Math 1107 E - Fall 2019 : Assignment 1

1. let $z = -5+3i$. Evaluate each of the following exactly. Your answers should be as simple as possible and in rectangular form. Show your work

a. $\bar{z}$

$= -5 - 3i$

b. z^{-1}

$= \frac{a}{a^2+b^2} - \frac{b}{a^2+b^2} i$

$= \frac{-5}{(-5)^2+(3)^2} - \frac{3}{(-5)^2+(3)^2} i$

$= -\frac{5}{34} - \frac{3}{34} i$

c. z^2

$= (-5+3i)(-5+3i)$

$= 25 - 15i - 15i + 9i^2$

$= 25 - 9 - 30i$

$= 16 - 30i$

d. $|z|$

$= \sqrt{a^2+b^2}$

$= \sqrt{(-5)^2+(3)^2}$

$= \sqrt{25+9}$

$= \sqrt{34}$

$= 5.8310$

2. Find all real numbers a such that $\left|\frac{a-2i}{5-i}\right| = 2$. Justify your answer

$\left|\frac{a-2i}{5-i}\right| = \left|\frac{z}{w}\right| = \frac{|z|}{|w|}$

$\hookrightarrow \frac{|a-2i|}{|5-i|} = \frac{\sqrt{a^2+2^2}}{\sqrt{5^2+1^2}} = 2$

$\frac{\sqrt{a^2+4}}{\sqrt{25+1}} = 2$

$\frac{\sqrt{a^2+4}}{\sqrt{26}} = 2$

$\sqrt{a^2+4} = 2\sqrt{26}$

$a^2+4 = (2\sqrt{26})^2$

$a^2+4 = 4(26)$

$a^2 = 100$

$a = \pm 10$

3. Let P_2 denote the set of polynomials in x with real coefficients having degree at most two. Let $f: P_2 \to \mathbb{R}^2$ be given by

$$f(ax^2+bx+c) = \begin{bmatrix} a+2c \\ b-c \end{bmatrix}$$

a. What is $f(-5x+3)$?

$-5x+3 = 0 \cdot x^2 - 5x + 3$

$\hookrightarrow a=0, b=-5, c=3$

$f(-5x+3) = \begin{bmatrix} 0+2(3) \\ -5-3 \end{bmatrix}$

$f(-5x+3) = \begin{bmatrix} 6 \\ -8 \end{bmatrix}$

b. Give an element $p \in P_2$ such that

$$f(p) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Justify your answer.

$f(ax^2+bx+c) = \begin{bmatrix} a+2c \\ b-c \end{bmatrix}$

$f(p)$ $\quad$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

so, $a+2c=1$ and $b-c=0$

$a+2b=1$ $\quad$ $b=c$

$a=1-2b$

thus, $p = ax^2+bx+c$

$p = (1-2b)x^2+bx+b$

4. Simplify $5i\begin{bmatrix} 1+i \\ -2 \end{bmatrix} - 3\begin{bmatrix} 1 \\ i \end{bmatrix}$ as much as possible. Show your work

$$5i\begin{bmatrix} 1+i \\ -2 \end{bmatrix} - 3\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$= \begin{bmatrix} 5i(1+i) \\ 5i(-2) \end{bmatrix} + \begin{bmatrix} -3(1) \\ -3(i) \end{bmatrix}$$

$$= \begin{bmatrix} 5i+5i^2 \\ -10i \end{bmatrix} + \begin{bmatrix} -3 \\ -3i \end{bmatrix}$$

$$= \begin{bmatrix} 5i+5i^2-3 \\ -10i-5i \end{bmatrix}$$

$$= \begin{bmatrix} 5i+5(-1)-3 \\ -10i-3i \end{bmatrix}$$

$$= \begin{bmatrix} 5i-5-3 \\ -7i \end{bmatrix}$$

$$= \begin{bmatrix} -8+5i \\ -7i \end{bmatrix}$$