University of Ottawa
MAT 1332 Pracice Final Exam
Duration: 3 hours.

Family Name:______________________

First Name:______________________

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 3 hours to complete this exam.
- The exam consists of two parts:
  - Part A contains 10 multiple choice questions. You must enter your answer in the table provided on page 2 of this exam. There are no part marks for multiple choice questions. There are extra pages at the end of the exam to solve multiple choice questions. These pages will not be marked, but you have to hand them in.
  - Part B contains 7 long-answer questions. The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Draw boxes around your final answer.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty-approved calculators (TI-30X, TI-34X, Casio FX-260X and Casio FX-300X) are allowed. All others will be confiscated.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- If you tear off any blank pages, they must still be handed in.
- Where it is possible to check your work, do so.
- Good luck!

Family Name:______________________ First Name:________________________

Student number:____________________ Total marks: ___________ out of 60

Your answers to multiple choice questions (two points for each correct answer):

Problem	1	2	3	4	5	6	7	8	9	10
Your answer										
Your marks										

Marks for long answer questions:

Problem	11	12	13	14	15	16	17
Points	5	5	3	6	3	8	10
Your marks							

PART A: MULTIPLE CHOICE QUESTIONS

Remember to record your final answer in the table on page 2.

Use the back or the attached pages at the end to work out your solutions, if necessary.

Question 1. Find whether the following integral converges or diverges. If it converges, give the value.

$$\int_0^{\pi/2} \frac{\cos(x)}{(\sin^2(x))^{1/3}} dx$$

A: 3; B: 2/3; C: 1/3; D: -1/2; E: -2/3; F: does not converge

Question 2. Find the value of following definite integral

$$\int_1^e x^2 \ln(x^3) dx$$

A: 1; B: $\frac{2}{3}e^3 - \frac{1}{3}$; C: $\frac{2}{3}e^3 + \frac{1}{3}$ D: $\frac{1}{3}e^3 + \frac{1}{3}$; E: $\frac{1}{3}e^3 - \frac{1}{3}$;

Question 3. Find the volume of the solid obtained by the rotation of the curve $y = 2\sqrt{x}$ for $0 \le x \le 4$ around the x axis.

A: 16π; B: $16\pi^2$; C: 32; D: $32\pi^2$; E: 32π

Question 4. Consider the separable differential equation

$$\frac{dx}{dt} = \frac{\pi(1+x^2)t}{12\sqrt{1-t^2}}, \qquad x(0) = 1.$$

Evaluate $x(1)$.

A: 0; B: 1; C: -1; D: $-\sqrt{3}$; E: $\sqrt{3}$

Question 5. Given the improper integral $\int_1^\infty x^p dx$, which of the following is true?
A: The integral converges for all $p > 0$
B: The integral converges for all $p < 0$.
C: The integral converges for $p \le -1$.
D: The integral diverges for $p \ge -1$.
E: The integral is divergent for all values of p.

Question 6. Which of the following are eigenvalues of the matrix

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

A : 3; B : 1; C : -2; D : -3; E : None of them are eigenvalues.

Question 7. Let

$$f(x,y) = \frac{1}{\sqrt{x^2 - 9y^2}}$$

be function of two real variables. What is the domain of f?

A: $\{(x,y) : x - 3y \neq 0\}$
B: $\{(x,y) : |x| > |3y|\}$
C: $\{(x,y) : |x| \geq |3y|\}$
D: $\{(x,y) : x - 3y \neq 0, x + 3y \neq 0\}$
E: $\{(x,y) : x - 3y \geq 0, x + 3y \geq 0\}$

Question 8. Evaluate the Jacobian for $f(x,y) = (x^2 + x + y,\ yx + x^2)$ at the point $(1,2)$.

A: $\begin{bmatrix} 4 & 2 \\ 5 & 1 \end{bmatrix}$ B: $\begin{bmatrix} 3 & 2 \\ 1 & 3 \end{bmatrix}$ C: $\begin{bmatrix} 3 & 3 \\ 2 & 1 \end{bmatrix}$ D: $\begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$ E: $\begin{bmatrix} 3 & 1 \\ 4 & 1 \end{bmatrix}$

Question 9. Consider the system of differential equations:

$$\begin{aligned} \frac{dx}{dt} &= x^2 - 2xy + y^2 \\ \frac{dy}{dt} &= e^{x^2-1} - 1 + \ln(y) \end{aligned}$$

Which of the following points is an equilibrium for the system?
A: $(x,y) = (0,2)$
B: $(x,y) = (5,2)$
C: $(x,y) = (3,3)$
D: $(x,y) = (1,1)$
E: $(x,y) = (1,3)$

Question 10. Find the tangent plane to the graph of the function

$$f(x,y) = \pi - 2\cos 3x + 4\sin 5y$$

at the point $(0,\pi)$.

A: $z = 20y - 19\pi + 2$; B: $z = -20y + 19\pi - 2$; C: $z = -20x - 20y + 21\pi - 2$;
D: $z = -20y + 21\pi - 2$; E: $z = -20y + 21\pi + 2$

PART B: LONG ANSWER QUESTIONS:

Question 11. [5 points] Calculate the indefinite integral

$$\int \frac{-2x^3 + 7x^2 + 78x - 37}{x^2 - 3x - 40} dx.$$

Question 12. [5 points]

Let A be the matrix

$$A = \begin{pmatrix} 5 & -5 \\ 2 & 3 \end{pmatrix}.$$

1. Show that the eigenvalues of A are $z_1 = 4 + 3i$ and $z_2 = 4 - 3i$.
2. Find an eigenvector corresponding to the eigenvalue $4 + 3i$.
3. Express z_1/z_2 in the form $z = a + ib$.
4. Express z_2 in the form $z_2 = re^{i\theta}$.
5. Find $z_1\bar{z}_2$.

Question 13. [3 points] Solve the system

$$\begin{aligned} 2x - 3y + 4z &= 7 \\ -5x - 12y + 3z &= 2 \\ -3x - 15y + 7z &= 9 \end{aligned}$$

Question 14. [6 points] Suppose that a fish population grows according to the logistic equation, and that a fixed number H of fish is removed per unit time. Then the number of fish is given by the differential equation

$$\frac{dN}{dt} = 2N\left(1 - \frac{N}{10}\right) - H$$

(a) [2] Find all biologically meaningful steady states and their stability for $H = 3.75$.
(b) [1] Draw the phase-line diagram for $H = 3.75$.
(c) [2] Determine the steady states in general, i.e., for any value of H.
(d) [1] What is the minimal H for which the population goes extinct?

Question 15. [3 points] Calculate the area between the curves $y = (x-2)^2$ and $y = 10 - x^2$.

Question 16. [8 points] Consider the following system of linear differential equations:

$$\begin{aligned} \frac{dx}{dt} &= -3x + y \\ \frac{dy}{dt} &= x - 3y \end{aligned}$$

(a) Find the eigenvalues and eigenvectors associated with the system.

(b) Find the equilibrium and determine its stability.

(c) Draw the x- and y-nullclines in the phase plane.

(d) Sketch the solution curve for the initial value $x(0) = 6, y(0) = 2$ in the phase plane.

Question 17. [10 points] Consider a disease that propagates according to the system

$$\frac{dx}{dt} = 12 - 3xy - 3x$$
$$\frac{dy}{dt} = 3xy - 6y$$

where x represents susceptible individuals, y represents infected individuals.

(a) Find the two biologically meaningful equilibria.

(b) Show that $J(x, y)$, the Jacobian matrix of this system, is given by

$$\begin{bmatrix} -3y - 3 & -3x \\ 3y & 3x - 6 \end{bmatrix}$$

(c) For each of the two biologically meaningful equilibria from (a), find the eigenvalues of the Jacobian matrix.

(d) Determine the stability of the two equilibria.

(e) Sketch trajectories in the region $x \geq 0$, $y \geq 0$.

(f) If 10 people are initially infected, what happens eventually?