Solution for homework # 6

Question 1:

Using the following parameters from example 4.1 in the book.

Composite density: $\rho = 1580\ kg/m^3$

Composite specific heat: $C_p = 870\ J/(kg.K)$

Composite thermal conductivity along thickness direction: $K_z = 0.69\, W/(m.K)$

Total heat generated during the whole curing cycle: $H_T = 150\, J/g = 2.37x10^8\ J/m^3$

Universal gas constant: $R = 8.31\, J/(mol.K)$

Aluminum thermal conductivity: $K_{Al} = 237\ W/(K.m)$

Thickness of aluminum mold: $t_1 = 12.7\, mm$

Thickness of bagging material on top: $t_2 = 1.0\, mm$

Glass cloth thermal conductivity: $K_{gc} = 0.26\ W/(K.m)$

Heat flux from Al mold to composite: $Q_1 = \dfrac{K_{Al}}{t_1}(T_\infty - T)$

Where T is the temperature of the composite, T∞ is the environment temperature.

Heat flux from the mold to composite via vacuum bag: $Q_2 = \dfrac{K_{gc}}{t_2}(T_\infty - T)$

Heat generated by the composite: $Q_c = \dfrac{d\alpha}{dt} H_T\ L$

Where L is the composite thickness, L = 2 mm.

Energy balance: $\rho C_p\, L \dfrac{dT}{dt} = Q_1 + Q_2 + Q_c$

Or $a\dfrac{dT}{dt} + bT + c = 0$

Where $a = \rho C_p\ L \qquad b = \dfrac{K_{Al}}{t_1} + \dfrac{K_{gc}}{t_2} \qquad c = -\left[Q_c + T_\infty\left(\dfrac{K_{Al}}{t_1} + \dfrac{K_{gc}}{t_2}\right)\right]$

Assuming: $T = C_1 e^{-mt} + C_2$ yields: $m = \dfrac{b}{a}$ and $C_2 = -\dfrac{c}{b}$

With initial conditions: When t = 0, $T = T_o$, $T_o = C_1 + C_2$. This yields: $C_1 = T_o + \frac{c}{b}$

Then: $T = \left(T_o + \frac{c}{b}\right)e^{-mt} - \frac{c}{b}$

Where $b = \frac{K_{Al}}{t_1} + \frac{K_{gc}}{t_2} = 18921 W/(m^2.K)$

$$m = \frac{b}{a} = \frac{b}{\rho C_p L} = \frac{18921}{(1580)(870)(2.0x10^{-3})}\sec^{-1} = 6.88\sec^{-1}$$

Therefore: $T - T_\infty = \left(T_o - \frac{Q_c + 18921T_\infty}{18921}\right)e^{-6.88t} + \frac{Q_c}{18921}$

The degree of cure at time increment (n+1):

$\alpha_{n+1} = \alpha_n + \left(\frac{d\alpha}{dt}\right)_n \Delta t$. Where Δt is the time increment.

Two types of curing cycles will be considered:

1. Linear increase in temperature: Temperature of autoclave increases from room temperature (20^{o}C) up to 180 ^{o}C at the rate of 5 ^{o}C/minute.
2. Two-step temperature increase: Temperature of autoclave increases from 20 ^{o}C to 110 ^{o}C at the rate of 5 ^{o}C/minute for 18 minutes. It is held constant at 110 ^{o}C for 20 minutes. Then it increases to 180 ^{o}C at the same rate. It is then held there for 60 minutes.

Solution:

1. ***Initial conditions:***

The initial degree of cure is assumed to be 0.1 to start.

Initial temperature of the autoclave: T_o = 20 ^{o}C = 293 ^{o}K.

Initial composite temperature: T = 293 ^{o}K.

$$K_1 = A_1 \exp\left(-\frac{\Delta E_1}{RT}\right) = (2.101x10^9)\exp\left(-\frac{8.07x10^4}{(8.31)(293)}\right)\min^{-1} = 8.48x10^{-6}\min^{-1}$$

. $$K_2 = A_2 \exp\left(-\frac{\Delta E_2}{RT}\right) = -(2.014x10^9)\exp\left(-\frac{7.78x10^4}{(8.31)(293)}\right)\min^{-1} = -2.67x10^{-5}\min^{-1}$$

$$K_3 = A_3 \exp\left(-\frac{\Delta E_3}{RT}\right) = (1.96x10^5)\exp\left(-\frac{5.66x10^4}{(8.31)(293)}\right)\min^{-1} = 1.57x10^{-5}\min^{-1}$$

$$\frac{d\alpha}{dt} = (K_1 + K_2\alpha)(1-\alpha)(B-\alpha) = 1.93x10^{-6}\ \min^{-1}$$

Viscosity: $\mu = \mu_\infty \exp\left(\frac{U}{RT} + k\alpha\right) = (7.93x10^{-14})\exp\left(\frac{9.08x10^4}{8.31x293} + 14.1x0.1\right) = 5098\,Pa - \sec$

2. ***Increment 1:*** (0 to 5 minutes), T∞ varies from 20 °C to 50 °C. Average T∞ = 305.5 °C.

$$K_1 = 3.29x10^{-5}\ \min^{-1} \qquad K_2 = -9.88x10^{-5}\ \min^{-1}$$

$$Q_c = \frac{(3.29 - 9.88x0.1)x10^{-5}\,x(1.0.1)x(0.47-0.1)}{60}x(2.37x10^8)(2.0x10^{-3}) = 0.0605W/m^2$$

$$T - 305.5 = \left[293 - \frac{0.0605 + 18921x305.5}{18921}\right]e^{-(6.88)(300)} + \frac{0.0605}{18921} = 305.5\ {}^oK$$

$$\frac{d\alpha}{dt} = (K_1 + K_2\alpha)(1-\alpha)(B-\alpha) = 7.66x10^{-6}\ \min^{-1}$$

$$\alpha_1 = 0.1 + (7.66x10^{-6})(\frac{300}{60}) = 0.1$$

$$\mu = \mu_\infty \exp\left(\frac{U}{RT} + k\alpha\right) = (7.93x10^{-14})\exp\left(\frac{9.08x10^4}{8.31x305.5} + 14.1x0.1\right) = 1109\,Pa - \sec$$

For subsequent time increments:

Using the same procedure for subsequent time increments, subsequent values of the temperature of the composite, degree of cure, and viscosity can be obtained. Table 1 shows the results for the linear increase in temperature. Table 2 shows the results for the two step cure cycle. Figures 1 and 2 show the variations of the viscosity as a function of time for the two curing cycles.

Whether the resin flow can go though the thickness of the laminate in time:

According to example 4.3, initial fiber volume fraction of V_o = 0.5, allowable fiber volume fraction V_a = 0.85, and final fiber volume fraction V_f = 0.68. fiber modulus E = 234 GPa. Fiber waviness ratio β = 300.

Stress in the fiber bundle:

$$\sigma_b = \frac{3\pi E}{\beta^4}\frac{1-\sqrt{\frac{V_f}{V_o}}}{\left(\sqrt{\frac{V_a}{V_f}}-1\right)^4} = \frac{3\pi(234GPa)}{300^4}\frac{1-\sqrt{\frac{0.68}{0.5}}}{\left(\sqrt{\frac{0.85}{0.68}}-1\right)^4} = 232.7\,KPa$$

Pressure in resin: $P_r = P_a - P_f = P_a - \sigma_b = 344 - 232.7 = 111.3\,KPa$

Assume $r_f = 3.5\,\mu m \qquad k_o = 18$ (example 4.3).

Permeability transverse to the fiber direction:

$$K = \frac{r_f^2}{4k_o}\frac{(1-v_f)^3}{v_f^2} = \frac{(3.5x10^{-6}m)^2}{4(18)}\frac{(1-0.68)^3}{0.68^2} = 1.2x10^{-14}m^2$$

Table 1 Progression of cure (linear increase in autoclave temperature)

Time (min)	T_a (°C)	T_∞ (°C)	T_∞ (K)	Q_c (W/m^2)	T (K)	Rate of Cure (min^{-1})	Degree of Cure(α)	Viscosity (Pa·sec)
0	20	20	293	9.16E-03	293	1.93E-06	0.100	5097.9
5	45	32.5	305.5	3.63E-02	305.5	7.66E-06	0.100	1109.1
10	70	57.5	330.5	4.16E-01	330.5	8.78E-05	0.100	74.6
15	95	82.5	355.5	3.37E+00	355.5	7.10E-04	0.104	7.7
20	120	107.5	380.5	2.02E+01	380.5	4.26E-03	0.125	1.4
25	145	132.5	405.5	8.54E+01	405.5	1.80E-02	0.215	0.8
30	170	157.5	430.5	1.70E+02	430.5	3.59E-02	0.395	2.2
35	180	175	448	1.40E+02	448	2.96E-02	0.543	6.5
40	180	180	453	1.25E+02	453	2.65E-02	0.675	32.3
45	180	180	453	8.91E+01	453	1.88E-02	0.769	121.5
50	180	180	453	6.33E+01	453	1.34E-02	0.836	311.6
55	180	180	453	4.50E+01	453	9.49E-03	0.883	608.5
60	180	180	453	3.20E+01	453	6.75E-03	0.917	979.1
65	180	180	453	2.27E+01	453	4.80E-03	0.941	1372.9
70	180	180	453	1.62E+01	453	3.41E-03	0.958	1745.7

During the period when the viscosities are low, their values are about 1 Pa-sec (see the figures)

Using Darcy's law: $u_z = \frac{K}{\mu}\frac{dp}{dz} = \frac{1.2x10^{-14}m^2}{1Pa-\sec}\frac{111.3KPa}{2.0mm} = 6.68x10^{-4}\,mm/\sec$

Time required for the resin to flow across the thickness of the laminate:

$$t = \frac{thickness}{u_z} = \frac{2mm}{6.68x10^{-4}\,mm/\sec} = 2994\sec = 49.9\min utes$$

Examining the curves in the figures, the duration where the viscosities are low is about 30 minutes (from 10 minutes to 40 minutes) for the linear increase cycle, and this duration is about

35 minutes (from 10 minutes to 45 minutes) for the two step cure cycle. As such, both schedules of cure are not adequate for the flow.

Table 2 Progression of cure (two-step increase in autoclave temperature)

Time (min)	T_a (°C)	T_∞ (°C)	T_∞ (K)	Q_c (W/m^2)	T (K)	Rate of Cure (min^{-1})	Degree of Cure (α)	Viscosity (Pa·sec)
0	20	20	293	9.16E-03	293	1.93E-06	0.100	5097.9
5	45	32.5	305.5	3.63E-02	305.5	7.66E-06	0.100	1109.1
10	70	57.5	330.5	4.16E-01	330.5	8.78E-05	0.100	74.6
15	95	82.5	355.5	3.37E+00	355.5	7.10E-04	0.104	7.7
18	110	102.5	375.5	1.43E+01	375.5	3.02E-03	0.113	1.7
23	110	110	383	2.24E+01	383	4.73E-03	0.137	1.3
28	110	110	383	1.88E+01	383	3.97E-03	0.157	1.8
33	110	110	383	1.61E+01	383	3.39E-03	0.173	2.2
38	110	110	383	1.39E+01	383	2.94E-03	0.188	2.8
42	135	122.5	395.5	3.64E+01	395.5	7.68E-03	0.195	1.2
45	160	147.5	420.5	1.18E+02	420.5	2.48E-02	0.319	1.4
50	180	170	443	5.32E+02	443	1.12E-01	0.768	207.0
55	180	180	453	6.35E+01	453	1.34E-02	0.809	211.8
60	180	180	453	5.25E+01	453	1.11E-02	0.864	462.4
65	180	180	453	3.73E+01	453	7.87E-03	0.903	805.6
70	180	180	453	2.65E+01	453	5.60E-03	0.931	1195.2
80	180	180	453	1.88E+01	453	3.98E-03	0.951	1581.9

increase and a two-step increase in temperature of the autoclave.

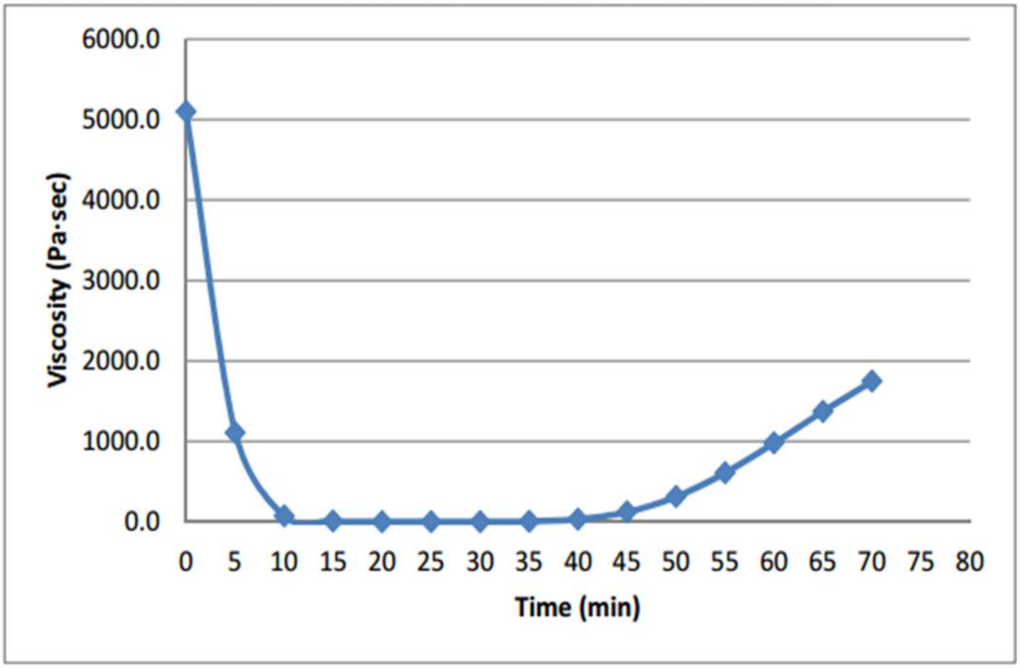

Fig. 1 Variation of viscosity as a function of time (linear increase in autoclave temperature)

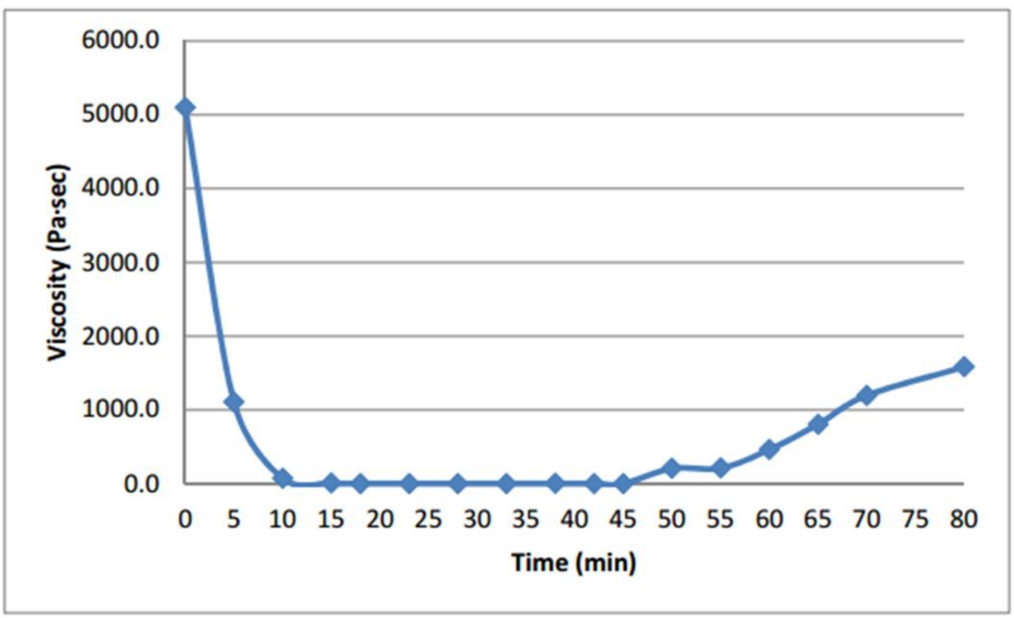

Fig. 2 Variation of viscosity as a function of time (two-step increase in autoclave temperature)