PHY1321 PHY1331 Fall 2019 Assignment 5. Due Oct 21, 6:00PM

LAST NAME: ____________________ ASSIGNED NUMBER______________

Assignment 5: ENTROPY

1. An ice tray contains 500 g of liquid water at 0°C. Calculate the change in entropy of the water as it freezes slowly and completely at 0°C. (It is a one-line problem so please fit this into the space provided below:

$$\Delta S = \frac{mL}{T} = \frac{0.5 \cdot (3.3 \cdot 10^5)}{273}\frac{J}{K} = 604.40\frac{J}{K}$$

2 A 1.00-kg iron horseshoe is taken from a forge at 900°C and dropped into 4.00 kg of water at 10.0°C. Assuming that no energy is lost by heat to the surroundings, determine the total entropy change of the horseshoe-plus-water system.

Q2 SOLUTION: $c_{\text{iron}} = 448\ \text{J/kg}\cdot°\text{C};\ c_{\text{water}} = 4\,186\ \text{J/kg}\cdot°\text{C}$ since $Q_{\text{cold}} = -Q_{\text{hot}}$:

we have: $4.00\ \text{kg}(4\,186\ \text{J/kg}\cdot°\text{C})(T_f - 10.0°\text{C}) = -(1.00\ \text{kg})(448\ \text{J/kg}\cdot°\text{C})(T_f - 900°\text{C})$

which yields $T_f = 33.2°\text{C} = 306.2\ \text{K}$

$$\Delta S = \int_{283\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{1173\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1173}\right)$$

$$\Delta S = (4\,186\ \text{J/kg}\cdot\text{K})(4.00\ \text{kg})(0.078\,8) + (448\ \text{J/kg}\cdot\text{K})(1.00\ \text{kg})(-1.34)$$

$$\Delta S = \boxed{718\ \text{J/K}}$$

3 What change in entropy occurs when a 27.9-g ice cube at –12°C is transformed into steam at 115°C?
We assume a constant specific heat for each phase. As the ice is warmed from –12°C to 0°C, its entropy increases by:

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{261\ \text{K}}^{273\ \text{K}} \frac{m\, c_{\text{ice}}\, dT}{T} = m\, c_{\text{ice}} \int_{261\ \text{K}}^{273\ \text{K}} T^{-1} dT = m\, c_{\text{ice}} \ln T\Big|_{261\ \text{K}}^{273\ \text{K}}$$

$$\Delta S = 0.027\,0\ \text{kg}(2\,090\ \text{J/kg}\cdot°\text{C})(\ln 273\ \text{K} - \ln 261\ \text{K}) = 0.027\,0\ \text{kg}(2\,090\ \text{J/kg}\cdot°\text{C})\left(\ln\left(\frac{273}{261}\right)\right)$$

$$\Delta S = 2.54\ \text{J/K}$$

As the ice melts its entropy change is $\Delta S = \frac{Q}{T} = \frac{m L_f}{T} = \frac{0.027\,0\ \text{kg}(3.33\times 10^5\ \text{J/kg})}{273\ \text{K}} = 32.9\ \text{J/K}$

As liquid water warms from 273 K to 373 K,

$$\Delta S = \int_i^f \frac{m\, c_{\text{liquid}}\, dT}{T} = m\, c_{\text{liquid}} \ln\left(\frac{T_f}{T_i}\right) = 0.027\,0\ \text{kg}(4\,186\ \text{J/kg}\cdot°\text{C})\ln\left(\frac{373}{273}\right) = 35.3\ \text{J/K}$$

As the water boils and the steam warms,

$$\Delta S = \frac{m L_v}{T} + m\, c_{\text{steam}} \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{0.027\,0\ \text{kg}(2.26\times 10^6\ \text{J/kg})}{373\ \text{K}} + 0.027\,0\ \text{kg}(2\,010\ \text{J/kg}\cdot°\text{C})\ln\left(\frac{388}{373}\right) = 164\ \text{J/K} + 2.14\ \text{J/K}$$

The total entropy change is $(2.54 + 32.9 + 35.3 + 164 + 2.14)\ \text{J/K} = \boxed{236\ \text{J/K}}$.

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4. A 1.00-mol sample of an ideal monatomic gas, initially at a pressure of 1.00 atm and a volume of 0.025 0 m^3, is heated to a final state with a pressure of 2.00 atm. and a volume of 0.040 0 m^3. Determine the change in entropy of the gas in this process.

$$\Delta S = nC_V ln\frac{T_f}{T_i} + nRln\frac{V_f}{V_i} = \frac{3}{2}Rln\frac{p_f V_f}{p_i V_i} + Rln\frac{V_f}{V_i} = \frac{3}{2}Rln\frac{p_f}{p_i} + \frac{5}{2}Rln\frac{V_f}{V_i} = 12.43ln2 + 20.78ln1.6 = 18.38\frac{J}{K}$$

5 A 1 500-kg car is moving at 20.0 m/s. The driver brakes to a stop. The brakes cool off to the temperature of the surrounding air, which is nearly constant at 20.0°C. What is the total entropy change?

$$\Delta S = \frac{\Delta E_{kin}}{T} = \frac{300000J}{293K} \cdot \frac{J}{K} = 1023.9 \cdot \frac{J}{K}$$

6 a) Find the speed of the fastest O_2 molecule that can be found in 1 mole of air at 20C. (use dv=1m/s). Provide the answers below:

One mole of air contains 0.21moles of oxygen.

We use the Maxwell-Boltzmann for 0.21 mole of O_2 and dv=1m/s

$$N_v = NP_v = 0.21 \cdot 1 \cdot P_v = 0.21 \cdot 4\pi \cdot \left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}} v^2 e^{-\frac{Mv^2}{2RT}} dv$$

Change v until N_v=1

A_ Ans: v_{max}=2812.8m/s=2813m/s

b) Express this speed in terms of v_{mp} at this temperature

B_ Ans $v_{mp}(O_2, 293K)$=390m/s, v_{max}=/ v_{mp} =7.21

c) C_ Find how many moles of the He atoms at T=20°C are needed to have 1 He atom reaching the escape velocity from Earth (11200m/s)

Probability of the He atom to reach this speed is given by: $P = 7.4 \cdot 10^{-46}$.

To find one of the atoms having this speed one needs roughly $1.33 \cdot 10^{45} atoms$.

This translates to $n = 2.2 \cdot 10^{21}$

7 Question 7 is supposed to be solved/answered individually – Help Centre has been instructed not to help students with it! Please treat this question as a puzzle – I invoke an honor system!

a) The photon of energy of 10eV is absorbed by the 200000 liters tank of water at temperature of 20C. What is resulting change in the entropy?

The tank will not change its temperature (293K) as result of absorbing the Q=10eV=16x10^{-19}J of heat. The entropy change is thus given by:

$$\Delta S = \frac{Q}{T} = \frac{16}{293} \cdot 10^{-19}\frac{J}{K} = 5.46 \cdot 10^{-21}\frac{J}{K}$$

b) As result of certain process the number of possible microstates realizing certain microstate changed from the initial value of 10^{27} to the final value of 10^{39}. What was the change of the entropy?

$$\Delta S = klnW_f - klnW_i = kln\frac{W_f}{W_i} = \frac{8.31}{6.02} \cdot 10^{-23} ln10^{12} = \cdot 10^{-22}\frac{J}{K}$$