m v_m$$

$$E_c = E_f v_f + v_m E_m \qquad w_f = \frac{V_f \rho_f}{V_f \rho_f + V_m \rho_m} \qquad w_f = \frac{v_f \rho_f}{v_f \rho_f + \rho_m (1 - v_f)}$$

$$\begin{bmatrix} e_1 \\ e_b \end{bmatrix} = \begin{bmatrix} F_{11} & F_{1b} \\ F_{b1} & F_{bb} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_b \end{bmatrix} \qquad F_{1b} = F_{b1} = -\frac{16}{\pi^3}\frac{\beta^2}{E}\varsigma(\varsigma - 1)^3 \qquad F_{11} = \frac{4}{\pi}\frac{1}{E}\varsigma^2[1 + 2(\varsigma - 1)^2]$$

$$F_{bb} = \frac{\beta^4}{3\pi E}(\varsigma - 1)^4 \qquad e_b = 1 - \sqrt{\frac{V_f}{V_o}} \qquad \zeta = \sqrt{\frac{V_a}{V_f}}$$

1. **Question 1 (15 marks):**

The shear strength between the fiber (glass, carbon or Kevlar) and epoxy is 25 MPa. Determine the mode of failure (slipping or fiber fracture) for the following situations:

a. (5 marks) Glass fiber with fiber length of 0.1 mm, 0.2 mm, 0.5 mm, 1mm or 6 mm.
b. (5 marks) Carbon fiber with fiber length of 0.1 mm, 0.2 mm, 0.5 mm, 1mm or 6 mm
c. (5 marks) Kevlar fiber with fiber length of 0.1 mm, 0.2 mm, 0.5 mm, 1mm or 6 mm.

Solution:

$$\frac{l}{d} = \frac{\sigma_f}{2\tau}$$

a. **Glass fibers:**

$$\frac{l}{d} = \frac{\sigma_f}{2\tau} = \frac{4544}{(2)(25)} = 90.88$$

$$l = (90.88)(10\mu m) = 908.8\mu m = 0.909mm$$

For length = 0.1 mm, failure by slipping

For length = 0.2 mm, failure by slipping

For length = 0.5 mm, failure by slipping

For length = 1mm, failure by fiber fracture.

For length = 6 mm, failure by fiber fracture.

b. **Carbon fibers:**

$$\frac{l}{d} = \frac{\sigma_f}{2\tau} = \frac{3280}{(2)(25)} = 65.6$$
$$l = (65.6)(7\mu m) = 459.2\mu m = 0.459mm$$

For length = 0.1 mm, failure by slipping

For length = 0.2 mm, failure by slipping

For length = 0.5 mm, failure by failure fracture

For length = 1 mm, failure b fiber fracture.

For length = 6 mm, failure by fiber fracture.

c. **Kevlar fibers:**

$$\frac{l}{d} = \frac{\sigma_f}{2\tau} = \frac{2900}{(2)(25)} = 58$$
$$l = (58)(12\mu m) = 696\mu m = 0.696mm$$

For length = 0.1 mm, failure by slipping

For length = 0.2 mm, failure by slipping

For length = 0.5 mm, failure by slipping

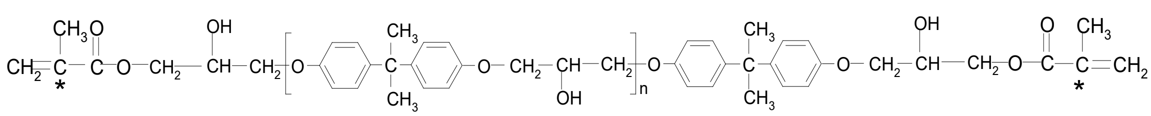
For length = 1 mm, failure by fiber fracture.

For length = 6 mm, failure by fiber fracture.

2. **Question 2 (15 marks):**

a. (8 marks) A vinyl ester resin with the chemical formula shown below (n = 2) is cross linked using styrene. For 100 g of the vinyl ester, how much total heat is generated?

b. (7 marks) Assume that the rate of cure is 0.01/minute. Assume also that there is 50% heat loss to the environment. What is the temperature of the composite after 5 minutes? (Specific heat capacity of the resin system is 1.045 J/(g.ºC).



BISPHENOL A EPOXY BASED (METHACRYLATE) VINYL ESTER RESIN (*n = 0-3*)

Solution:

a. Total heat generated:

Mass of the vinyl ester molecule:

One way to avoid mistake is to divide the molecule into three parts. One part on the left of the bracket, one part on the right of the bracket, and the bracketted part.

Left of the bracket:

7C + 3O + 11H = 7(12) + 3(16) +11(1) =143 g/mole

Right of the bracket:

22 C + 5O + 25 H = 22(12) + 5(16) + 25(1) = 369 g/mole

The bracket part:

2(18C + 3O + 20 H) = 2(18 x12 + 3 x16 + 20x 1) = 568 g/mole

Total mass = 143 + 369 + 568 = 1080 g/mole

Mass of styrene molecule:

8 C + 8 H = 8(12) + 8(1) = 104 g/mole

In the reaction, 1 vinyl ester molecule corresponds to 2 styrene molecule.

Consider 1 vinyl ester molecule, 4 double bonds are broken, and 8 single bonds are formed.

Energy associated with the broken double bonds: 4(680 kJ/mole) =2720 kJ/mole

Energy associated with the formed single bonds: 8(370) = 2960 kJ/mole

Net energy generated per 1 vinyl ester molecule: 2960 – 2720 = 240 kJ/mole

Energy associated with 100 g vinyl ester:

(240 kJ/mole) (100g/1080g/mole) = **22.22 kJ**

b. Temperature after 5 minutes:

Mass of styrene corresponding to 100g of vinyl ester:

(104/1080)(100g) = 9.62 g.

Total mass of the vinyl ester resin system:

100 + 9.62 = 109.62 g

The rate of cure is 1% per minute. After 5 minutes, the amount of cure is 5%. As such the amount of heat generated after 5 minutes is (22.22 kJ) (0.05)_= 1.111 kJ.

There is 50% heat loss to the environment. As such the amount of heat left to heat up the sample is: (1.111 kJ) (0.5) = 0.556 kJ.

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc} = \frac{0.556kJ}{(109.62g)(1.045J/g.C)} = 4.84^{o}C$$

Temperature of the sample after 5 minutes: **24.84 ºC**, assuming room temperature = 20 ºC.

Question 3 (15 marks):

a. (7 marks) It is desired to make a composite piece having the cross section as shown in the figure in page 1. It is also desirable for the composite to have a modulus of 45 GPa along the fiber direction. Among three types of composites (glass/epoxy, carbon/epoxy, Kevlar/epoxy) choose the type of composites that can be used to make the piece.
b. (8 marks) For the chosen composite, if one uses 100 g of fibers, how many grams of epoxy should be used?

Solution:

a. Which fiber?

The smaller radius is the determining factor. It is 0.15 mm.

$$|\varepsilon_{\max}| = \frac{d}{2\rho}$$

For glass fiber: $|\varepsilon_{\max}| = \dfrac{d}{2\rho} = \dfrac{10\mu m}{2(150\mu m)} = 0.0333 = 3.33\%$

For carbon fibers: $|\varepsilon_{\max}| = \dfrac{d}{2\rho} = \dfrac{7\mu m}{2(150\mu m)} = 0.0233 = 2.33\%$

For Kevlar fibers: $|\varepsilon_{\max}| = \dfrac{d}{2\rho} = \dfrac{12\mu m}{2(150\mu m)} = 0.04 = 4.00\%$

Compare with the strain limits for the fibers, only **glass fibers** can be used.

b. Grams of epoxy:

$$E_c = E_f v_f + E_m v_m$$
$$45GPa = (70GPa)v_f + 4.5GPa(1 - v_f)$$

$$v_f = 0.618$$

Assume that there are 100 cm^3 of composites, the amount of fibers if 61.8 cm^3 and the amount of resin is 38.2 cm^3.

Weight of fibers = (61.8 cm^3) (2.5 g/cm^3) = 154.5 g

Weight of resin = (38.2 cm^3) (1.20 g/cm^3) = 45.84 g.

Total weight of composite = 200.34 g.

Weight fraction of fibers: 154.5/200.34 = 0.77

Weight of epoxy resin corresponding to 100g of fibers:

(100g)((45.84/154.5) =**29.67 g.**

3. **Question 4 (15 marks):b**

A bed of prepregs made of carbon/epoxy are placed inside a mould where the ends of the fibers are fixed from motion. The initial fiber volume fraction is 0.50. The fibers are compressed along the thickness direction. The thickness of the fiber beds at the beginning of compression is 3 mm and its desired final thickness is 2.8 mm. Assuming that the fibers arrange in a square array. Assuming the fiber waviness ratio of 250, determine the stress along the fiber direction.

Solution:

$$\begin{bmatrix} e_1 \\ e_b \end{bmatrix} = \begin{bmatrix} F_{11} & F_{1b} \\ F_{b1} & F_{bb} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_b \end{bmatrix}$$

From the statement of the problem, $e_1 = 0$

$$0 = F_{11}\sigma_1 + F_{1b}\sigma_b \quad or$$

$$\sigma_1 = -\frac{F_{1b}\sigma_b}{F_{11}}$$

$$e_b = F_{b1}\sigma_1 + F_{bb}\sigma_b = (F_{bb} - \frac{F_{1b}^2}{F_{11}})\sigma_b$$

$$e_b = \frac{2.8 - 3.0}{3.0} = -0.067$$

$$e_b = 1 - \sqrt{\frac{V_f}{V_o}}$$

From $V_o = 0.50$, $V_f = 0.569$

$$\zeta = \sqrt{\frac{V_a}{V_f}}$$

For square array, $V_a = \pi/4 = 0.785$

$$\zeta = 1.175$$

$$F_{1b} = F_{b1} = -\frac{16}{\pi^3}\frac{\beta^2}{E}\varsigma(\varsigma - 1)^3 = -\frac{16}{\pi^3}\frac{250^2}{234GPa}(1.175)(0.175)^3 = -0{,}869GPa^{-1}$$

$$F_{bb} = \frac{\beta^4}{3\pi E}(\varsigma - 1)^4 = \frac{250^4}{3\pi(234GPa)}(0.175)^4 = 1662.1GPa^{-1}$$

$$F_{11} = \frac{4}{\pi}\frac{1}{E}\varsigma^2[1+2(\varsigma-1)^2] = \frac{4}{\pi}\frac{1}{234GPa}(1.175)^2[1+2(0.175)^2] = 0.00798GPa^{-1}$$

From

$$e_b = F_{b1}\sigma_1 + F_{bb}\sigma_b = (F_{bb} - \frac{F_{1b}^2}{F_{11}})\sigma_b$$

$$-0.067 = (1662.1 - \frac{0.869^2}{0.00798})\sigma_b$$

$$\sigma_b = -0.0427MPa = -42.7KPa$$

From

$$\sigma_1 = -\frac{F_{1b}\sigma_b}{F_{11}} = -\frac{-(0.869)(-0.0427x10^{-3})}{0.00798} = -4.65MPa$$

True/False questions (each question is worth 1 mark). Circle the correct answer

1. The cross section perpendicular to the fiber direction in a unidirectional glass/epoxy composite (fiber volume fraction 0.60) has the dimensions of 1 mm by 2 mm. The number of fibers in this cross section is between 20,000 and 25,000. **T** **F**

2. The cross section perpendicular to the fiber direction in a unidirectional glass/epoxy composite (fiber volume fraction 0.60) has the dimension of 1 mm by 2 mm. The length of the sample is 5 mm. The amount of interface between fibers and the matrix is between 0.002 m^2 and 0.003 m^2 . **T** **F**

3. The numerical value for the fiber volume fraction of carbon fibers is smaller than the numerical value for the mass (or weight) fraction of carbon fibers. **T** **F**

4. The larger is the diameter of a fiber, the larger is the radius of curvature it can be bent. **T** **F**

5. The aspect ratio of spherical particles is smaller than the aspect ratio of fibers such as glass fibers or carbon fibers **T** **F**

6. In the curing process for epoxy using amines, reaction with primary amines is more difficult as compared to reaction with secondary amines. **T** **F**

7. In composites, the material would have larger mechanical properties when the fiber volume fraction is high. As such it is better to have the fibers touch each other since this would produce highest fiber volume fraction. **T** **F**

8. The aspect ratio of most fibers used in composites is around 1. **T** **F**

9. The reason why there are more thermoset composites as compared to thermoplastic composites is due to the compatibility criterion. **T** **F**

10. Glass fiber is anisotropic while carbon fiber is isotropic. **T** **F**

Answers:

1. F: $n = 15{,}286\ (1.2/78.5\text{x}10^{-6})$

2. T $0.0024\ m^2$

3. T

4. T

5. T

6. F

7. F

8. F

9. F

10. F