Chapter 1

BINARY SYSTEMS

Topics

- → Digital Systems
- → Numbering systems
  - → Base, representation, converting between numbering systems
- → Arithmetic Operations in the Binary Numbering System
  - → Addition, subtraction, multiplication, division, modulo
  - → Defining 1's complement, 2's complement
  - → Using the complement for subtraction
  - → Signed numbers
- → Binary Codes

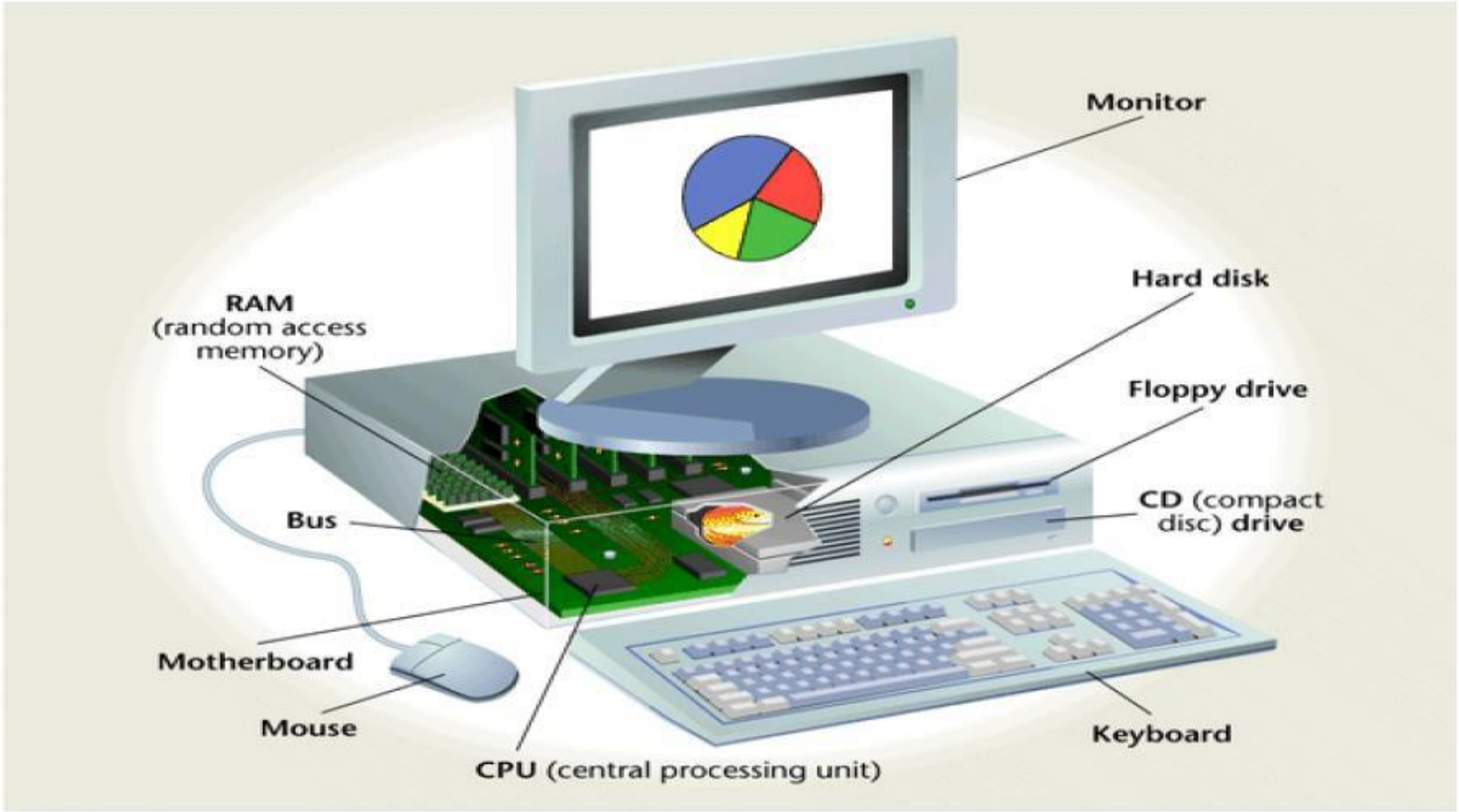
Digital age


Visualize Summit
Lucidchart
Visualize Summit for Sales
THE NEW
DIGITAL AGE OF SELLING
FT.
BOB PERKINS — AA-ISP
+
MARIO MARTINEZ, JR. — VENGRESO
Inside Sales reps being hired 3x faster than field sales
Technology shifts
FACTORS
RISE OF INSIDE SALES
The way customers are engaging
Virtual collaboration
"LET'S NOT FORGET THE HUMAN PIECE"
2008 24% of people in the USA were on social media.
In 2017 it was 81%!
3.64
"CONNECTED DEVICES" Per person worldwide
"BUYERS ARE VIDEO HUNGRY."
-Mario
TIP: Include a video conference link with every calendar invite.
New tech: ADAPT+ADOPT
"Are you selling with your customers tech preferences/habits in mind?"
I'M HERE TO STEAL YOUR JOB!
NOT TRUE!!!
A.I. will help you sift through the noise and find better-qualified leads.
Sample A.I. tools:
CRYSTAL
crystalknows.com
• Free + paid versions
• A.I. suggestions on best ways to connect w/ a prospect.
NUDGE
• Helps you know who to reach out to, and when.
phone
email
OMNI-CHANNEL
Fig (1) A Brief History of contact methods
EXAMPLE CADENCE
DAY 1 Follow on LinkedIn
DAY 2 Like
DAY 3 Comment
DAY 4 tag
Warm up for the phone call
They will get email notifications!
"I'M DECLARING 2018 the YEAR of PERSONALIZATION"
-Bob
H2H
"Human to human relationships are not dead."
People still buy from people they KNOW, LIKE, +TRUST
"All things being equal, RELATIONSHIPS WIN.
All things being unequal... RELATIONSHIPS STILL WIN."
-Anthony Iannarino
ILLUSTRATION: libby thomas
@GoVengreso
vengreso

The Central Tool of Modern Information Systems

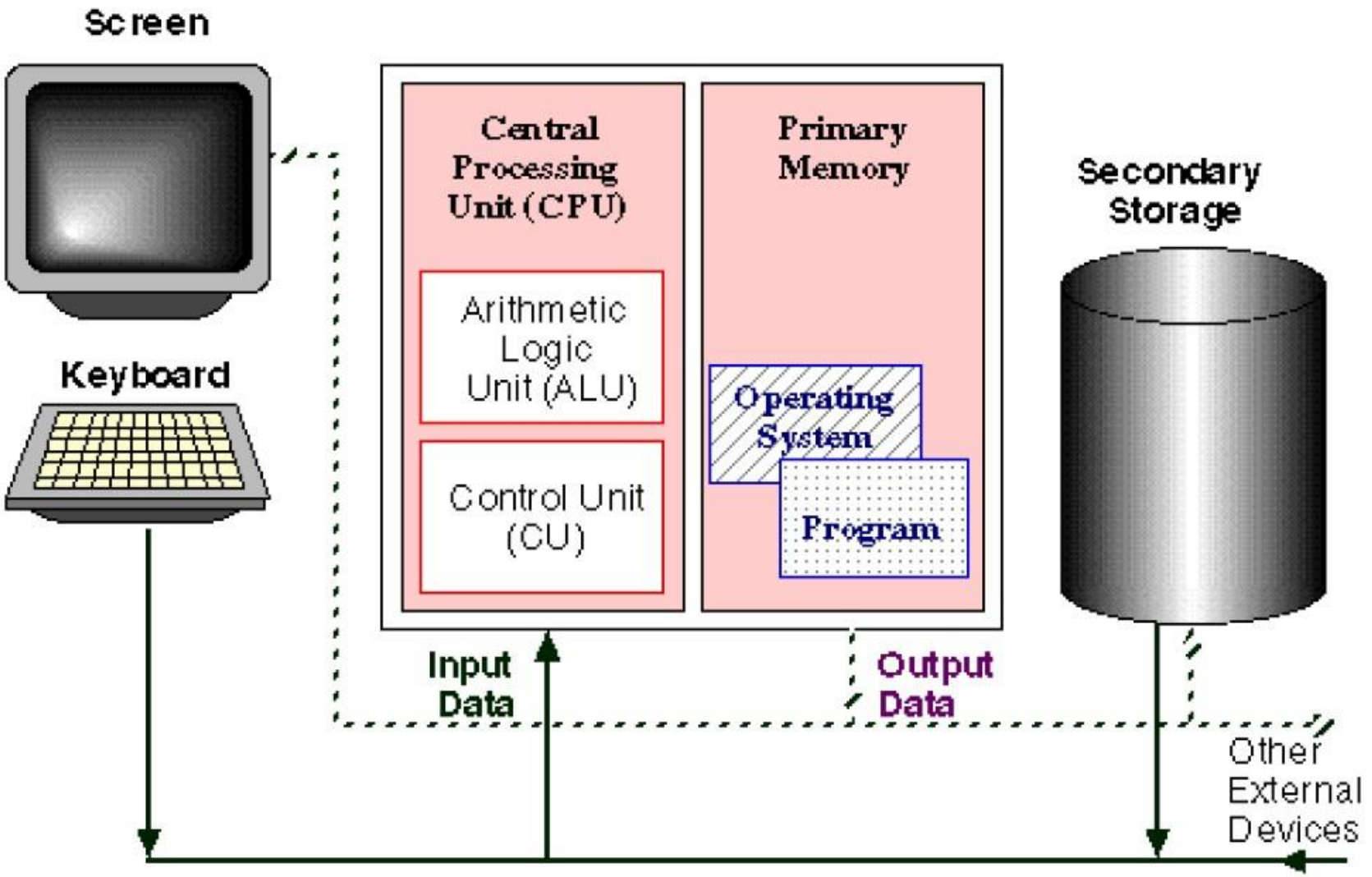
All computers have the same basic components.

Inside the computer



A look inside a computer

Block Diagram of a Digital Computer



Binary System and Logic Circuits

- What kind of data do computers work with?
  - Deep down inside, it's all 1s and 0s
- What can you do with 1s and 0s?
  - Boolean algebra operations
  - These operations map directly to hardware circuits (logic circuits)

Different Numbering Systems

- Decimal (Arabic): (0,1,2,3,4,5,6,7,8,9):
  Example: **$(452968)_{10}$**
- Octal: (0,1,2,3,4,5,6,7):
  Example **$(4073)_{8}$**
- Hexadecimal(0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F)
  Example: $(2BF3)_{16}$
- Binary: (0,1):
  Example: $(100111000101 1)_{2}$

Base in Numbering systems

- The decimal numbering system uses **base 10**. The values of the positions are calculated by taking 10 to some power.

1	6	2	.	3	7	5	Digits
100	10	1		1/10	1/100	1/1000	Weights

1	6	2	.	3	7	5	Digits
10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}	Weights

- Base 10 for decimal numbers?

  It uses 10 digits: The digits 0 through 9.

Base in Numbering systems [2]

- The binary system is called binary because it uses **base 2**.The values of the positions are calculated by taking 2 to some power.
- Base 2 for binary numbers :

  It uses 2 digits. The digits 0 and 1.

Representation of Numbers

→ ***There are two possible ways of writing a number in a given system:***

1- Positional Notation

2- Polynomial Representation

Positional Notation

$$N = (a_{n-1}a_{n-2} \ldots a_1a_0 \,.\, a_{-1}a_{-2} \ldots a_{-m})_r$$

Where

. = radix point

r = radix or base

n = number of integer digits to the left of the radix point

m = number of fractional digits to the right of the radix point

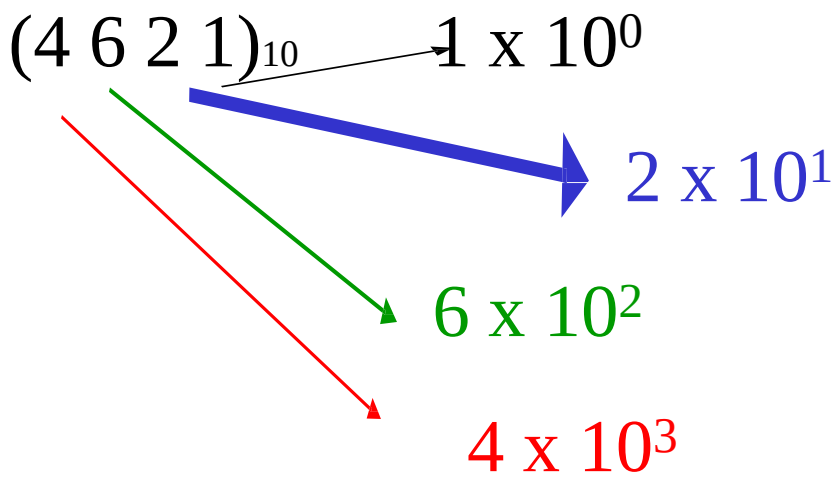
a_{n-1} = most significant digit (MSD)

a_{-m} = least significant digit (LSD)

Positional Notation

The Decimal Numbering System

- The decimal numbering system is a positional number system.
- Example:



Positional Notation

Binary Numbering System

- The Binary Numbering System is also a positional numbering system.
- Instead of using ten digits, 0 - 9, the binary system uses only two digits, the 0 and the 1.

→ Example of a binary number & the values of the positions.

						1	0	1	0	1	0	1
						2^6	2^5	2^4	2^3	2^2	2^1	2^0

1	1	0	1	.	0	1	Binary digits, or bits
2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	Weights (in base 10)

Polynomial Notation

$$N = a_{n-1} \times r^{n-1} + a_{n-2} \times r^{n-2} + \ldots + a_0 \times r^0 + a_{-1} \times r^{-1} \ldots + a_{-m} \times r^{-m}$$

$$= \sum_{i=-m}^{n-1} a_i r^i$$

Example:

Positional (N) ***Polynomial (N)***

$$N = (651.45)_{10} = 6 \times 10^2 + 5 \times 10^1 + 1 \times 10^0$$

$$+ 4 \times 10^{-1} + 5 \times 10^{-2}$$

Important number systems

→ **There are three important number systems**

- Binary Number System
- Octal Number System
- Hexadecimal Number System

Binary numbers

Digits = {0, 1}

Positional Polynomial

$(11010.11)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$

- **1 K (kilo) = 2^{10} = 1,024**
- **1M (mega) = 2^{20} = 1,048,576**
- **1G (giga) = 2^{30} = 1,073,741,824**

Converting Decimal to Binary

$$N = (a_{n-1}a_{n-2} \ldots a_1a_0 . a_{-1}a_{-2} \ldots a_{-m})_r$$

Integer ← → . ← *Fractional* →

Radix point

→ **Integer** part and **Fractional** part are **converted differently**

Converting the Integer Part

- keep dividing by 2 until the quotient is 0. Collect the remainders *in reverse order*.
- Example: $(162)_{10}$.

```
162 / 2  = 81   rem 0
 81 / 2  = 40   rem 1
 40 / 2  = 20   rem 0
 20 / 2  = 10   rem 0
 10 / 2  = 5    rem 0
  5 / 2  = 2    rem 1
  2 / 2  = 1    rem 0
  1 / 2  = 0    rem 1
```

- Then $(162)_{10} = (10100010)_2$

Converting the Fraction Part

- keep multiplying the *fractional part* by 2 until it becomes 0. Collect the integer parts (in forward order).
  - – However this may not terminate!
  - –Example: $(0.375)_{10}$

$$0.375 \times 2 = 0.750$$
$$0.750 \times 2 = 1.500$$
$$0.500 \times 2 = 1.000$$

$\rightarrow$ So, $(.375)_{10} = (.011)_2$

And $(162.375)_{10} = (10100010.011)_2$

Why does this work?

- This method can be applied to convert from decimal to *any* base
- Try converting 162.375 from decimal to decimal.

```
162 / 10  = 16 rem 2
 16 / 10  = 1  rem 6
  1 / 10  = 0  rem 1
```

- Each division "strips off" the rightmost digit (the remainder). The quotient represents the remaining digits in the number.

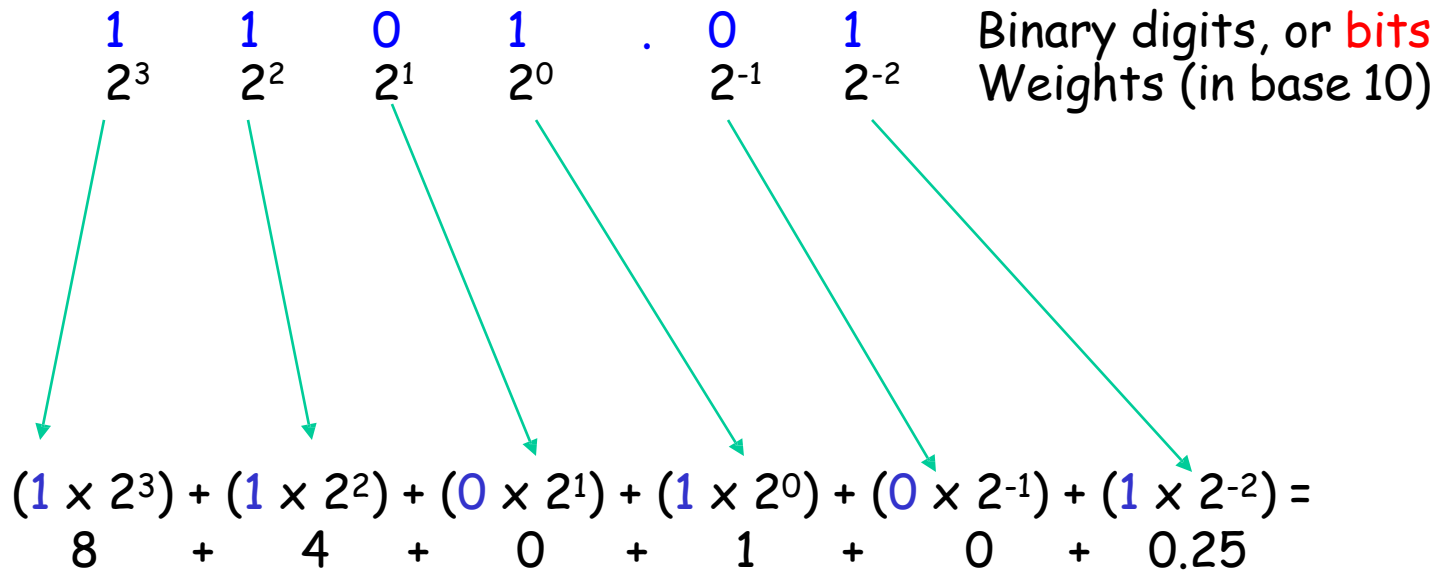
Why does this work? [2]

$0.375 \times 10 = 3.750$
$0.750 \times 10 = 7.500$
$0.500 \times 10 = 5.000$

- Each multiplication "strips off" the leftmost digit (the integer part). The fraction represents the remaining digits.

Converting binary to decimal

- To convert binary, or base 2, numbers to decimal we first obtain the polynomial representation of the number, then sum the products.
  - Example: $(1101.01)_2$



→ The decimal value is: $(13.25)_{10}$

Octal number system

–Digits = {0, 1, 2, 3, 4, 5, 6, 7}

Positional = Polynomial

$(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1}$

- **Octal (base 8) digits range from 0 to 7.**
  Since $8 = 2^3$, one octal digit is equivalent to 3 binary digits.

Converting Decimal to Octal

→ Integer part: keep dividing by 8 until the quotient is 0. Collect the remainders *in reverse order*.

→ Fr*actional part* parts

$$(3564)_{10} \div 8 = 445 \quad \text{remainder is } 4 \quad \text{LSB}$$
$$445 \div 8 = 55 \quad \text{remainder is } 5$$
$$55 \div 8 = 6 \quad \text{remainder is } 7$$
$$6 \div 8 = 0 \quad \text{remainder is } 6 \quad \text{MSB}$$

Therefore,

$$(3564)_{10} = (6754)_8$$

e *fractional* e integer

Same method as for the decimal to binary

Converting Octal to Decimal

- To convert Octal, or base 8, numbers to decimal we first obtain the polynomial representation of the number, then sum the products.

Example

$$(127.4)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} = (87.5)_{10}$$

Converting Binary to Octal

•To convert from binary to octal, make groups of 3 bits, starting from the binary point. Add 0s to the ends of the number if needed. Then convert each group of bits to its corresponding octal digit.

Example

$(10110100.001011)_2$ =(010 110 100 . 001 011 $)_2$

= (2 6 4 . 1 3 $)_8$

Converting Octal to Binary

- To convert from octal to binary, replace each Octal digit with its equivalent 3-bit binary sequence.
- Example

$$(261.35)_8 = (2\ \ 6\ \ 1\ .\ 3\ \ 5)_8$$
$$= (010\ \ 110\ \ 001\ .\ 011\ \ 101)_2$$

Hexadecimal numbers

-Digits = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

Positional Polynomial

- $(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0$

•*Hexadecimal* (base 16) digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, and F. Since $16 = 2^4$, one hexa digit is equivalent to 4 binary digits.

–It's often easier to work with a number like B5 instead of 10110101.

Converting Decimal to Hexadecimal

→ **Integer part**: keep dividing by 16 until the quotient is 0. Collect the remainders *in reverse order*.

→ **Fractional Part**: keep multiplying the *fractional part* by 16 until it becomes 0. Collect the integer parts (in forward order).

Same method as for the decimal to binary conversion

$$
\begin{aligned}
(37,822)_{10} \div 16 &= 2363 \quad \text{remainder is } 14 \text{ or E} \quad \text{LSB} \\
2,363 \div 16 &= 147 \quad \text{remainder is } 11 \text{ or B} \\
147 \div 16 &= 9 \quad \text{remainder is } 3 \\
9 \div 16 &= 0 \quad \text{remainder is } 9 \quad \text{MSB}
\end{aligned}
$$

Therefore,

$$(37,822)_{10} = (\text{EB } 39)_{16}$$

Converting Hexadecimal to Decimal

- To convert Hexadecimal, or base 16, numbers to decimal, first obtain the polynomial representation of the number, then sum the products.

Example

$$(B65F)_{16} = 11 \times 16^3 + 6 \times 16^2 + 5 \times 16^1 + 15 \times 16^0$$
$$= (46{,}687)_{10}$$

Converting Hexadecimal to Binary

- To convert from hexadecimal to binary, replace each hex digit with its equivalent 4-bit binary sequence.
- Example

261.35_{16} = (2 6 1 . 3 5)$_{16}$

= (0010 0110 0001 . 0011 0101)$_2$

Converting Binary to Hexadecimal

- To convert from binary to hex, make groups of 4 bits, starting from the binary point. Add 0s to the ends of the number if needed. Then convert each group of bits to its corresponding hex digit.
- Example

$(10110100.001011)_2 = (1011\ 0100.\ 0010\ 1100)_2$

$= (B\ 4\ .\ 2\ C)_{16}$

Decimal	Binary	Octal	Hex
0	0 0 0 0	0	0
1	0 0 0 1	1	1
2	0 0 1 0	2	2
3	0 0 1 1	3	3
4	0 1 0 0	4	4
5	0 1 0 1	5	5
6	0 1 1 0	6	6
7	0 1 1 1	7	7
8	1 0 0 0	1 0	8
9	1 0 0 1	1 1	9
1 0	1 0 1 0	1 2	A
1 1	1 0 1 1	1 3	B
1 2	1 1 0 0	1 4	C
1 3	1 1 0 1	1 5	D
1 4	1 1 1 0	1 6	E
1 5	1 1 1 1	1 7	F

Summary

- Converting from numerical system with base ***r*** to decimal

$$N = a^{n-1} \times r^{n-1} + a^{n-2} \times r^{n-2} + \ldots + a^{0} \times r^{0} + a^{-1} \times r^{-1} + \ldots + a^{-m} \times r^{-m}$$

- Converting from decimal to number of base ***r:***

$N_{10} = I_1.F_1$

Integer Part	Fractional Part
$I_1 / r = Q_1$ rem r_1	$F_1 * r = D_1 . f_1$
$Q_1 / r = Q_2$ rem r_2	$f_1 * r = D_2 . f_2$
...	...
$Q_{n-2} / r = Q_{n-1}$ rem r_{n-1}	$f_{n-2} * r = D_{n-1} . f_{n-1}$
$Q_{n-1} / r = 0$ rem r_n	$f_{n-1} * r = D_n . 0$
$I_r = (r_n r_{n-1} \ldots r_2 r_1)_r$	$F_r = (D_1 D_2 \ldots D_{n-1} D_n)_r$

- Number base r is $N_r = I_r.F_r$

Summary (continued)

- Converting from Binary to Octal
  - Divide the bits into groups of 3 – translate each 3-bits into an octal digit (see slide 31)
- Converting from Octal to binary
  - Convert each octal digit to its 3-bit value (see slide 31)
- Converting from Binary to Hexadecimal
  - Divide the bits into groups of 4 – translate each 4-bits into a hexadecimal digit (see slide 31)
- Converting from Hexadecimal to binary
  - Convert each hexadecimal digit to its 4-bit value (see slide 31)

ARITHMETIC OPERATIONS IN A BINARY SYSTEM

Binary Addition

	0	0	1	1
	+0	+ 1	+0	+1
	--	--	--	--
sum	0	1	1	10

(sum of 0 and carryover of 1)

Apply carry-over when adding multiple bit numbers

Examples:

1 0 0 1	0 0 0 1	1 1 0 0
+ 0 1 1 0	+ 1 0 0 1	+ 0 1 0 1
1 1 1 1	1 0 1 0	1 0 0 0 1

carryover

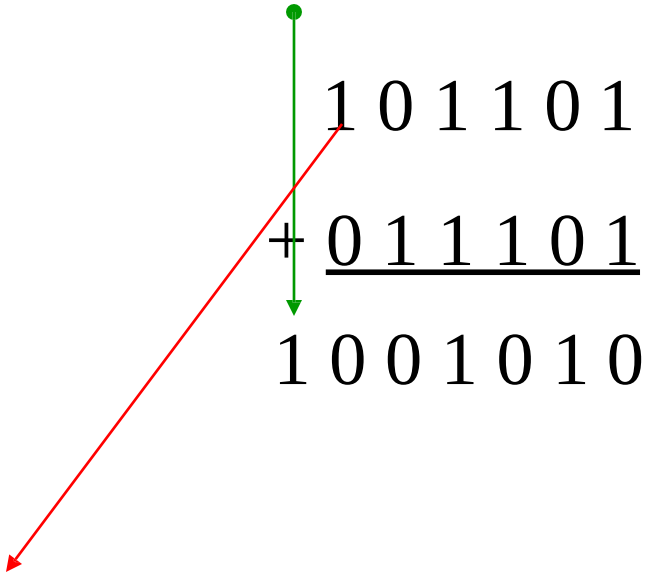
Binary Addition-Examples

Carry 1 1 1 1 0 1

1 0 1 1 0 1

\+ 0 1 1 1 0 1

Sum 1 0 0 1 0 1 0



Check your work

$(45)_{10}$

\+ $(29)_{10}$

= 74

$1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 32 + 0 + 8 + 4 + 1 = 45$

Binary Addition- Examples

Addition of three Binary Digits

x	y	CarryIn	Sum	CarryOut
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

Addition of Large Binary Numbers

- Example showing larger numbers:

	1 0 1 0	0 0 1 1	1 0 1 1	0 0 0 1
+	0 1 1 1	0 1 0 0	0 0 0 1	1 0 0 1
1	0 0 0 1	0 1 1 1	1 1 0 0	1 0 1 0

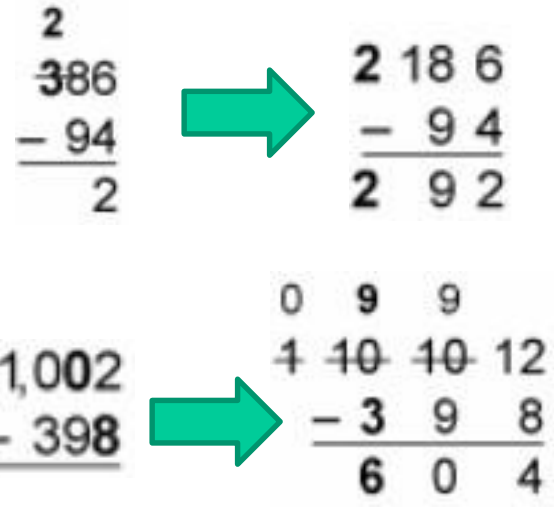
Binary Subtraction

	0	^{1}0	1	1
	- 0	- 1	- 0	- 1
	--	--	--	--
difference	0	1	1	0

(Requires a borrow, note that the value 10 has a decimal value of 2)

$$\begin{array}{r} 0\,1\,0\,1\,0\,0 \\ 1_0\,0_0\,1_0\,0\,1\,1 \\ -\;0\,1\,0\,1\,0\,1 \\ \hline 0\,1\,0\,1\,1\,0 \end{array}$$

Decimal Examples



Binary Multiplication

	0	0	1	1
	x 0	x 1	x 0	x 1
	--	--	--	--
Product	0	0	0	1

```
    1010      ---->  Multiplicand
  × 1011      ---->  Multiplier
 ------
    1010      ---->  Partial product 1
   1010       ---->  Partial product 2
  0000        ---->  Partial product 3
 1010         ---->  Partial product 4
 ------
 1101110
 ------
```

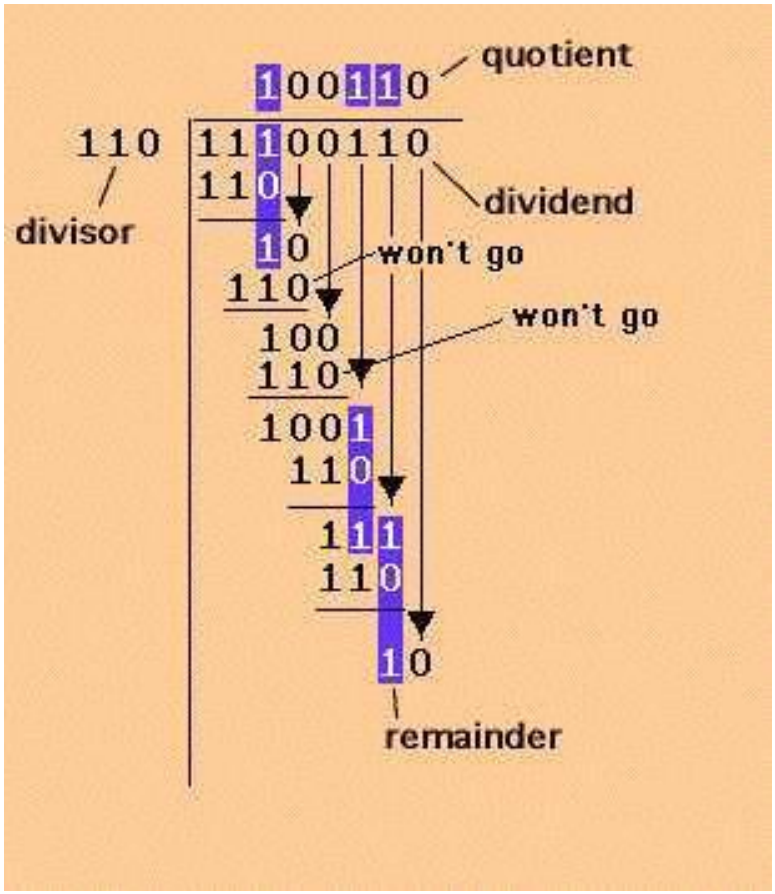
```
    1011.01
  x   110.1
 ----------
    101101
         0
   101101
  101101
 ----------
 1001001.001
```

Decimal Example

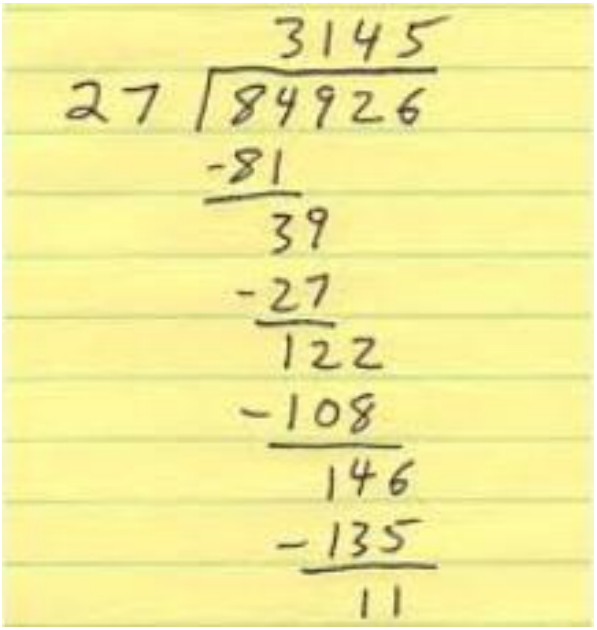
423 x 211

```
   423
 x 211
 -----
   423
  423
+ 846
 -----
 89253
 -----
```

Binary Division



Decimal Example



Complements in Numbering Systems

•Complements are used in digital systems (computers) for simplifying the Subtraction operation and for logical manipulation

• There are two types of complements for each ***base 'r' system:***

1- Radix complement ✉ r's complement

Ex. ***base 10 → 10's complement***

base 2 → 2's complement

2- Diminished radix complement ✉ (r-1) complement

Ex. ***base 10 → 9's complement***

Base 2 → 1's complement

Radix complement (*r*'s complement)

$$[N]_r = r^n - (N)_r$$

where n is the number of digits in $(N)_r$.

Example

- **What 2's complement of $(N)_2 = (101001)_2$**

$$[N]_2 = 2^6 - (101001)_2 = (1000000)_2 - (101001)_2 = (010111)_2$$

- **10's complement of $(N)_{10} = (72092)_{10}$**

$$[N]_{10} = (100000)_{10} - (72092)_{10} = (27908)_{10}.$$

Obtaining 2's complement

- **Can be obtained directly from the given number by**
  1 copying each bit of the number starting at the least significant bit and proceeding forward the most significant bit until the first 1 has been copied.
  2 After the first 1 has been copied replace each of the remaining 0s and 1s by 1s and 0s respectively

(a) $[1010100]_2 = 2^7 - (1010100)_2 = (10000000)_2 - (1010100)_2 = (\mathbf{0101100})_2$

(b) $[101001]_2 = 2^6 - (101001)_2 = (1000000)_2 - (101001)_2 = \mathbf{(010111)_2}$

(a) **1 0 1 0 1 0 0** (b) **1 0 1 0 0 1**

2 's → **0 1 0 1 1 0 0** **0 1 0 1 1 1**

Diminished radix complement (*r-1*'s complement)

$$[N]_{r-1} = (r^n - 1) - (N)_r$$

-9's complement of $[546700]_9$

$$= 999999 - 546700 = 453299$$

-1's complement of [1011000]

$$= (10000000 - 1)_2 - (1011000)_2 = (0100111)_2$$

Obtaining 1's complement

- ***1 's complement can be obtained directly from the given number by replacing each of 0s and 1s by 1s and 0s of the number (i.e. complement each bit)***

$$\mathbf{[1011000]} = (10000000 - 1)_2 - (1011000)_2$$

$$= \mathbf{(0100111)_2}$$

1 0 1 1 0 0 0

1 's complement → **0 1 0 0 1 1 1**

Summary of Complements

$[N]_r = r^n - (N)_r$

- 10's Complement
  - Subtract from 10^n
  - Take 9's complement and add 1
- 2's Complement
  - Subtract from 2^n
  - Start from LSB, copy bit until 1 is copied, then change bit up to MSB
  - Flip each bit and add 1

$[N]_{r-1} = (r^n - 1)_r - (N)_r$

- 9's Complement
  - Subtract from 10^n-1
  - Subtract each digit from 9
- 1's Complement
  - Subtract from 2^n-1
  - Flip each bit

Subtraction with 2's Complement

- **2's complement are used to convert subtraction to addition, which reduces hardware requirements (only adders are needed).**

$$A - B = A + (-B) = A + 2's\{B\}$$

(add *2*'s complement of *B* to *A*)

- 2's Complement has the properties of the minus sign

A + (-A) = 0 **A + 2's{A} = 0**

- (- A) = A **2's{ 2's{A}} = A**

A - B = A+ (-B) **A - B = A +2's{B}**

Subtraction with 2's Complement [2]

Examples:

A= 1010100 B= 1000011

- 2's complement

A - B = A + (-B)= A+ [B]

= (1010100) + (0111101) = (0010001)

End carry = 1, means that B is less than A, and result is a positive number

$$\begin{array}{rr} & 1010100 \\ + & 0111101 \\ \hline \cancel{1} & 0010001 \end{array}$$

End carry = 0, means that B is larger than A, and result is 2's complement number (a negative value), try B-A in our example.

Subtraction with 1's Complement

Examples: note: same proprieties minus sign as 2's complement.

A= 1010100 B= 1000011

- 1's complement

A - B = A+ [B]= (1010100) + (0111100) = (0010001)

```
          1010100
+         0111100
    ---------------
Add end carry   1  0010000
             └──→ +  1
    ---------------
          0010001
```

Subtraction with 10s/9'sComplements

$(72)_{10}$ - $\mathbf{(32)_{10}}$ = $\mathbf{(40)_{10}}$

10's Complement

[32]= 10^2 – $\mathbf{(32)_{10}}$ = $\mathbf{(68)_{10}}$

$\mathbf{(72)_{10}}$ + $\mathbf{(68)_{10}}$ = ~~**1**~~ $\mathbf{(40)_{10}}$

9's Complement

[32]= (10^2 -1) – $\mathbf{(32)_{10}}$ = $\mathbf{(67)_{10}}$

$(72)_{10}$ + $\mathbf{(67)_{10}}$ = $\mathbf{(1 + 39)_{10}}$ = $\mathbf{(40)_{10}}$

Signed Binary numbers

• Recall that digital Systems are made with devices that take on exactly two states : 0 and 1.

•**The only states are “1” and “0”. There is no “-” state!**

→ because of hardware limitations computers represent negative numbers by using the leftmost bit for the sign bit.

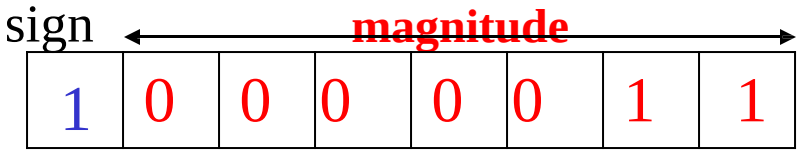
-- “0” indicates a positive number,

-- while a “1” indicates a negative number

Signed Magnitude

The leftmost bit indicates the sign of the number. The remaining bits give the magnitude of the number

→ Using 8 bits to represent binary number the value in the example is:

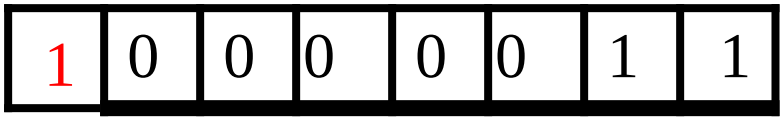


-3 = 10000011 = 1/ (sign bit) 0000011

→ Sign Magnitude representation is good for having the ability for a human to read and understand what number is represented

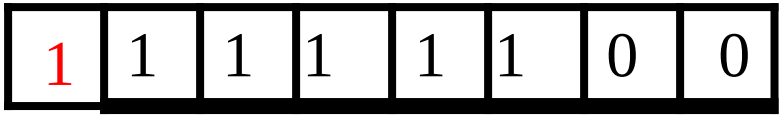
Signed Complement

(a) Signed Magnitude representation



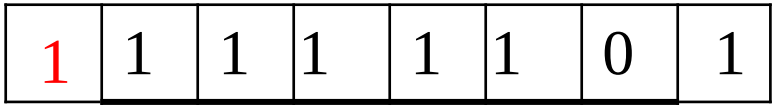
-3 = 10000011 = 1/ (sign bit) 0000011

(b) Signed 1's representation



-3 = 10000011 = 1/ (sign bit) 1111100

(c) Signed 2's representation



-3 = 10000011 = 1/ (sign bit) 1111101

Signed Binary Numbers.

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

Fixed-Length Registers

- All practical digital devices have fixed-length registers
- This means that numbers in a computer are represented by a fixed number of bits
  - The earliest microprocessors were 4-bit devices
  - Intel 8080 and the 6502 (Apple II) chips were 8-bit
  - Intel 8088 (IBM PC) and Motorola 68000 (Mac) are 16-bit devices
  - Pentium chips and PowerPC chips are 32-bit

Range of a number Overflow during addition

- A fixed-length register can only hold a ***Range*** of numbers
  - For a 4-bit device, the *range* of positive integers is 0 - 15
  - For an 8-bit device the *range* of positive integers is 0 – 255
  - When adding positive integers, ***Overflow*** occurs when the sum falls outside the range of the register

Overflow in Signed Complements

•when numbers are treated as signed complement, a "carry" of 1 from the addition of the most significant bits **DOES NOT** indicate an overflow,

3 00011
+ (-3)+11101
= 00000, with a carry of "1" (2'complement) : We know that addion operation in 2's complement the end-carry is discarded !

•For signed complement, overflow occurs when:

→ ***The addition of two positive numbers results in a negative number***

OR → ***The addition of two negative numbers results in a positive number***

Overflow Examples

- In a 6-bit register with signed 2's complement

+ 17 = 010001

+ 16 = +010000

= 100001

100001 = - (11111) = -$(31)_{10}$ instead of **+ $(33)_{10}$**

- Same with a 7-bit register

+ 17 = 0 010001

+ 16 = +0 010000

= 0100001

0100001 = + 33 No Overflow

Binary codes: *BCD* (1)

- To represent information as strings of alpha-numeric characters.
- ***Binary Coded Decimal (BCD)***
  - Used to represent the decimal digits 0 - 9.
  - 4 bits are used.
  - Each bit position has a weight associated with it (***weighted code***).
  - Weights are: 8, 4, 2, and 1 from MSB to LSB (called 8-4-2-1 code).

Binary codes: *BCD* (2)

– BCD Codes:

0→ 0000	1→ 0001	2→ 0010
3→ 0011	4→0100	5→ 0101
6→ 0110	7→0111	8→ 1000
9→ 1001		

– Used to encode numbers for output to numerical displays

–***Example***: $(9750)_{10} = (1001011101010000)_{BCD}$

Binary codes: *ASCII* [2]

- ***ASCII*** (American Standard Code for Information Interchange) (see table 1.7 of textbook)
  - Most widely used character code.
  - ***Example***: ASCII code representation of the word ‘***Digital’***

Character	Binary Code	Hexadecimal Code
D	1000100	44
i	1101001	69
g	1100111	67
i	1101001	69
t	1110100	74
a	1100001	61
l	1101100	6C

Practice Problems Solved in the Class

Examples: Signed Complements 2's

	Sign-bit	
$(9)_{10}$	0	1001
$+(6)_{10}$	0	0110
	0	1111
$(9)_{10}$	0	1001
$-(6)_{10}$	1	1010
	0	0011
$(6)_{10}$	0	0110
$-(9)_{10}$	1	0111
	1	1101

Examples: Signed Complements 1's

	Sign-bit	
$(9)_{10}$	0	1001
$+(6)_{10}$	0	0110
	0	1111

$(9)_{10}$	0	1001
$-(6)_{10}$	1	1001
1	0	0010 = (0010) + (0001) = (0011)

$(6)_{10}$	0	0110
$-(9)_{10}$	1	0110
	1	1 1 0 0

Problems

Question

(a) Convert le following binary number into (i) Octal, (ii) Decimal, (iii) hexadecimal

10101101.10110

(b) Convert A= 16.25 and B = 8.25 into binary, use 7 bits to represent the integer part and 3 bits to represent the fractional part, then perform the following operations

I) C= A + B
ii) D = A – B

Note: Compute C and D
(a) using non-signed binary numbers and without complements
(b) using signed 2's complement

(c) Convert le following number into (i)Decimal, (ii) Octal, (iii) binary

$$(FD8.C2B)_{16}$$

Problems

Answers: **10101101.10110**

i) Octal→ **(010 101 101.101 100)**

(2 5 5 . 5 4)$_8$

iii) Hexa → **(1010 1101.1011 0000)2**

(A D . B 0)16

ii) Decimal

$(10101101.10110)_2 = 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2}\ 1 \times 2^{-3} + 1 \times 2^{-4} + 0 \times 2^{-5}$

$= (173.6875)_{10}$

Problems

Integer part

- Conversion de $(16)_{10}$.

```
16 / 2  = 8   remainder 0
 8 / 2  = 4   remainder 0
 4 / 2  = 2   remainder 0
 2 / 2  = 1   remainder 0
 1 / 2  = 0   remainder 1
```

- Then $(16)_{10} = (10000)_2$

Fractional part

– Converting: $(0.25)_{10}$

$0.25 \times 2 = 0.50$
$0.50 \times 2 = 1.00$ ↓

•then, $(.25)_{10} = (.01)_2$

and $(16.25)_{10} = (10000.01)_2$

Same as for A

→ B = 8.25: $(8.25)_{10} = (1000.01)_2$

→ Representation using 7 bits and 3 bits

A= $(16.25)_{10}$ = $(0010000.010)_2$

B= $(8.25)_{10}$ = $(0001000.010)_2$

Problems

	Non Signed Binary
A	0010000.010
+ B	0001000.010
	0 011000.100

A	0010000.010
- B	0001000.010
	0001000.000

Problems

Signed 2' complement

A	0 010000.010
+ B	0 001000.010
	0 011000.100

A	0 010000.010
+ (-B)	1 110111.110
	0 001000.000

Question 1:

Convert A = $(00010010.0101)_{BCD}$ and B= $(2.25)_{10}$ into pure binary format employing 8 bits for the integer part and 3 for the fractional part, including the sign bit. Perform the following operations in specific signed complement as indicated for each operation.

(i) C = -A - B using signed 2's complement
(ii) D = – A + B using signed 1's complement
(iii) E= A – B using 9's complement (show all intermediate steps)

Problems

A= $(12.50)_{10}$ B= $(2.25)_{10}$ $(12.50)_{10}$ - $(2.25)_{10}$ using 9's complement

9's complement of (2.25)
First step use the same number of position to represent the two numbers (02.25) then obtain the 9's complement

$[02.25]_9 = (10^4 - 1)_{10} - (02.25)_{10} = (99.99 - 02.25)_{10} = (97.74)_{10}$

E = A- B = A+ $[B]_9$ =

```
          12.50
    +     97.74
(end carry) 1  10 .2 4
                  + 1
          10.  25
```