MATH 104 PRACTICE EXAM

1. Suppose that $f(x)$ is a function such that $f(3)=1$ and $f'(3)=-2$. Let $g(x)=\ln\left(x^2+\sqrt{f(x)}\right)$. Find $g'(3)$. [10 Marks]

2. Find an equation for the tangent line to the curve $(x^3+y^3)^2=4xy^2$ at the point $(1,1)$. [10 Marks]

3. Let $f(x)=x\ln x$. Compute the 4th degree Taylor polynomial of $f(x)$ about $a=5$. [10 Marks]

4. The demand q for a product is a function of the unit price p. The *elasticity of demand* $E(p)$ is defined by

$$E(p)=-\frac{p}{q}\frac{dq}{dp}.$$

Assume that p and q satisfy the demand equation $q=4\sqrt{\frac{60-p}{p}}$ for $0<p<60$.

(a) Find the elasticity of demand when $p=\$50$. [5 Marks]

(b) Use the result of part (a) to estimate the percentage change of demand when the price increases 0.7% from \$50. [5 Marks]

5. Let $f(x)=x^2-5x+2-3\ln x$. Find all values of x at which $f(x)$ attains a local maximum, and all values of x at which $f(x)$ attains a local minimum. Justify in each case that you have found a local maximum or local minimum. [10 Marks]

6. Let $f(x)=x^4e^{-x}$. Determine where the graph of $f(x)$ is concave up. [10 Marks]

7. A company sells q desks per month. The monthly cost is $C(q)=600+30q$ and the demand equation is $p=180-30\ln q$ where q is the number of desks sold per month at price p. Find the maximum profit, the production level that will realize the maximum profit, and the price the company should charge for each desk. [10 Marks]

8. (a) Use a linear approximation to obtain an estimate of $\sqrt{26}$. [5 Marks]

(b) Use a quadratic approximation to obtain a better estimate. [5 Marks]

9. A zero-coupon bond is a bond that is sold for less than its face value and has no periodic interest payments. Instead, the bond is redeemed for its face value at maturity. Thus, in a sense, interest is paid at maturity. Suppose that a zero-coupon bond sells for \$400 and can be redeemed in 16 years for its face value of \$1,000.

(a) What is the nominal interest rate of the bond under continuous compounding? (A calculator-ready answer is enough.) [5 Marks]

(b) What is the doubling time under the interest rate scheme in part (a)? (Again, a calculator-ready answer is enough.) [5 Marks]

10. It is desired to use Newton's Method to find the value of x where the function $f(x)=x^4-x^3-2x+1$ has its absolute maximum. Find x_1 if $x_0=1$. Note that you are only required to find x_1. [10 Marks]