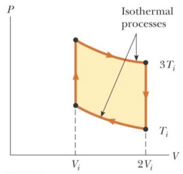
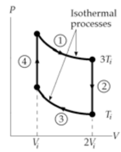
PHY1321 PHY1331 Fall 2019 Assignment 2. Due Oct 4 , 6:00PM

LAST NAME (TYPED): ____________________ ASSIGNED NUMBER(BRIGHSPACE)____________

First Law of Thermodynamics, Heat and Work in Gas Processes, Kinetic Theory of Gases

1 In 1816 Robert Stirling, a Scottish clergyman, patented the *Stirling engine*, which has found a wide variety of applications ever since. Fuel is burned externally to warm one of the engine's two cylinders. A fixed quantity of inert gas moves cyclically between the cylinders, expanding in the hot one and contracting in the cold one. Figure below represents a model for its thermodynamic cycle. Consider *n* mol of an ideal monatomic gas being taken once through the cycle, consisting of two isothermal processes at temperatures $3T_i$ and T_i and two constant-volume processes. Determine, in terms of *n*, *R*, and T_i, (a) the net energy transferred by heat to the gas. (b) the ratio of the **work performed by the system** to the **heat absorbed** by it in one cycle.
/A Stirling engine is easier to manufacture than an internal combustion engine or a turbine. It can run on burning garbage. It can run on the energy of sunlight and produce no material exhaust.**/**
SOLUTION:





a) In the isothermal process: $Q = nRTln\frac{V_2}{V_1}$. Therefore $Q_3 = nR(3T_i)\ln(2)$ and $Q_3 = nRT_i\ln\left(\frac{1}{2}\right)$

For the constant volume processes, $Q = nC_V(T_f - T_i)$ so $Q_2 = n\frac{3}{2}R(T_i - 3T_i)$ and $Q_4 = n\frac{3}{2}R(3T_i - T_i)$

The net energy by heat transferred is $Q = Q_1 + Q_2 + Q_3 + Q_4 = 2nRT_i ln2$

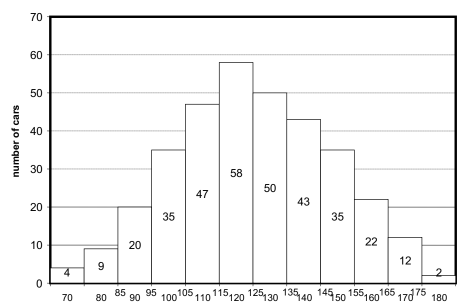
(b)A positive value for heat represents energy transferred into the system. $Q = Q_1 + Q_4 = 3nRT_i(1 + ln2)$

Since the change in temperature for the complete cycle is zero, $\Delta E_{int} = Q + W = 0 \;\; so \; Q = -W$

Therefore, the ratio in question is: $\frac{|W|}{|Q_h|} = \frac{Q}{Q_h} = \frac{Q}{Q_1+Q_4} = \frac{nRT_i(ln2)}{3nRT_i(1+ln2)} = 0.273$

2 Given is distribution of speeds of cars at 417 Highway as measured by OPP.
a) Is this a discrete or continuous distribution?
b) Find the Vmp, Vrms, Vavg.
c) Find the probability that a randomly picked car will have speed lower than 125km/h.
d) Find the probability that a randomly picked car will have speed larger than 95km/h and less then 135km/h.

Answers:
a) discrete
b) V_{mp}=120km/h; V_{rms} =127km/h; V_{avg}=125km/h
c) P(v<125) =
d) P(95<v<135)



3 Fill the table below: A, B, C D and E represent different gases. Fill the table below:

	Degrees of Freedom	AVG Energy of single molecule	Cv	Cp	Gamma
A	5	5/2kT	5/2 R	7/2 R	7/5
B	1	1/2 kT	1/2 R	3/2 R	3
C	9	9/2kT	9/2R	11/2 R	11/9
D	11	11/2 kT	11/2R	13/2 R	13/11
E	3	3kT	3R	4R	4/3

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4 **A)** One mole of an ideal gas do 5 000 J of work on its surroundings as it expands isothermally to a final pressure of 2.00 atm and volume of 25.0 L. Determine (i) the initial volume and (ii) the temperature of the gas.

(i) $W = -nRTln\frac{V_f}{V_i} = -p_fV_fln\frac{V_f}{V_i}$ (i) so $V_i = V_f e^{\frac{W}{p_fV_f}}$ so that

$V_i = 0.025e^{\frac{-5000}{(0.025)(2)(101300)}} = 0.025e^{-0.9872}$ =0.00932m³ = 9.32*l*

(ii) $T_i = T_f = \frac{p_fV_f}{nR} = \frac{2(101300)(0.025)}{1(8.314)} = 609.2K$

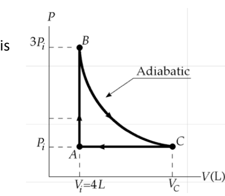
B) As a 1.00-mol sample of a monatomic ideal gas expands adiabatically, the work done on it is –3 500 J. The initial temperature and pressure of the gas are 500 K and 3.60 atm. Calculate (iii) the final temperature, and (iv) the final pressure.

(iii) $W = nC_V(T_f\text{-}T_i)$ so that $-3500 = (1)(1.5)(8.314)(T_f - 500)$ and T_f =219K

(iv) $p_iV_i^{\gamma} = p_fV_f^{\gamma}$ and thus $p_i(\frac{nRT_i}{p_i})^{\gamma} = p_f(\frac{nRT_f}{p_f})^{\gamma}$ leading to $T_i^{\gamma}p_i^{\gamma-1} = T_f^{\gamma}p_f^{\gamma-1}$ and $(\frac{T_i^{\gamma}p_i^{\gamma-1}}{T_f^{\gamma}})^{\frac{1}{\gamma-1}} = p_f$

and $p_f = p_i(\frac{T_i^{\gamma}}{T_f^{\gamma}})^{\frac{1}{\gamma-1}}$ $and\ so: p_f = (3.60\ atm)\ (\frac{219}{500})^{\frac{5}{2}} = 0.457atm$

5 **A** 4 liter sample of a diatomic gas with γ =1.4 confined to a cylinder, is carried through a closed cycle. The gas is initially at 1.00atm. and 300K. First, its pressure is tripled under constant volume. Then it expands adiabatically to its original pressure. Finally the gas is compressed isobarically to its original volume.
a) draw pV diagram of this cycle
b) determine the volume of the end of the adiabatic expansion
c) find the temperature of the gas at the start of the adiabatic expansion
d) find the temperature at the end of the cycle
e) what was the net work done on the gas for this cycle



(a) See the diagram at the right.

(b) $P_BV_B^{\gamma} = P_CV_C^{\gamma}$ so that $3P_iV_i^{\gamma} = P_iV_C^{\gamma}$

so that $V_C = \left(3^{1/\gamma}\right)V_i = \left(3^{5/7}\right)V_i = 2.19V_i$ $\quad V_C = 2.19(4.00\text{ L}) = \boxed{8.77\text{ L}}$

(c) $P_BV_B = nRT_B = 3P_iV_i = 3nRT_i$ which gives $T_B = 3T_i = 3(300\text{ K}) = \boxed{900\text{ K}}$

(d) After one whole cycle, $T_A = T_i = \boxed{300\text{ K}}$

(e) In AB, $Q_{AB} = nC_V\Delta V = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$, $Q_{BC} = 0$ as this process is adiabatic

$P_CV_C = nRT_C = P_i(2.19V_i) = (2.19)nRT_i$ so $T_C = 2.19T_i$,

$Q_{CA} = nC_P\Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = (-4.17)nRT_i$

For the whole cycle

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = (0.829)nRT_i$$
$$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$
$$W_{ABCA} = -Q_{ABCA} = -(0.829)nRT_i = -(0.829)P_iV_i$$
$$W_{ABCA} = -(0.829)\left(1.013\times10^5\text{ Pa}\right)\left(4.00\times10^{-3}\text{ m}^3\right) = \boxed{-336\text{ J}}$$

6 Using the approach demonstrated during the lecture show that for pV^{γ} =const. for adiabatic gas process. (Present your derivation on the opposite site of this page). DETAILS OF THIS CALCULATION WERE GIVEN IN LECTURE.