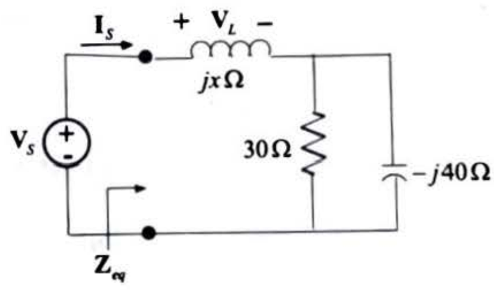
1) AC Circuit analysis

Express final answers for phasors and complex impedances in polar form. The voltage is expressed as peak magnitude and the angular frequency of operation is 2000 rad/s.
Assume $V_S = 200\angle 45° \ V$



a) Write a time domain expression for the voltage source [1]

$v_s(t)$= $200\cos(2000t + 45°)$

b) Calculate x such that the equivalent impedance as seen by the source $\mathbf{Z_{eq}}$ is purely real. [3]

c) Calculate the L and C component values. [2]

d) Calculate the phasor currents $\mathbf{I_S}$ [2]

e) Calculate the phasor voltage $\mathbf{V_L}$ [2]

b) $$Z_{eq} = jX + (30\Omega \,||\, -j40\Omega) = jX + \frac{30\cdot(-j40)}{30-j40} = jX + \frac{30\cdot(40\angle -90°)}{50\angle -53°}$$

$$= jX + 24\angle -37° = jX + 19.2 - j14.4 \longrightarrow \boxed{X = 14.4}$$

c) $$\frac{1}{j\omega C} = -j40 \longrightarrow C = \frac{1}{40\times(2000)} = 12.5\,\mu F$$

$$j\omega L = j14.4 \longrightarrow L = \frac{14.4}{2000} = 7.2\,mH$$

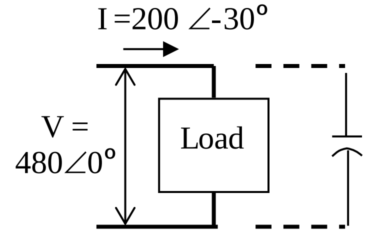
d) $$I_S = \frac{V_S}{Z_{eq}} = \frac{200\angle 45°}{19.2\angle 0°} = 10.4\angle 45° \ A$$

e) $$V_L = Z_L\cdot I_S = j14.4\cdot(10.4\angle 45°) = 14.4\angle 90°\cdot(10.4\angle 45°)$$

$$= 149.8\angle 135° \ V$$

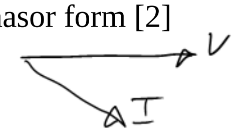
2) Power Factor and Correction

A load is supplied at 480V (rms) at 60 Hz. The current into the load is 200A (rms) and is lagging the voltage by 30°.



a) Determine the power factor [1]
b) Determine the real power, reactive power, and apparent power [3], showing an appropriately labeled power triangle [2]
c) How many kVARs of capacitance must be placed in parallel to bring the power factor to 0.95 lagging? [2]
d) With the capacitance added as in part c), what is the new input current in phasor form [2]

a) Power Factor angle $\theta = \angle V - \angle I = 30°$

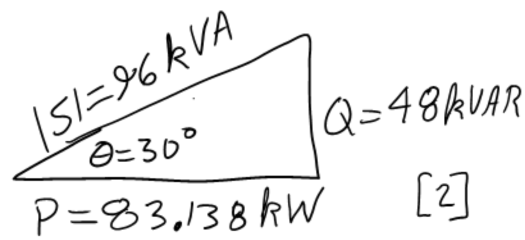


$PF = \cos(\theta) = 0{,}866$, Lagging [1]

b) $P = VI\cos\theta = 480 \times 200 \times 0{,}866 = 83.138\,kW$ [1]

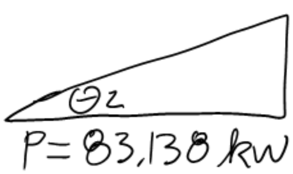
$Q = VI\sin\theta = 480 \times 200 \times 0{,}5 = 48\ kVAR$ [1]

$|S| = VI$ or $\sqrt{P^2+Q^2} = 480 \times 200 = 96\ kVA$ [1]



[2]

c)



$Q_2 = P\tan(\cos^{-1}[0{,}95])$

$= 83.138\,kW \tan(18.19°) = 27.326\,kVAR$

$\theta_2 = \cos^{-1}(0.95) = 18{,}19°$

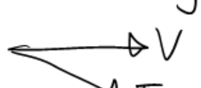
kVAR of capacitance

$Q_{cap} = 27{,}326\,kVAR - 48\,kVAR = -20{,}674\ kVAR$

[2]

d) $P = V \cdot I_2 \cos\theta_2 \Rightarrow I_2 = \frac{P}{V\cos\theta_2} = \frac{83.138\,kW}{480 \times 0{,}95} = 182.32\ A$

phase is the negative of the PF angle



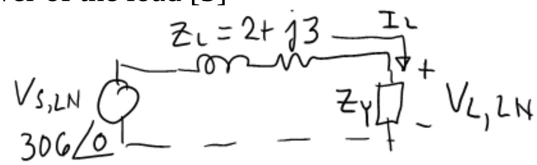
$I = 182{,}32\angle{-18.19°}$

[2]

3) 3-Phase Circuit

A Wye connected 3-phase source is connected to a Delta connected load. At the source, the rms line-to-neutral voltage V_{LN} is 306V. Each line between the source and the load has an impedance Z_{Line} of (2+j3) Ω and each branch of the load has an impedance Z_{Δ} of (21 + j12) Ω.

a) Determine the rms line current including the phase [3]
b) Find the rms line-to-neutral voltage at the load including the phase [2]
c) Determine the voltage regulation at the load [1]
d) Determine the total real power, reactive power, and apparent power of the load [3]
e) Determine the load power factor [1]



a) $Z_L = 2+j3$, $Z_\Delta = 21+j12$,

$Z_Y = \frac{Z_\Delta}{3} = \frac{21+j12}{3} = 7+j4 = 8.062\angle 29.745$

$I_L = \frac{V_{S,LN}}{Z_L + Z_Y} = \frac{306\angle 0}{2+j3+7+j4} = \frac{306\angle 0}{9+j7} = \frac{306\angle 0}{11.402\angle 37.87} = 26.838\angle -37.87$ [3]

b) $V_{L,LN} = I_L \cdot Z_Y = 26.838\angle -37.87 \times 8.062\angle 29.745 = 216.4\angle -8.13°$ [2]

(could also have used $V_S - V_{Line}$, or voltage divider)

c) $V.Reg = \frac{V_{NOLOAD} - V_{LOAD}}{V_{LOAD}} \times 100\% = \frac{306-216.4}{216.4} \times 100\% = 41.42\%$ [1]

d) $P = 3 I_L^2 \cdot Re\{Z_Y\} = 3 \times 26.838^2 \times 7 = 15.126\, kW$ [1]
$Q = 3 I_L^2 \cdot Im\{Z_Y\} = 3 \times 26.838^2 \times 4 = 8.643\, kVAR$ [1]
$|S| = \sqrt{P^2 + Q^2} = \sqrt{15.126k^2 + 8.643k^2} = 17.421\, kVA$ [1]

Alternate also correct:
$P = 3 V_{L,LN} I_L \cos\theta$
$Q = 3 V_{L,LN} I_L \sin\theta$
$\theta = \angle V_{L,LN} - \angle I_L = -8.13 - -37.87 = 29.74°$
$|S| = 3 V_{L,LN} \cdot I_L$

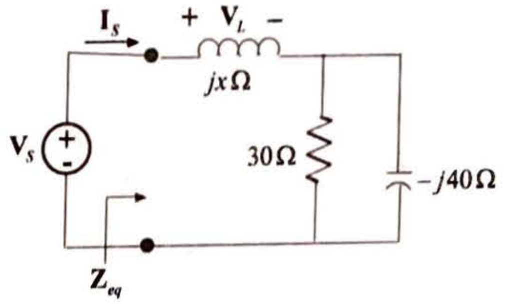
e) Power Factor $= \frac{P}{|S|} = \frac{15.126\, kW}{17.421\, kVA} = 0.8682$ Lagging [1]

or directly from Z_Y or from Z_Δ e.g., $|Z| = \sqrt{65}$, $x = 4$, $R = 7$, $\frac{7}{\sqrt{65}} = 0.8682$ Lagging

or from V, I: $P.F. = \cos\theta = \cos(29.74°) = 0.8682$ Lagging

1) AC Circuit analysis

Express final answers for phasors and complex impedances in polar form. The voltage is expressed as peak magnitude and the angular frequency of operation is 2000 rad/s. Assume $V_S = 250\angle 30° \ V$



a) Write a time domain expression for the voltage source [1]

$v_s(t)$= $250\cos(2000t + 30°)$

b) Calculate x such that the equivalent impedance as seen by the source $\mathbf{Z_{eq}}$ is purely real. [3]

c) Calculate the L and C component values. [2]

d) Calculate the phasor currents $\mathbf{I_S}$ [2]

e) Calculate the phasor voltage $\mathbf{V_L}$ [2]

b) $$Z_{eq} = jX + (30\Omega \parallel -j40\Omega) = jX + \frac{30\cdot(-j40)}{30-j40} = jX + \frac{30\cdot(40\angle -90°)}{50\angle -53°}$$

$$= jX + 24\angle -37° = jX + 19.2 - j14.4 \rightarrow \boxed{X = 14.4}$$

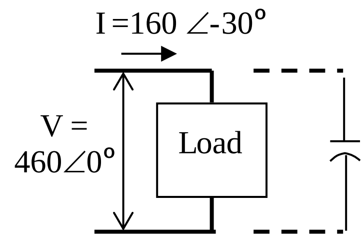
c) $$\frac{1}{j\omega C} = -j40 \longrightarrow C = \frac{1}{40\times(2000)} = \boxed{12.5\ \mu F}$$

$$j\omega L = j14.4 \longrightarrow L = \frac{14.4}{2000} = \boxed{7.2\ mH}$$

d) $$I_s = \frac{V_s}{Z_{eq}} = \frac{250\angle 30°}{19.2\angle 0°} = \boxed{13\angle 30°\ A}$$

e) $$V_L = Z_L \cdot I_s = j14.4\cdot(13\angle 30°) = 14.4\angle 90° \cdot (13\angle 30°) = \boxed{187.2\angle +120°\ V}$$

2) Power Factor and Correction



A load is supplied at 460V (rms) at 60 Hz. The current into the load is 160A (rms) and is lagging the voltage by 30°.

a) Determine the power factor [1]
b) Determine the real power, reactive power, and apparent power [3], showing an appropriately labeled power triangle [2]
c) How many kVARs of capacitance must be placed in parallel to bring the power factor to 0.95 lagging? [2]
d) With the capacitance added as in part c), what is the new input current in phasor form [2]

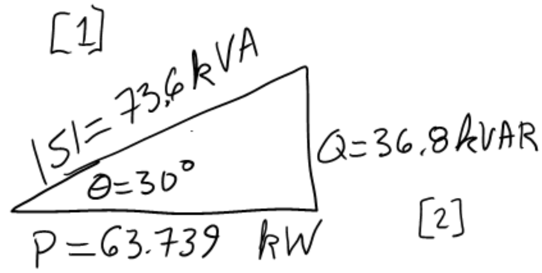
a) Power Factor angle $\theta = \angle V - \angle I = 30°$

$PF = \cos(\theta) = 0,866$, Lagging [1]

b) $P = VI\cos\theta = 460 \times 160 \times 0,866 = 63.739$ kW [1]

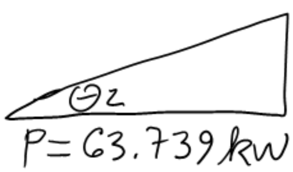
$Q = VI\sin\theta = 460 \times 160 \times 0,5 = 36,800$ kVAR [1]

$|S| = VI$ or $\sqrt{P^2+Q^2} = 460 \times 160 = 73.600$ kVA [1]



[2]

c)



$Q_2 = P\tan(\cos^{-1}[0,95])$

$= 63.739 \text{ kW} \tan(18.19°) = 20.950$ kVAR

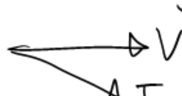
$\theta_2 = \cos^{-1}(0,95) = 18,19°$

kVAR of capacitance

$Q_{cap} = 20,950 \text{ kVAR} - 36,8 \text{ kVAR} = -15.850$ kVAR [2]

d) $P = V \cdot I_2 \cos\theta_2 \Rightarrow I_2 = \frac{P}{V\cos\theta_2} = \frac{63.739 \text{ kW}}{460 \times 0,95} = 145.86$ A

phase is the negative of the PF angle

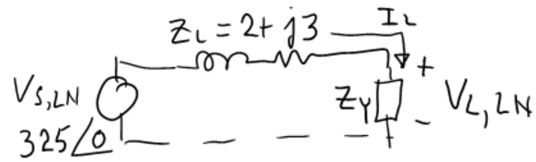


$I = 145.86\angle -18.19°$ [2]

3) 3-Phase Circuit

A Wye connected 3-phase source is connected to a Delta connected load. At the source, the rms line-to-neutral voltage V_{LN} is 325V. Each line between the source and the load has an impedance Z_{Line} of (2+j3) Ω and each branch of the load has an impedance Z_Δ of (21 + j12) Ω.

a) Determine the rms line current including the phase [3]
b) Find the rms line-to-neutral voltage at the load including the phase [2]
c) Determine the voltage regulation at the load [1]
d) Determine the total real power, reactive power, and apparent power of the load [3]
e) Determine the load power factor [1]



a) $Z_L = 2+j3$, $Z_\Delta = 21+j12$,

$Z_Y = \frac{Z_\Delta}{3} = \frac{21+j12}{3} = 7+j4 = 8.062\angle 29.745$

$I_L = \frac{V_{S,LN}}{Z_L + Z_Y} = \frac{325\angle 0}{2+j3+7+j4} = \frac{325\angle 0}{9+j7} = \frac{325\angle 0}{11.402\angle 37.87} = 28.504\angle -37.87$ [3]

b) $V_{L,LN} = I_L \cdot Z_Y = 28.504\angle -37.87 \times 8.062\angle 29.745 = 229.8\angle -8.13^\circ$ [2]

(could also have used $V_S - V_{Line}$, or voltage divider)

c) $V.Reg = \frac{V_{NOLOAD} - V_{LOAD}}{V_{LOAD}} \times 100\% = \frac{325 - 229.8}{229.8} \times 100\% = 41.42\%$ [1]

d) $P = 3 I_L^2 \cdot Re\{Z_Y\} = 3 \times 28.504^2 \times 7 = 17.063\,kW$ [1]

$Q = 3 I_L^2 \cdot Im\{Z_Y\} = 3 \times 28.504^2 \times 4 = 9.750\,kVAR$ [1]

$|S| = \sqrt{P^2 + Q^2} = \sqrt{17.063k^2 + 9.750k^2} = 19.652\,kVA$ [1]

Alternate also correct:

$P = 3 V_{L,LN} I_L \cos\theta$

$Q = 3 V_{L,LN} I_L \sin\theta$

$\theta = \angle V_{L,LN} - \angle I_L = -8.13 - -37.87 = 29.74^\circ$

$|S| = 3 V_{L,LN} \cdot I_L$

e) Power Factor $= \frac{P}{|S|} = \frac{17.063\,kW}{19.652\,kVA} = 0.8682$ Lagging [1]

or directly from Z_Y or from Z_Δ e.g. $|Z| = \sqrt{65}$, $X = 4$, $R = 7$, $\frac{7}{\sqrt{65}} = 0.8682$ Lagging

or from V, I: $P.F. = \cos\theta = \cos(29.74^\circ) = 0.8682$ Lagging