Name: ____________________

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CARLETON UNIVERSITY
DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2004-C (Fall 2019)
Test 4 (Tuesday, October 29, 2019)

Time: 50 minutes (no cellphones, notes, books, talking). Use the back of the paper to show your work for long answer questions and only write the final solution in the space under the question.

MARKS

(1) 1. Which of the followings is a potential function for the vector filed $\mathbf{F} = (2x\sin(xy^2)+x^2y^2\cos(xy^2), 2x^3y\cos(xy^2))$?

(a) $x^2\sin(xy^2)$

(b) $xy\cos(xy^2)$

(c) $y^2\sin(xy^2)$

(1) 2. Let $\mathbf{F} = \bigtriangledown f$ be a vector filed for a function f. Which of the following is NOT right about the line integral of the vector filed $\mathbf{F}$ over a curve $\mathcal{C}$?

(a) It only depends on the start and end points of the curve $\mathcal{C}$.

(b) It depends on the parametrization of $\mathcal{C}$.

(c) It is zero if the curve $\mathcal{C}$ is closed.

(1) 3. For which function $f(x,y)$ the line integral $\int_{\mathcal{C}} f d\mathbf{r}$ gives the length of $\mathcal{C}$ parametrized by $\mathbf{r}(t)$?

(a) $f(x,y)$ is constantly equal to 1 (b) $f(x,y) = x+y$ (c) $f(x,y) = |\mathbf{r}(t)|$

(1) 4. Let c be the average of the function $f(x,y)$ over the curve $\mathcal{C}$. Then

(a) $\int_{\mathcal{C}} d\mathbf{r} = c\int_{\mathcal{C}} f d\mathbf{r}$ (b) $\int_{\mathcal{C}} f d\mathbf{r} = c\int_{\mathcal{C}} d\mathbf{r}$ (c) $c = \frac{\int_{\mathcal{C}} d\mathbf{r}}{\int_{\mathcal{C}} f d\mathbf{r}}$

(4) 5. Find $\int_{\mathcal{C}} x^2+y^2 d\mathbf{r}$ where $\mathcal{C}$ is the circle $x^2+y^2 = 4$.

(4) 6. Let $\mathcal{C}$ be the curve parametrized by $\mathbf{r}(t) = (t, \sqrt{1-t^2})$, $-1 \leq t \leq 1$. Find the following line integral over $\mathcal{C}$.

$$\int_{\mathcal{C}} x^2dx + y^2dy.$$

(4) 7. Use line integrals of vector fields to find a function f such that $\bigtriangledown f = (y+z, x+z, x+y)$.

Q5

$$C: r(t) = (2\cos t, 2\sin t) \quad 0 \le t \le 2\pi$$
$$r'(t) = (-2\sin t, 2\cos t)$$
$$\Rightarrow |r'(t)| = 2$$
2 marks

$$\int_C x^2+y^2\,dr = \int_0^{2\pi} (4\cos^2 t + 4\sin^2 t)\,2\,dt$$
2 marks
$$= \int_0^{2\pi} 8\,dt = 16\pi$$

Q6

2 marks
$$\int_C x^2 dx + y^2 dy = \int_C x^2 x'(t) + y^2 y'(t)\,dt$$
$$= \int_{-1}^{1} t^2 + (1-t^2)\frac{-2t}{2\sqrt{1-t^2}}\,dt$$
1 mark
$$= t^3/3\Big|_{-1}^{1} + \int_{-1}^{1} -t\sqrt{1-t^2} = 2/3 + \frac{(1-t^2)^{3/2}}{3}\Big|_{-1}^{1} = 2/3 + 0$$
1 mark

Q7

$$C: r(t) = (tx, ty, tz) \quad 0 \le t \le 1 \qquad r'(t) = (x, y, z)$$
1.5 mark
$$f = \int_C \nabla f\,dr = \int_0^1 (yt + zt, xt + zt, xt + yt)\cdot(x, y, z)\,dt$$
1.5 marks
$$= 2\int_0^1 xyt + xzt + zyt = xy + xz + yz$$
1 mark