

**MAT 2371**  
**Solutions for the midterm 2019**

**Part A: Multiple choice questions**

Question	1	2	3	4	5
Réponse	E	D	C	A	E

1. Let  $X$  be the number of defective objects in the sample of size  $n = 3$ .  $X$  has a hypergeometric distribution with  $N = 20$ ,  $N_1 = 4$ , and  $n = 3$ . We want

$$P(X = 2) = \frac{\binom{N_1}{2} \binom{N_2}{n-2}}{\binom{N}{n}} = \frac{\binom{4}{2} \binom{16}{1}}{\binom{20}{3}} = \frac{96}{1140}.$$

The answer is E.

2. Let  $X$  be the required number of people that the journalist meets to obtain someone that is willing to speak.  $X$  has a geometric distribution with  $p = 2/3$ . We want

$$P(X = 4) = (1 - p)^3 p = (1/3)^3 (2/3) = 2/81.$$

The answer is D.

3. Let  $X$  be the number of abandoned cars next week.  $X$  has a Poisson distribution with mean  $\mu = 1.7$  cars. We want

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - P(X = 0) - P(X = 1) = 1 - e^{-1.7} \frac{1.7^0}{0!} - e^{-1.7} \frac{1.7^1}{1!} = 0.507$$

The answer is C.

4. Let  $A$  be the event of receiving the packet without error, and let  $B$  be the event that the least reliable channel is used. We have  $P(B) = 1 - 0.75 = 0.25$ , and  $P(A|B) = 0.7$ . We want

$$P(A \cap B) = P(A|B)P(B) = 0.7(0.25) = 0.175.$$

The answer is A.

5. Let  $X$  be the total number of tosses of the 6-sided die.  $X$  has a geometric distribution with  $p = 1/6$ . My net gain is  $W = 2(X - 1) - 10 = 2X - 12$ . We want

$$E[2X - 12] = 2X - 12 = 2(1/p) - 12 = 2(6) - 12 = 0.$$

The answer is E.

## Part B: Short Answer Questions

1. a) The events  $A, B, C$  are not mutually exclusive. If they were, this would imply for example that  $A \cap B \neq \emptyset$ , and thus  $P(A \cap B) = P(\emptyset) = 0$ . But, by independence, we have  $P(A \cap B) = P(A)P(B)$ . Thus,  $P(A \cap B) = (0.5)(0.2) = 0.10 \neq 0$ . Therefore, the events cannot be mutually exclusive.

b) We want

$$\begin{aligned}P(A \cup B \cup C) &= 1 - P(A' \cap B' \cap C') \\&= 1 - P(A')P(B')P(C') \quad (\text{by independence}) \\&= 1 - (0.5)(0.8)(0.3) = 0.72.\end{aligned}$$

Alternatively, we could use the addition rule:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

By independence,

$$\begin{aligned}P(A \cap B) &= P(A)P(B) = (0.5)(0.2) = 0.10 \\P(A \cap C) &= P(A)P(C) = (0.5)(0.3) = 0.15 \\P(B \cap C) &= P(B)P(C) = (0.2)(0.3) = 0.06 \\P(A \cap B \cap C) &= P(A)P(B)P(C) = (0.5)(0.2)(0.3) = 0.03\end{aligned}$$

Thus,

$$P(A \cap B \cap C) = 0.5 + 0.2 + 0.3 - 0.10 - 0.15 - 0.06 + 0.03 = 0.72.$$

c) Let  $D = A \cup B \cup C$ . We want

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{P(A)}{P(D)} = \frac{0.5}{0.72} = \frac{5}{72}$$

We use the fact that  $A \cap D = A$ .

2. a) The support of  $X$  is  $\{1, -1\}$ .

b) We have

$$P(\text{both are red}) = P(\text{both are blue}) = \frac{\binom{5}{2}}{\binom{10}{2}} = \frac{10}{45} = \frac{2}{9}.$$

Thus,

$$f_X(1) = P(X = 1) = P(\text{both are red}) + P(\text{both are blue}) = \frac{4}{9}.$$

Since  $f_X(-1) + f_X(1) = 1$ , then  $f_X(-1) = 1 - 4/9 = 5/9$ . Thus, the pmf is

$$f_X(x) = \begin{cases} 5/9 = 0.5556, & x = -1 \\ 4/9 = 0.4444, & x = 1 \end{cases}$$

c) The expected gain is

$$E[X] = (-1)(5/9) + (1)(4/9) = -1/9 = -0.1111.$$

3. (a) Since  $\int_{-\infty}^{\infty} f(x) dx = 1$ , then

$$1 = \int_0^a f(x) dx = \int_0^a 4x^3 dx = x^4 \Big|_0^a = a^4.$$

Thus,  $a$  is 1 or -1. But, since  $f(x) \geq 0$  for all  $x$ , then  $a$  must be positive. Thus,  $a = 1$ .

(b) We want

$$P(X < 1/2) = \int_0^{1/2} 4x^3 dx = x^4 \Big|_0^{1/2} = (1/2)^4 = 1/16 = 0.0625.$$

(c) We want

$$E[X] = \int_{-\infty}^{\infty} x f(x) dx = \int_0^1 4x^4 dx = (4/5)x^5 \Big|_0^1 = 4/5 = 0.8.$$

4. (a)  $X$  has a binomial distribution with  $n = 5$  and  $p = 0.8$ .

(b) We want

$$P(X \leq 1) = \binom{5}{0} (0.8)^0 (0.2)^5 + \binom{5}{1} (0.8)^1 (0.2)^4 = (0.2)^5 + 5(0.8)(0.2)^4 = 0.00672.$$

(c) The mean of  $X$  is  $\mu_X = E[X] = np = (5)(0.8) = 4$  and its standard deviation is

$$\sigma_X = \sqrt{np(1-p)} = \sqrt{5(0.8)(0.2)} = 0.8944.$$

5. Let  $A$  be the event that a paint customer buys latex paint, and let  $B$  be the event that a paint customer purchases a roller. We know that  $P(A) = 0.75$ ,  $P(A') = 0.25$ ,  $P(B|A) = 0.6$ , and  $P(B|A') = 0.3$ .

(a) We want

$$P(B) = P(B|A)P(A) + P(B|A')P(A') = (0.6)(0.75) + (0.3)(0.25) = 0.525.$$

(b) We want

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{(0.6)(0.75)}{0.525} = 0.8571.$$