The University of British Columbia

MATH 104 (102)

Mock Midterm 1

24 September 2008

Time: 50 minutes

FULL NAME: _____ STUDENT # : _____

SIGNATURE: _____

This Examination paper consists of 7 pages (including this one). Make sure you have all 7.

instructions:

No memory aids allowed. No calculators allowed. No communication devices allowed.

marking:

Q1	/10
Q2	/10
Q3	/10
Q4	/4
Q5	/6
Q6	/10
TOTAL	/50

Name of Instructor: Mark MacLean

Q1 [10 marks]

Compute the following limits.

(a) $\lim_{t \to 2} \frac{t^2 - 4}{t^3 - 8}$

(b) $\lim_{x \to 0} \frac{1 - \sqrt{1 - x^2}}{x}$

(c) $\lim_{x \to \infty} \frac{5x + 3}{3x - 2}$

Q2 [10 marks]

(*a*) Use the notion of continuity to show that $x^7 = x - 2$ has at least one real root (i.e. there is at least one value of x that solves this equation).

(*b*) Find the constant c that makes

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4, \\ cx + 20 & \text{if } x \geq 4. \end{cases}$$

a continuous function.

Q3 [10 marks]

(*a*) Use the definition of the derivative to compute $f'(1)$ for $f(x) = \sqrt{3x+1}$. No credit will be given for any other method of computing this derivative.

(*b*) Use the result of (a) to find the equation of the tangent line to the curve $y = \sqrt{3x+1}$ at $x = 1$.

Q4 [4 marks]

The function $f(x) = b/(x^2 + ax + 2)$ at $x = 1$ has value of 2 and its derivative is 0. Find the values of a and b.

Q5 [6 marks]

Find a and b to make $f(x)$ differentiable

$$f(x) = \begin{cases} ax + b & \text{if } x > 0 \\ 2e^x & \text{if } x \leq 0 \end{cases}$$

Q6 [10 marks]

Mondo Inc. estimates that it can sell 3400 of its premium espresso machine, the Caffeinator, at a price of \$900 each, but it will lose 100 sales for each \$50 increase in price of the machine. Let p be the price and q be the number of machines sold.

(a) Write q as a linear function of p.

(b) Write the revenue R as a function of p, and find the marginal revenue, MR, with respect to p.

(c) Suppose it costs \$400 to manufacture the espresso machines. Write the cost C as a function of p and then find the marginal cost, MC, with respect to p.

(*d*) Find the marginal profit, MP, with respect to p.

(*e*) Last year, Mondo Inc. sold the Caffeinator for \$1050. If it increases its price this year, will it increase or decrease its profit? Explain your answer.