

Sample Midterm Test

Problem 1. Find the equation(s) of the tangent line(s) to the curve given with parametric equation

$$x = 3t^2 + 1, \quad y = t^3 - t$$

at the point $(4, 0)$.

Problem 2. Find the length of the polar curve

$$r = \theta^2, \quad 0 \leq \theta \leq 2\pi.$$

Problem 2'. Find the area of the planar region that lies inside the curve $r = 4 \sin(\theta)$ and outside the curve $r = 2$.

Problem 3. Find a Cartesian equation for the hyperbola with vertices $(-3, -4)$, $(-3, 6)$ and foci $(-3, -7)$, $(-3, 9)$.

Problem 4. Find an equation of the plane that passes through the point $(1, 5, 1)$ and is perpendicular to the planes $2x + y - 2z = 2$ and $x + 3z = 4$.

Problem 5. Find parametric equations for the tangent line to the curve given with parametric equations

$$x = t \cos(t), \quad y = t, \quad z = t \sin(t),$$

at the point $(-\pi, \pi, 0)$.

Bonus question. Find the point(s) on the curve

$$\mathbf{r}(t) = 2 \cos(t)\mathbf{i} + 2 \sin(t)\mathbf{j} + e^t\mathbf{k}, \quad 0 \leq t \leq \pi,$$

where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.

Bonus question. Find a vector equation for the curve that is the intersection of the surfaces $z = y^2$ and $x^2 + y^2 = 1$.