

# MAT1330 A : Instructor Monica Nevins

## Wednesday, November 8, 2017 : Test #2

Duration: 75 minutes

Family name: \_\_\_\_\_

First name: \_\_\_\_\_

Student number : \_\_\_\_\_

**For picking up your graded test:**

Circle the DGD you will attend to pick up your test (whether you are registered or not). Tests take at least one week to grade; watch for announcements in class.


# :	DGD1	DGD2	DGD3	DGD4
Day :	Tuesday	Tuesday	Wednesday	Thursday
Start time:	10	1	1	11:30
Room :	SMD 425	TBT 0019	MNT 207	FTX 137
TA :	Ruaridh	Luca	Rabib	Ruaridh


Please read the following instructions carefully.

- You have 75 minutes to complete this exam.
- This is a closed book exam. Except for Faculty-approved calculators (models: Texas Instruments TI-30\* and TI-34\*, Casio FX-260\* and Casio FX-300\*), no notes, cell phones, smartwatches or related devices of any kind are permitted. All such devices, including cell phones, **must be stored in your bag under your desk for the duration of the exam.**
- Read each question carefully — you will save yourself time and grief later on.
- Questions 1 through 7 are multiple choice, worth a total of six points plus one bonus point. **Record your answers to the multiple choice questions in the boxes provided.**
- Questions 8 through 10 are long answer, with number of points as indicated. **You must show your work, your work must be legible and well-justified, and you must record your answers in the spaces provided.**
- Where it is possible to check your work, do so.
- Good luck!


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**Marker's use only:**

Question	Marks
1-7 (/6) +bonus	
8 (/5)	
9 (/5)	
10 (/8)	
Total (/24)	

1. (1 point) If  $f(x) = e^x \arctan(x)$  then  $f'(1) =$

A. 0

C.  $e\pi/4 + e/2$

E. 1

B.  $e\pi/4$

D.  $e/2 - e\pi$

F.  $e$

Your answer:

2. (1 point) Which of the following is equal to the derivative of  $f(x) = 2^{\sin(x)}$ ?

A.  $2^{\sin(x)} \cos(x)$

C.  $2^{\cos(x)} \sin(x) \ln(2)$

E.  $2^{\sin(x)}$

B.  $\ln(2) 2^{\sin(x)} \cos(x)$

D.  $\frac{2^{\sin(x)}}{\ln(2)}$

F.  $\sin(x) \cos(x) 2^{\sin(x)-1}$

Your answer:

3. (1 point) Which of the following is equal to the derivative of  $g(x) = \ln(e^x x^3)$ ?

A.  $\frac{e^x}{x^3}$

C.  $3x^2 e^x + x^3 e^x$

E.  $1 + 3/x$

B.  $e^x \ln(x^3) - 3e^x/x$

D.  $(3x^2 + x^3)e^x \ln(e^x x^3)$

F.  $3/x$

Your answer:

4. (1 point) Suppose  $f$  is a function and we know that

$$\tan(f(x)) + f(x) = x^2$$

for all  $x$  in the domain of  $f$ . Then

A.  $f'(x) = \frac{2x}{\sec(x)\tan(x) + 1}$

B.  $f'(x) = 2x - \sec(x)\tan(x)$

C.  $f'(x) = \frac{2x}{\sec^2(x) + 1}$

D.  $f'(x) = \frac{2x}{\sec(f(x))\tan(f(x)) + 1}$

E.  $f'(x) = 2x - \sec^2(f(x))$

F.  $f'(x) = \frac{2x}{\sec^2(f(x)) + 1}$

Your answer:

5. (1 point) Which of the following is the equation of the tangent line to the curve

$$y = x^{1/3} - 16/x$$

at the point  $(8, 0)$ ?

A.  $y = \frac{16}{3}x - \frac{128}{3}$

C.  $y = \frac{2}{3}x - \frac{16}{3}$

E.  $y = -\frac{2}{3}x + \frac{16}{3}$

B.  $y = \frac{1}{3}x - \frac{8}{3}$

D.  $y = \frac{1}{3}x^{1/3} + \frac{16}{x} - \frac{8}{3}$

F.  $y = x - 8$

Your answer:

6. (1 point) If  $h(x) = \sqrt{e^{2x} + x^3}$  then

A.  $h'(x) = e^x + \frac{3}{2}\sqrt{x}$

C.  $h'(x) = \frac{2e^{2x} + 3x^2}{2\sqrt{e^{2x} + x^3}}$

E.  $h'(x) = \frac{1}{2\sqrt{2e^{2x} + 3x^2}}$

B.  $h'(x) = \frac{2e^{2x} + 3x^2}{2(e^{2x} + x^3)}$

D.  $h'(x) = \frac{1}{2\sqrt{e^{2x} + x^3}}$

F.  $h'(x) = \frac{2e^{2x} + 3x^2}{2\sqrt{x}}$

Your answer:

7. (1 point) Suppose we are given

$$f(x) = \frac{-3 + 6x - x^2}{x}, \quad f'(x) = \frac{-x^2 + 3}{x^2}, \quad f''(x) = -\frac{6}{x^3}.$$

Which one of the following statements is true of the value

$$x = -\sqrt{3} ?$$

- A. It is not a critical point of  $f$ , but it is an inflection point.
- B.  $f$  has a local minimum there, but it is not a global minimum.
- C.  $f$  has a local maximum there, but it is not a global extremum.
- D.  $f$  has a local and global minimum there.
- E.  $f$  has a local maximum and a global minimum there.
- F. It is neither a critical point nor an inflection point of  $f$ .

Your answer:

8. (1+3+1=5 points) In this question, you will use the definition of the derivative to find the derivative of a function, and then you will check your answer using the quotient rule.

(a) Give the definition of the derivative of a differentiable function  $f(x)$  at a point  $x$  in its domain.

Answer:  $f'(x) = \lim_{\square} \square$

(b) Using the definition of the derivative, find the derivative of  $f(x) = \frac{2x}{3-x}$  at a point  $x$  in its domain.

(c) Compute the derivative of  $f(x) = \frac{2x}{3-x}$  using the quotient rule.

9. (2 + 2 + 1 = 5 points) A population of snails for a nice restaurant grows at a logistic rate, and is harvested regularly. Denoting the fraction of snails harvested each month by the parameter  $h$ , the DTDS governing the growth of this population is given by

$$N_{t+1} = N_t(4.2 - N_t) - hN_t,$$

where  $t$  is in months,  $N_t$  is the population (in thousands of snails) at time  $t$ , and  $0 \leq h \leq 2$ .

(a) Find the equilibria  $N^*$  of this system, showing your work below. Your answers may be formulas using the parameter  $h$ .

$N^* =$   and  $N^* =$

If you were unable to answer (a), use the (incorrect) formula  $\frac{4}{17}(4 - 3h)$  for  $N^*$  in parts (b) and (c).

(b) The *positive* equilibrium  $N^*$  is stable, and the *equilibrium harvest*  $S$  is  $S = hN^*$ . Using your formula in (a), find the value of  $h$  that maximizes  $S$ , justifying your answer with Calculus.

Optimal rate of harvest  $h$ :

(c) Find the maximum value of the equilibrium harvest  $S$  that the restaurant can expect.

Maximum value of  $S$ :

10. (8 points) We would like to sketch the graph of  $y = f(x)$ . We have computed for you:

$$f(x) = \frac{x}{x^3 - 1}, \quad f'(x) = \frac{-2x^3 - 1}{(x^3 - 1)^2} \quad \text{and} \quad f''(x) = \frac{6x^2(x^3 + 2)}{(x^3 - 1)^3}.$$

Fill in the answers below and complete the following tables and then sketch the graph of the function using this information. **Round your answers to three decimal places.**

$$\lim_{x \rightarrow 1^+} f(x) = \boxed{\phantom{000}} \quad \text{and} \quad \lim_{x \rightarrow 1^-} f(x) = \boxed{\phantom{000}}$$

$$\lim_{x \rightarrow -\infty} f(x) = \boxed{\phantom{000}} \quad \text{and} \quad \lim_{x \rightarrow \infty} f(x) = \boxed{\phantom{000}}$$

Critical points of  $f$ :

Intervals of $x$	
Sign of $f'(x)$	
Behaviour of $y = f(x)$	

$x$ -values where concavity of  $f$  could change:

Intervals of $x$	
Sign of $f''(x)$	
Behaviour of $y = f(x)$	

On the following page, sketch a graph of the function  $y = f(x)$ . **Mark all critical points and inflection points; indicate all asymptotes with a dashed line.**

On this page, sketch a graph of the function  $y = f(x)$  from Question 10. Mark all critical points and inflection points; indicate all asymptotes with a dashed line.

