BIT Calculus I: Exam with Answers December 2018

Answers: ceacb cdbab cddac cbcba addba cbb

1. The domain of the function $f(x) = \sqrt{(x-1)(x-3)}$ is

   (a) $[1,3]$, or equivalently, $\{x : \ 1 \leq x \leq 3\}$.

   (b) $(1,3)$, or equivalently, $\{x : \ 1 < x < 3\}$.

   (c) $(-\infty, 1] \bigcup [3, +\infty)$, or equivalently, $\{x : \ x \leq 1, \ \ x \geq 3\}$.

   (d) $(-\infty, 1) \bigcup (3, +\infty)$, or equivalently, $\{x : \ x < 1, \ \ x > 3\}$.

   (e) None of the above.

2. The domain of the function $g(x) = 2^x$ is

   (a) $[0, \infty)$, or equivalently, $\{x : \ x \geq 0\}$.

   (b) $(0, \infty)$, or equivalently, $\{x : \ x > 0\}$.

   (c) $(-\infty, 0]$, or equivalently, $\{x : \ x \leq 0\}$.

   (d) $(-\infty, 0)$, or equivalently, $\{x : \ x < 0\}$.

   (e) $(-\infty, \infty)$, or equivalently, $\mathbb{R}$, all real numbers.

3. Let $f(x) = 2x^3 - 1$ and $g(x) = \dfrac{1}{x}$. Then the value of the composition $f(g(1))$ is

   (a) 1. (b) $-\dfrac{1}{3}$. (c) -1. (d) -3. (e) None of the above.

4. If $e^{x+2} = 3$, what is x ?

   (a) $\ln 2$. (b) $3 - \ln 2$. (c) $\ln 3 - 2$. (d) $\dfrac{\ln 3}{2}$. (e) None of the above.

5. The expression $\dfrac{(x^{0.4})^2 \cdot x^{-3.8}}{x^2}$ simplifies to

(a) x^5. (b) x^{-5}. (c) x. (d) x^{-1}. (e) None of the above.

6. Which of the following is equal to $\log_2 8$?

(a) $\dfrac{1}{3}$. (b) $-\dfrac{1}{3}$. (c) 3. (d) -3. (e) None of the above.

7. Which of the following is equal to $e^{2\ln 3}$?

(a) $\dfrac{1}{9}$. (b) $-\dfrac{1}{9}$. (c) -9. (d) 9. (e) None of the above.

8. The statement $3\ln x + \ln 5$ written as a single logarithm is

a) $\ln(3x+5)$. (b) $\ln(5x^3)$. (c) $\ln(x^3+5)$. (d) $\ln(15x)$. (e) None of the above.

9. What is $\lim\limits_{x\to\infty} \dfrac{4x^3+x^2-3}{1+2x-x^3}$?

(a) -4. (b) -3. (c) 0. (d) ∞. (e) None of the above.

10. Evaluate $\lim\limits_{x\to 3} \dfrac{x-3}{x^2-9}$.

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{6}$ (c) 0 (d) ∞ (e) None of the above

11. What is the slope of the curve $y = \log_2 x$ at $x = 1$?

(a) 0. (b) 1. (c) $\dfrac{1}{\ln 2}$. (d) $\ln 2$. (e) None of the above.

12. Let $f(x) = (2x^2 + 3)^4$. Evaluate $f'(x)$, that is, find the derivative of f.

(a) $4(2x^2 + 3)^3$. (b) $4x(2x^2 + 3)^3$. (c) $16(2x^2 + 3)^3$.

(d) $16x(2x^2 + 3)^3$. (e) None of the above

13. What are the critical numbers of the function $f(x) = \dfrac{1}{x+2}$?

(a) -2. (b) 0. (c) 2. (d) No critical numbers. (e) None of the above.

14. The **second** derivative f'' of $f(x) = e^{3x-1}$ is

(a) $9e^{3x-1}$. (b) $3e^{3x-1}$. (c) $\ln(3)e^{3x-1}$. (d) $3\ln(3)e^{3x-1}$.

(e) None of the above.

15. The graph of the function $y = \dfrac{x^2}{x^2+1}$ has

(a) no horizontal asymptote and a vertical asymptote $x = -1$.

(b) a horizontal asymptote $y = 1$ and a vertical asymptote $x = -1$.

(c) a horizontal asymptote $y = 1$ and no vertical asymptote.

(d) neither horizontal no vertical asymptote.

(e) none of the above.

16. Determine the interval(s) where the function $f(x) = x^3 - 3x^2 + 1$ is INCREASING.

(a) $x \in (0, 2)$, or equivalently, $\{0 < x < 2\}$.

(b) $x \in (0, \infty)$, or equivalently, $\{x > 0\}$.

(c) $x \in (-\infty, 0) \bigcup (2, \infty)$, or equivalently, $\{x < 0,\ x > 2\}$.

(d) $x \in (-\infty, \infty)$, or equivalently, $x \in \mathbb{R}$, all real x.

(e) None of the above.

17. Let $f(x,y) = \dfrac{x^4}{6} - x^2 + \dfrac{5}{6}$. What are the inflection points of the function (if any)?

(a) $(0,-1)$ and $(0,1)$. (b) $(-1,0)$ and $(1,0)$. (c) $(1,0)$.

(d) No inflection points. (e) None of the above.

18. The graph of the function $f(x) = x^3 - 9x^2 + 12x + 23$ is concave downwards (DOWN) over the interval

(a) $(3,+\infty)$. (b) $(-3,3)$. (c) $(-\infty,3)$. (d) $(-\infty,+\infty)$. (e) None of the above.

19. The absolute maximum value of the function $f(x) = x^2 + x - 2$ on the interval $[-3,0]$ is:

(a) 8. (b) 4. (c) 0. (d) -2. (e) None of the above.

20. The function $y = y(x)$ is defined implicitly by the relation $x^2 - xy + y^3 = 0$. Then the derivative $y' = \dfrac{d\,y}{d\,x}$ is given by

(a) $\dfrac{y-2x}{3y^2-x}$. (b) $\dfrac{2x}{3y^2-x}$. (c) $\dfrac{2x}{3y-x}$. (d) $\dfrac{y-2x}{3y-x}$.

(e) None of the above.

21. Suppose that a population of 300 species is growing **exponentially** with growth constant 0.5, where t is measured in months. What is the size of the population in 4 months? (Choose the best approximation.)

(a) 2217. (b) 2483. (c) 2011. (d) 1946. (e) None of the above.

22. The value of $\displaystyle\int_{-1}^{2} (x+1)\,dx$ is

(a) -2.5. b) -1. (c) 3. (d) 4.5. (e) None of the above.

23. The integral $\int \left(3^x + \frac{1}{x}\right) dx$ evaluates to

(a) $\frac{3^x}{\ln 3} + \ln x + C.$ (b) $3^x \ln 3 + \ln|x| + C.$ (c) $e^x \ln 3 + \ln|x| + C.$

(d) $\frac{3^x}{\ln 3} + \ln|x| + C.$ (e) None of the above.

24. The value of $\int_0^1 2x(x^2-1)^5\, dx$ is

(a) $\frac{1}{6}.$ (b) $-\frac{1}{6}.$ (c) $\frac{1}{3}.$ (d) $-\frac{1}{3}.$ (e) None of the above.

25. The area of the region bounded by the x axis and the parabola $y = -x^2 + 4$, between the lines $x = 0$ and $x = 3$, is represented by

(a) $\int_0^2 (-x^2+4)\, dx - \int_2^3 (-x^2+4)\, dx.$ (b) $\int_0^3 (-x^2+4)\, dx.$

(c) $\int_0^2 (-x^2+4)\, dx + \int_2^3 (-x^2+4)\, dx.$ (d) $\int_0^3 (x^2-4)\, dx.$

(e) None of the above.

26. If $f(x) = \cos(2x+3)$, then $f'(x)$, that is, the derivative of f, is given by

(a) $\sin(2x+3).$ (b) $-\sin(2+3).$ (c) $-2\sin(2x+3).$ (d) $2\cos(2x+3).$

(e) None of the above.

27. The integral $\int \sec^2\left(\frac{x}{5}\right) dx$ evaluates to

(a) $\frac{1}{5}\tan\left(\frac{x}{5}\right).$ (b) $5\tan\left(\frac{x}{5}\right).$ (c) $-\frac{1}{5}\cot\left(\frac{x}{5}\right).$ (d) $-5\cot\left(\frac{x}{5}\right).$

(e) None of the above.

28. The value of $\int_0^{\pi/4} \sin(2x)\, dx$ is

(a) $-\frac{1}{2}.$ (b) $\frac{1}{2}.$ (c) $-1.$ (d) $1.$ (e) None of the above.