# Assignment #2

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## Problem 3.14 pg 120

a) $S: \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

b) $P = 1/36$ for all the sample points

c) $P(A) = 1/36$ $P(B) = 1/2$ $P(C) = 1/6$ $P(D) = 11/36$ $P(E) = 1/6$

## Problem 3.15 pg 120

; Do this problem in two ways i) From first principles, starting by numbering the marbles, and ii) by using your knowledge of counting rules

a) $(B_1, B_2), (B_1, R_1), (B_1, R_2), (B_1, R_3), (B_2, R_1), (B_2, R_2), (B_2, R_3), (R_1, R_2), (R_1, R_3), (R_2, R_3)$

b) $1/10$ for each sample point

c)

| | First Principles | Counting Rules |
|---|---|---|
| Two 2 marbles e drawn | $P(A)$: Only option with two blue marbles $\rightarrow (B_1, B_2)$ $\therefore 1/10$ | $P(A)$: $\frac{2C_2\, 3C_0}{5C_2} = \frac{1}{10}$ |
| 1 red and 1 marble drawn | $P(B) =$ 6 options with b & r marbles $(B_1,R_1)(B_1,R_2)(B_1,R_3)(B_2,R_1)(B_2,R_2)(B_2,R_3) \Rightarrow 6/10 \Rightarrow 3/5$ | $P(B) = \frac{2C_1 \times 3C_1}{5C_2} = \frac{3}{5}$ |
| two red marbles drawn | $P(C) =$ 3 options with two red marbles $(R_1,R_2)(R_1,R_3)(R_2,R_3) \Rightarrow 3/10$ | $P(C) = \frac{2C0 \times 3C2}{5C2} = \frac{3}{10}$ |

## Problem 3.20 page 121

; Do not answer parts a) and b). Answer the following questions. If 5 rhinos are selected at random w/o replacement, what is the probability that: i) Exactly 2 would be white rhinos?

$P(2R) = \frac{3610C2 \times 11330C3}{14940C5} \approx 0.2546 \approx 25.5\%$

$0.2546 + 0.0811 + 0.0129 + 8.219 \times 10^{-4}$

ie: 2 or 3 or 4 or 5

ii) At least two would be white rhinos?

$\frac{3610C2 \times 11330C3}{14940C5} + \frac{3610C3 \times 11330C2}{14940C5} + \frac{3610C4 \times 11330C1}{14940C5} + \frac{3610C5 \times 11330C0}{14940C5}$

$\approx 0.3495 \approx 34.95\%$

## Problem 3.32 page 123

Let N represent navy and B represent black

Possible order socks could be in:

BBNN BNBN BNNB NNBB NBNB NBBN

P(correct pairing) = 2/6 → (BBNN), (NN,BB)

P(incorrect pairing) = 4/6 → (BNBN), (BNNB), (NBNB), (NBBN)

∴ the 50/50 statement is incorrect

## Problem 3.34 page 123

a)

| | B | b |
|---|---|---|
| B | BB | Bb |
| b | Bb | bb |

$\Rightarrow \boxed{1/4}$

$P(bb) = \frac{\text{\# of favorable outcomes}}{\text{\# of possible outcomes}} = \frac{1}{4}$

b)

| | B | b |
|---|---|---|
| b | Bb | bb |
| b | Bb | bb |

$\Rightarrow 2/4 = \boxed{1/2}$

c) If one parent has the gene pair BB then one of those genes will be given to the child so they could only have the gene pairs BB or Bb => both brown. ∴ child cannot have blue eyes. [P(bb) = 0]

## Problem 3.44 page 131

a) A = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

B = {(1,4), (2,4), (3,4), (5,4), (6,4), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6)}

A ∩ B = {(3,4), (4,3)} A ∪ B = {(1,6), (2,5), (5,2), (6,1), (1,4), (2,4), (3,4), (5,4) (6,4) (4,1) (4,2) (4,3) (4,4), (4,5), (4,6)}

a) continued...

$A^c = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (2,6), (3,1), (3,2), (3,3), (3,5), (3,6), (4,1), (4,2), (4,4), (4,5), (4,6), (5,1), (5,4), (5,3), (5,5), (5,6), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

b) $P(A) = 6 \times \frac{1}{36} = \frac{1}{6}$  $P(A \cap B) = 2 \times \frac{1}{36} = \frac{1}{18}$

$P(B) = 11 \times \frac{1}{36} = \frac{11}{36}$  $P(A^c) = 30 \times \frac{1}{6} = \frac{5}{6}$

$P(A \cup B) = 15 \times \frac{1}{36} = \frac{5}{12}$

c) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 1/6 + 11/36 - 1/18$

$= 6/36 + 11/36 - 2/36$

$= 15/36 = 5/12$

$\therefore P(A \cup B) = 5/12$ → matches answer from part b)

d) Since there is a value for $P(A \cap B)$ ($1/18$) it means that A and B are not mutually exclusive. ie: they have at least one outcome in common.
A and B both share: (3, 4) and (4, 3).

Problem 3.58 page 134

a) Total = 160

85/160 → color code 5  85/160 ⇒ 17/32  $P(CC5) = 17/32$

b) 35/160 → color code 0

$\frac{85}{160} + \frac{35}{160} = \frac{120}{160} = \frac{3}{4}$  $P(CC5 \text{ or } CC0) = \frac{3}{4}$

c) 15/160 → model 2 and CC 0

$\frac{15}{160} = \frac{3}{32}$  $P(m2 \text{ and } CC0) = \frac{3}{32}$

## Problem 3.60 page 134

a) A = Absenteeism  T = turnover

$P(A) = 55\% = 0.55$  $P(T) = 41\% = 0.41$  $P(\text{both}) = 0.22$

$P(A \cup T) = P(A) + P(T) - P(A \cap T)$

$= 0.55 + 0.41 - 0.22$

$= 0.74 = 74\%$

b) $P(A^c) = 1 - P(A)$

$= 1 - 0.55$

$= 0.45 = 45\%$

c) $P(A \cup B)^c) = 1 - 0.74$

$= 0.26 = 26\%$

## Problem 3.62 page 134

Event A: The sum on dice is equal to 9

Event B: The sum on dice is equal to 10

Event A: 25 sample points

Event A: (1,2,6) (1,3,5) (1,4,4) (1,5,3) (1,6,2) (2,1,6) (2,2,5) (2,3,4) (2,4,3) (2,5,2) (2,6,1) (3,1,5) (3,2,4) (3,3,3) (3,4,2) (3,5,1) (4,1,4) (4,2,3) (4,3,2) (4,4,1) (5,1,3), (5,2,2), (5,3,1) (6,1,2) (6,2,1)

Event B: 27 sample points

(1,3,6) (1,4,5) (1,5,4) (1,6,3) (2,2,6) (2,3,5) (2,4,4) (2,5,3) (2,6,2) (3,1,6) (3,2,5) (3,3,4) (3,4,3) (3,5,2) (3,6,1) (4,1,5) (4,2,4) (4,3,3) (4,4,2) (4,5,1) (5,1,4) (5,2,3) (5,4,1) (6,1,3) (6,2,2) (6,3,1)

The probability of all the sample points in each event are equal because we are tossing 3 fair dice. Since Event A contains 25 sample points and event B contains 27 the probability of the event A occuring is less than the probability of event B occuring thus the Grand Duke was incorrect because there are more ways in which you can roll a 10 with 3 fair die than a 9.

## Problem 3.90 pg 148

a) Let

$A$ = System A detects $P(A|C) = 0.9$

$B$ = System B detects $P(B|C) = 0.95$

$C$ = intruder $P(A|C^c) = 0.2$

$C^c$ = no intruder $P(B|C^c) = 0.1$

b) $P(A \cap B|C) = P(A|C) \times P(B|C) = 0.9 \times 0.95$
$= 0.855$

c) $P(A \cap B|C^c) = P(A|C^c) \times P(B|C^c) = 0.2 \times 0.1$
$= 0.02$

d) $P(A^c|C) = 1 - P(A|C) = 1 - 0.9 = 0.1$

$P(B^c|C) = 1 - P(B|C) = 1 - 0.95 = 0.05$

P(Neither sounds alarm): $0.1 \times 0.05 = 0.005$

P(at least one): $1 - 0.005 = 0.995 = 99.5\%$