THE UNIVERSITY OF BRITISH COLUMBIA

Mathematics MATH 104 (104)

Mock Midterm #2

26 October 2009

TIME: 50 MINUTES

FULL NAME: ______________________ STUDENT # : ______________

SIGNATURE: ______________________

This Examination paper consists of 7 pages (including this one). Make sure you have all 7.

INSTRUCTIONS:

No memory aids allowed. ONE Sharp EL-510R calculator allowed. No communication devices allowed.

MARKING:

Q1	/9
Q2	/7
Q3	/10
Q4	/12
Q5	/12
TOTAL	/50

NAME OF INSTRUCTOR: Mark Mac Lean

Q1 [9 marks]

Find the derivatives of the following functions:

(a) $f(x) = x^2 e^{\frac{1}{x}}$

$$f'(x) = e^{1/x}(2x - 1)$$

(b) $f(x) = 2^{ex} e^{2x}$

$$f'(x) = 2^{ex} e^{2x}(e \ln 2 + 2)$$

(c) $f(x) = \dfrac{x^3 + 2x + 6}{e^x + \ln x}$

$$f'(x) = \frac{(3x^2 + 2)(e^x + \ln x) - (x^3 + 2x + 6)(e^x + 1/x)}{(e^x + \ln x)^2}$$

Q2 [7 marks]

Find the equation of the tangent line to the curve given by $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at the point $\left(0, \frac{1}{2}\right)$.

$$2x + 2y\frac{dy}{dx} = 2(2x^2 + 2y^2 - x)(4x + 4y\frac{dy}{dx} - 1)$$

$$\frac{dy}{dx} = 1$$

$$y = x + 1/2$$

Q3 [10 marks]

Let $f(x) = (x^2 - 1)^{2/3}$.

(a) (2 marks) Find the domain of $f(x)$. All real x

(b) (4 marks) Determine the x-coordinates of the local maxima and minima (if any) and intervals where $f(x)$ is increasing or decreasing.

$$f'(x) = \frac{4x}{3\sqrt[3]{x^2-1}}$$

Increase $x \in (-1, 0)$ and $(1, \infty)$.
Decrease $x \in (0, 1)$ and $(-\infty, -1)$.
Local min at $x = \pm 1$
Local max at $x = 0$

(c) (2 marks) Determine intervals where $f(x)$ is concave upwards or downwards, and the x-coordinates of inflection points (if any). You may use, without verifying it, the formula $f''(x) = \frac{4}{9}\frac{(x^2-3)}{(x^2-1)^{4/3}}$.

Concave up $x \in (-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$.
Concave down $x \in (-\sqrt{3}, -1)$, $(-1, 1)$ and $(1, \sqrt{3})$.
Inflection points $x = \pm\sqrt{3}$

(d) (2 marks) Find any x-intercepts and y-intercepts.

Intercepts: x: (-1,0) and (1,0).
y: (0,1)

(e) (2 marks) Find any vertical, horizontal, or slant asymptotes for $y = f(x)$.

No asymptotes.

(f) (4 marks) Sketch the graph of $y = f(x)$, showing the features given in items (a) to (e) above and giving the (x, y) coordinates for all points occurring above.

Q4 [12 marks]

Liquid is being poured into a parabolic bowl at a constant rate of 60π cm^3 s^{-1}. The volume of the bowl is given by $V = \frac{\pi x^4}{2}$, where the equation of the parabola is $y = x^2$, where y is the height of liquid in the bowl. Find the rate of increase of the height of the liquid in the bowl when the height is 10 centimetres.

Answer:

$$V = \frac{\pi x^4}{2} = \frac{\pi y^2}{2}$$

$$\frac{dV}{dt} = \pi y \frac{dy}{dt}$$

$$60\pi = \pi y \frac{dy}{dt}$$

$$\frac{dy}{dt} = 6 \text{ cm/s}$$

Q5 [12 marks]

A store manager wants to establish an optimal inventory policy for an item. Sales are expected to be at a steady rate and should total 10 000 items sold during the year. Each time an order is placed, a cost of \$49 is incurred. Carrying costs for the year will be \$2 per item, to be figured on the average number of items in storage during the year. How many items should the store manager order each time she orders to minimize the total inventory cost (which is the total ordering cost plus the total carrying cost)?

Answer:

Ordering cost = $49r$ and Carrying cost = $2(x/2) = x$. Cost is,

$$C = 49r + x \qquad rx = 10000$$

$$C(x) = \frac{490000}{x} + x$$
$$C'(x) = -\frac{490000}{x^2} + 1$$

$$x = \sqrt{490000} = 700$$