Name: _____

Student ID: _____

CARLETON UNIVERSITY
DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2004-C (Fall 2019)
Test 5 (Tuesday, November 12, 2019)

Time: 50 minutes (no cellphones, notes, books, talking). Use the back of the paper to show your work for long answer questions and only write the final solution in the space under the question.

MARKS

(1) 1. Which of the following is NOT correct about the surface $x^2 + y^2 = z^2 + a$?

(a) Hyperboloid of one sheet if $a = 1$

(b) Hyperboloid of two sheets if $a = -1$

(c) Hyperboloid of one sheet if $a \neq 0$

(d) Elliptic cone if $a = 0$

(1) 2. Find the Jacobian of the change of variables $x = u - v^2$ and $y = u + v^2$

(a) $\frac{1}{2}$ (b) 1 (c) $4v$ (d) $4u$

(1) 3. Evaluate $\int_0^1 \int_0^1 xy^2 dydx$

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

(4) 4. First sketch the region $\mathcal{R}$, enclosed by the lines $x = 0$, $y = x$, and $y = 2$ then use the right order for dA to evaluate the integral.

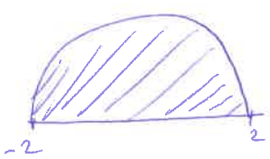
$$\int\int_{\mathcal{R}} \sqrt{1 - y^2} dA$$

$$\int_0^2 \int_0^y \sqrt{1-y^2}\, dx\, dy = \int_0^2 y\sqrt{1-y^2}\, dy = -\frac{1}{3}(1-y^2)^{3/2}\Big|_0^2$$

$$= -\frac{1}{3}(-3) + \frac{1}{3}$$

(4) 5. Find the following integral

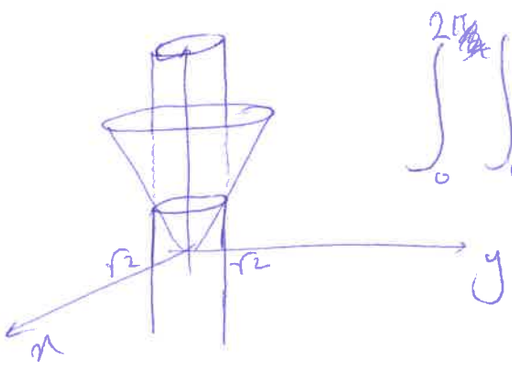
$$\int_{-2}^{2} \int_{0}^{\sqrt{4-x^2}} \sqrt{9 - x^2 - y^2} dydx.$$



$$\int_0^{\pi} \int_0^2 r\sqrt{9-r^2}\, dr\, d\theta = \int_0^{\pi} -\frac{1}{3}(9-r^2)^{3/2}\Big|_0^2 d\theta = \frac{1}{3}\pi\left(5^{3/2} - 27\right)$$

(5) 6. Sketch surfaces $\mathcal{S}_1 : z^2 = x^2 + y^2$ and $\mathcal{S}_2 : x^2 + y^2 = 2$. Find the volume under the surface $\mathcal{S}_1$ inside the cylinder Sc_2.

Hint: Use double integral to st up the integral, then solve it by polar coordinates.



$$\int_0^{2\pi} \int_0^{\sqrt{2}} \sqrt{r^2}\, r\, dr\, d\theta = \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$