

Chapter 5 - Homework

1. Question 1:

It is desired to filament wind a 300mm diameter pipe with a winding angle of +/- 54°. The length of the pipe is 1300 mm. The band width of the fibers is 6 mm. Assume a dwell angle of 180° at each end to start out with. Assume that there is no overlap nor gap between adjacent bandwidths.

Determine:

- *The number of circuits per pattern,*
- *The number of patterns per layer,*
- *The number of circuits per layer.*

You are free to adjust the dwell angle and the bandwidth slightly to obtain your results.

Solution:

First, define the *reference circle* as the circle of the cross section area of the cylinder at one end, say the left end. Assume that winding starts from a point B on that circle.

In one circuit, the feed moves twice the length of the mandrel. This means that forward once and backward once along the length of the mandrel.

When a pattern is complete, a set of circuits have been made such that the fiber path returns to the initial position as before the beginning of the pattern.

Equation (5.2) gives the number of revolutions required for the fiber feed to move a distance h which is equal to the length of the cylinder. For a circuit, two cylinder lengths need to be traveled. The corresponding number of revolutions will therefore be:

$$2n = \frac{2h \tan \alpha}{\pi D} = \frac{(2)(1300 \text{ cm}) \tan 54}{\pi(300 \text{ cm})} = 3.8 \quad (\text{a})$$

The corresponding number of degrees is:

$$(3.8)(360) = 1368^\circ \quad (\text{b})$$

In addition to the number of degrees in equation (b), one has to add two times the dwell angle in order to obtain the total number of degrees required to make a circuit. This gives:

$$\theta = 1368 + 2(180) = 1728^\circ \quad (c)$$

If one subtracts the above number by a whole multiple of 360° , one would obtain the angular advance of starting point (new point B1') as compared to starting point B1 on the reference circle. This angular advance is $1728 - (360)(4) = 288^\circ$.

In order to make a pattern, one needs to have a multiple of the advance angles such that this multiple will be equal to a multiple of 360° . This can be expressed as:

$$(m)(288) = (n)(360) \quad (d)$$

where m and n are integers and should be as small as possible.

Equation (d) shows that m and n can be quite large before the equation is satisfied. This may not be practical. In order to reduce the numbers m and n , one needs to adjust the operation to make the advance angle a good whole number. One good whole number close to 288 is 360. This can be done by adjusting the dwell angle to be $180 + 72/2 = 216^\circ$. (This can be done by adjusting the machine setting). If this is done, equation (d) becomes:

$$\begin{aligned} 360m &= 360n \\ m &= n = 1 \end{aligned} \quad (e)$$

What this means is that it takes 1 time the advance angle (or 1 circuit) to make a pattern.

Notes:

In the pattern calculated above, the fiber band will go back exactly to the same position on the reference circle as at the beginning of the winding process. This may not be desirable since if one continues with this process, the fiber will follow the same path as before and one may not be able to cover the whole surface of the mandrel. It is desirable to advance the position B1' one bandwidth distance along the circumferential direction after one pattern. This distance in angular

value can be calculated to be (Note that the circumferential coverage of a band width b is $b/\cos\alpha$):

$$\Delta\theta = \frac{b}{\pi D \cos \alpha} (360) = \frac{6}{\pi(300) \cos(54)} (360) = 3.9^\circ \quad (\text{f})$$

This advanced angular value is accumulated over 1 circuit. This angle is then divided by two dwell angles. The dwell angle is then adjusted to be: $216 + 3.9/2 = 218^\circ$.

The number of circuits/patterns per layer:

From equation 5.4:

$$C = \frac{\pi(300) \cos 54}{6} = 92.2$$

Since this is not a whole number, it needs to be adjusted. One way to do that is by adjusting the bandwidth. If the number C is 92, then the bandwidth needs to be: 6.02 mm.

Summary:

The dwell angle needs to be 218° . The bandwidth needs to be 6.02 mm. With this, there are 92 circuits to make up a layer. There are also 92 patterns per layer.

2. Question 2:

A pipe of 300 mm in diameter is filament wound with fiber glass. Two types of wind angle will be used. One is 30° and the other is 45° . Assume that there is 1/3 of the thickness of the 30° types and 2/3 of the thickness of the 45° type. Assume that the strength of the fiberglass used in this case is 1360 MPa. Determine the thickness of each type of winding to contain an internal pressure of 41 MPa.

Solution:

$$\sigma_y = \frac{PR}{t} = \frac{(45\text{MPa})(0.25\text{m})}{(6.36 \times 10^{-3} \text{m})} = 1772\text{MPa}$$

Strength of the pipe:

Along axial direction:

$$\sigma_a t = \sigma_f t_1 \cos^2 \alpha_1 + \sigma_f t_2 \cos^2 \alpha_2$$

$$\sigma_a = (1360MPa)(1/3)\cos^2 30 + (1360MPa)(2/3)\cos^2 45 = 340 + 453 = 793MPa$$

Along hoop direction:

$$\sigma_c t = \sigma_f t_1 \sin^2 \alpha_1 + \sigma_f t_2 \sin^2 \alpha_2$$

$$\sigma_c = (1360MPa)(1/3)\sin^2 30 + (1360)(2/3)\sin^2 45 = 113.3 + 453.3 = 566.6MPa$$

Pipe thickness considering the axial direction:

$$\sigma_x = \frac{PR}{2t} = \frac{(41MPa)(0.15m)}{2t} 793MPa$$

$$t = 0.00387m = 3.87mm$$

Pipe thickness considering the hoop direction:

$$\sigma_y = \frac{PR}{t} = \frac{(41MPa)(0.15m)}{t} 566.6MPa$$

$$t = 0.0109m = 10.9mm$$

The pipe thickness needs to be big enough in order for the pipe not to fail along either the axial or hoop direction. As such, the total wall thickness of the pipe should be 10.9 mm.

Thickness of the 30° layers: 3.63 mm

Thickness of the 45° layers: 7.27 mm