Name: ______________________

Student ID: ______________________

CARLETON UNIVERSITY
DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2004-C (Fall 2019)
Test 3 (Tuesday, October 15, 2019)

Time: 50 minutes (no cellphones, notes, books, talking). Use the back of the paper to show your work for long answer questions.

MARKS

(1) 1. Let $f(x, y) = x^2 \sin(xy^2)$. What is gradient of f?

(a) $(2x\sin(y^2), x^2\sin(2xy))$
(b) $(2x\sin(y^2), x^2\cos(2xy))$
(c) $(2x\sin(xy^2) + x^2y^2\cos(xy^2), 2x^3y\cos(xy^2))$

(1) 2. Let a surface be given by $f(x, y) = x^2 - y^2$. What is $D_{\vec{u}} \nabla f$ at $P = (2, -1)$ where $\vec{u} = \frac{1}{\sqrt{5}}(1, 2)$.

(a) $\frac{8}{\sqrt{5}}$ (b) 8 (c) 0

(1) 3. Consider the equation $zx + xy + yz = xyz$. Find z_x evaluated at the point $(1, 0, -1)$. $-\frac{z+y-yz}{x+y-xy}$

(a) $z + y + yz = yz$ (b) 0 (c) 1

(1) 4. What is the length of the curve $C : r(\vec{t}) = (3, \frac{t^2}{2}, \frac{t^3}{3})$ from $t = 0$ to $t = 1$?

(a) 2 (b) $\frac{\sqrt{8}-1}{3}$. (c) $\sqrt{8}$

(3) 5. Let $z = y\sqrt{x} - x\sqrt{y}$, $x = t - 2$ and $y = t^2$. Then find z_t.

$$\boxed{z_t = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = \left(\frac{y}{2\sqrt{x}} - \sqrt{y}\right) + \left(\sqrt{x} - \frac{x}{2\sqrt{y}}\right)(2t)}$$ 2 marks

$$= \frac{t^2}{2\sqrt{t-2}} - \sqrt{t^2} + 2t\sqrt{t-2} - 2t\frac{t-2}{2\sqrt{t^2}}$$ 1 mark

$r = 5$

(4) 6. Consider a perfect sphere shape balloon whose volume is changing at the rate of $10cm^3/s$. Find the rate of the change of its area. (For a sphere with radius r, volume is $\frac{4}{3}\pi r^3$ and area is $4\pi r^2$)

$V = \frac{4}{3}\pi r^3 \Rightarrow V' = \frac{4}{3}\pi(3r^2)r' \Rightarrow r' = \frac{V'}{4\pi r^2} = \frac{10}{4\pi r^2}$ 2 marks

$A = 4\pi r^2 \Rightarrow A' = 8\pi r r' = \frac{8\pi r \times 10}{4\pi r^2} = \frac{20}{r} = 4\,cm^2/s$ 2 marks

(5) 7. Let S be a surface given by $x^2 + yx + y^2 = z$. Find the equation of the tangent plane to the surface at the origin. 1 mark

$f = x^2 + yx + y^2 - z = 0 \Rightarrow \nabla f = (2x + y, x + 2y, -1)$ 1.5 mark

$\Rightarrow \nabla f|_0 = (0, 0, -1)$ 1 mark

$ax + by + cz = d \Rightarrow -z = d$ 1 mark

$d = 0 \Rightarrow z = 0$