Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Midterm: MAT 2371 (Fall 2019)

Professor: G. Lamothe
Duration: 80 minutes **October 29, 2019**

Name: ____________________ Student Number: ____________________

You must sign below.

Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: you will be asked to leave immediately the exam and academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

By signing below, you acknowledge that you have ensured that you are complying with the above statement.

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- This is a closed-book examination. However, one sheet (2-sided) is permitted.
- A non-graphic and non-programmable calculator is permitted.
- This examination has two parts: A (multiple choice) and B (short answer). For part A, provide your answer for each question in the given table found in the next page. For part B, give your answer in the provided space of this questionnaire.
- Each question is worth 10 points.
- At the end of the exam, please submit the questionnaire in its entirety including your sheet.

Part A: Multiple Choice Questions 10 points par question.

Record your answer to each question in the table below:

Question	1	2	3	4	5
Answer					

1. A sample of 3 objects is chosen randomly without replacement from a box containing 20 objects, where 4 of these objects are defective. Give the probability that the number of defective objects in the sample is exactly 2.

   A) 2/4 B) 6/1140 C) 2/190 D) 27/520 E) 96/1140

2. A journalist has to interview a number of people. Potential interviewees agree to speak with a probability of 2/3, independently of each other. What is the probability that the journalist should interview exactly 4 people to meet a person who agrees to speak?

   (A) 8/27 (B) 8/81 (C) 1/27 (D) 2/81 (E) 9/56

3. Assume that the number of cars abandoned each week on a certain highway is a Poisson variable with a mean of 1.7 cars. Calculate the probability that there will be least 2 abandoned cars next week.

   (A) 0.594 (B) 0.761 (C) 0.507 (D) 0.345 (E) 0.243

4. Consider two transmission channels. In the most reliable channel, a packet will be received without error with a probability of 0.99. However, if we use the least reliable channel, this probability of receiving the packet without error is only 0.7. Suppose that 75% of packets are transmitted in the most reliable channel. Calculate the probability that a packet will be received without error and that it was transmitted in the least reliable channel.

   (A) 0.175 (B) 0.25 (C) 0.7 (D) 0.7425 (E) 0.2475

5. My friend and I are playing the following game: I successively throw a 6-sided die and my friend give me 2 dollars for each toss of the die that does not result in a 6. When the toss of the die gives a 6 for the first time, I must pay my friend 10 dollars, and the game stops. Let W be my net gain at the end of the game, where W can be negative if I lost money. Note that W is the amount (in dollars) given to me by my friend minus 10 dollars. Determine my expected net gain, that is compute $E[W]$.

   (A) 6 (B) 2 (C) 1/6 (D) -10 (E) 0

Part B: Short Answer Questions 10 points par question.
Give your answer directly on the questionnaire. You must properly and clearly define your notation, and show your work.

1. Let A, B and C be independent events such that $P(A) = 0.5$, $P(B) = 0.2$ and $P(C) = 0.3$.
   a) (2 points) Are A, B and C mutually exclusive events?
   b) (6 points) Give the probability that at least one of A, B or C will occur.
   c) (2 points) What is the probability that A will occur, given that A, B or C has occurred?

2. A box contains 5 red marbles and 5 blue marbles. Two marbles are chosen at random (without replacement). If they are the same colour, you win \$1; if they are of different colors, you lose \$1. Let X be the gain for this game (that is the amount won minus the amount lost).

   a) (1 points) Give the support for X.
   b) (6 points) Give the probability mass function for X.
   c) (3 points) Give the expected gain for this game.

3. Consider a continuous random variable X with the following probability density function:

$$f(x) = 4\,x^3, \quad 0 < x < a.$$

(a) (2 points) Give the value of a.
(b) (4 points) Compute $P(X < a/2)$.
(c) (4 points) Compute $E[X]$

4. Let X be a random variable with the following moment generating function

$$M(t) = (0.2 + 0.8\, e^t)^5.$$

(a) (2 points) Give the distribution of X.
(b) (4 points) Compute $P(X \leq 1)$.
(c) (4 points) Compute the mean and the standard deviation of X.

5. A paint-store chain produces and sells latex and semigloss paint. Based on long-range sales, the probability that a paint customer will purchase latex paint is 0.75, while the probability is 0.25 for semigloss. Of those that purchase latex paint, 60% also purchase rollers. But only 30% of semigloss paint buyers purchase rollers. We randomly select a customer that purchased paint.
   (a) (5 points) What is the probability that this customer has purchased a roller.
   (b) (5 points) The selected paint customer has purchased a roller. What is the probability that this customer has purchased latex paint?

Scratch Paper

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