PHY1321 PHY1331 Fall 2019 Assignment 4. Due Oct 11, 6:00PM

LAST NAME: ____________________ ASSIGNED NUMBER______________

Maxwell Boltzmann Distribution Heat Engines,

1 Given is 1mole of nitrogen molecules at atmospheric pressure and temperature of 27^{o}C.

a) Write down the correct expression for N(500,502) , the number of molecules having speeds in the interval (500m/s 502/m/s).

b) Calculate the number of molecules having their speed between 500m/s and 502m/s. (provide the number corresponding to part a (do not show your calculations!)

SOLUTION:

a) $P_v = \frac{N_v}{N} = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv 4\pi\left(\frac{M}{2\pi RT}\right)^{\frac{3}{2}} v^2 e^{-\frac{Mv^2}{2RT}} dv = 4\pi\left(\frac{0.028}{2\pi(8.31)300}\right)^{\frac{3}{2}} (501)^2 e^{-\frac{0.028\cdot 501^2}{2(8.31)300}}(2)$

b) P_v=0.00364$N_v \Longrightarrow P_v N = 2.22 \cdot 10^{21}$

2 Using the arguments based on quantum mechanics and the Boltzmann energy distribution explain briefly:

a) Why real molecules behave (from the thermodynamics point of view) as the ideal gas molecules?

In general they don't ! But in a very low temperatures(<150K) all gases behave as ideal gases because their internal energy structure (higher lying energy levels) are inaccessible!

b) Why are the rotational degrees of freedom activated at relatively low temperatures for most molecules?

At relatively low Temperatures (150K< 500K) only the rotational energy levels are accessible via collisions!

c) Why are the vibrational degrees of freedom activated at very high temperatures for most molecules?

It takes very High Temperatures (5000K and higher) to activate the vibrational energy levels via collisions!

d) Why are the noble gases well approximated by the ideal gas?

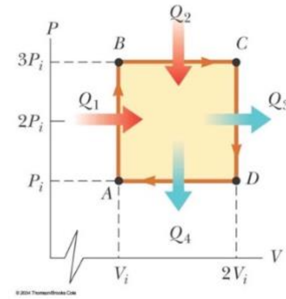
Noble gases are monoatomic, and atoms don't have oscillatory or rotational structure. Additionally the energy gaps in electronic structure require temperatures of the order of 100000 K to be populated.

3. Using Maxwell=Boltzmann Distribution of speeds for Ideal Gas obtain the Boltzmann Distribution of Energies for Ideal Gas. (Follow Lecture Discussions).
/Present your work on the opposite side of this page/

PHY1321 PHY1331 Fall 2019 Assignment 4. Due Oct 11, 6:00PM

LAST NAME: ______________________ ASSIGNED NUMBER_______________

4. A 1.00-mol sample of a monatomic ideal gas is taken through the cycle shown. At point *A*, the pressure, volume, and temperature are P_i, V_i, and T_i, respectively. In terms of *R* and T_i, find (a) the total energy entering the system by heat per cycle, (b) the total energy leaving the system by heat per cycle, (c) the efficiency of an engine operating in this cycle,
(HINT: efficiency of the engine= |Work performed|/|Heat absorbed|



At point *A*, $P_iV_i = nRT_i$ and $n = 1.00$ mol
At point *B*, $3P_iV_i = nRT_B$ so $T_B = 3T_i$
At point *C*, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$
At point *D*, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$

The heat for each step in the cycle is found using $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$
$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$
$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$
$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

(a)Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$ (b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c) Actual efficiency, $e = \frac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

5. A refrigerator has a coefficient of performance of 4.00. The ice tray compartment is at –20.0°C, and the room temperature is 22.0°C. The refrigerator can convert 30.0 g of water at 22.0°C to 30.0 g of ice at -20.0°C each minute. What input power is required? Give your answer in watts.

The Power required is equal to $P_W = \frac{W}{\Delta t}$ For the refrigerator, the COP=$\frac{|Q_c|}{|W|}$

The total heat Q_c removed by the refrigerator is given by:

$$Q_c = Q_1 + Q_2 + Q_3 = mc_{ice}(-20 - 0) - mL + mc_{water}(0 - 22))$$
$$Q_c = -0.03 \cdot (20 \cdot 2090 + 333000 + 22 \cdot 4186) = -14007J$$
$$COP = \frac{|Q_c|}{|W|} \Longrightarrow |W| = \frac{14007J}{4} = 3502J \Longrightarrow P_W = \frac{W}{\Delta t} = \frac{3502J}{60s}$$

ANS: To accomplish this task the required power is 58.4 W

6 A heat engine operates between two reservoirs at T_2 = 500 K and T_1 = 300 K. It takes in 2 000 J of energy from the higher-temperature reservoir and performs 400 J of work. Find (a) the entropy change of the Universe ΔS_U for this process and (b) the work *W* that could have been done by an ideal Carnot engine operating between these two reservoirs. (c) Show that the difference between the amounts of work done in parts (a) and (b) is $T_1 \Delta S_U$.

a) $\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} = \frac{-2000}{500} + \frac{1600}{300}\frac{J}{K} = 1.333\frac{J}{K}$

b) e $= \frac{|W|}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|}$ so for the Carnot engine $e = 1 - \frac{|T_c|}{|T_h|} = 1 - \frac{3}{5} = 0.4$ and thus

$W = (0.4)\ 2000J = 800J$ $\Delta S = \frac{Q_h}{T_h} + \frac{Q_c}{T_c} = \frac{-2000}{500} + \frac{1600}{300}\frac{J}{K} = 1.333\frac{J}{K}$

Work difference is 400J, while $T_c\Delta S = T_1\Delta S = 300(1.333) = 400\ J$