

Name (print):

ID number:

Section (circle): 001 002 003



University of British Columbia
APRIL EXAMINATION for MATH 110

Date: *April 21, 2011*

Time: *8:30 a.m. to 11:00 a.m.*

Number of pages: *13 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Possible marks
1		6
2		8
3		8
4		8
5		8
6		8
7		8
8		16
9		4 (bonus)
Total		70

1. Determine whether each statement is true. If it is, explain why; if not, give a counterexample.

(a) [**2 marks**] A linear approximation L to a function f satisfies $L(x) = f(x)$ for exactly one value of x .

(b) [**2 marks**] If $f(0) = 1$ and $f(1) = 1$, then there exists some c in the interval $(0, 1)$ satisfying $f'(c) = 0$.

(c) [**2 marks**] A point on a function may be both an inflection point and a critical point.

2. Evaluate the following limits, if they exist.

(a) [3 marks] $\lim_{x \rightarrow \pi^+} \frac{\sin x}{x - \pi}$

(b) [5 marks] $\lim_{x \rightarrow 0^+} \frac{\ln(\sin x)}{\ln(\cos x)}$

3. In this question you will state and then apply the Mean Value Theorem.

(a) [4 marks] Complete the statement of the Mean Value Theorem:

Suppose a function f is continuous on the interval $[a, b]$. Suppose further that f is differentiable on the interval . Then there exists some c in the interval satisfying

$$f'(c) = \text{$$

(b) [2 marks] What does the Mean Value Theorem tell us about the function $f(x) = \sin x$ on the interval $[0, b]$, where b is any positive number?

(c) [2 marks] Use your answer in part (b) to explain why we may conclude that $\sin b \leq b$ for any positive value b .

4. Given a curve, its *curvature* K is a measure of how much it deviates from a straight line. It can be computed using the following formula:

$$K = \frac{y''}{(1 + (y')^2)^{3/2}}.$$

In this question you will calculate the curvature of the curve $x^4 + y^4 = 2$ at a certain point.

- (a) [**3 marks**] Given the curve $x^4 + y^4 = 2$, find y' .

- (b) [**3 marks**] Use your answer in part (a) to calculate y'' .

- (c) [**2 marks**] Find the curvature K of the curve at the point $(1, 1)$.

5. [8 marks] In the 1986 Chernobyl reactor explosion, substantial amounts of the isotope strontium-90 (^{90}Sr) contaminated the area around the nuclear plant. ^{90}Sr undergoes exponential radioactive decay with a half-life of 29 years. Write down a “calculator-ready” expression for the percentage of the original ^{90}Sr remaining today.

6. [8 marks] Stalagmites are cone-shaped mineral deposits which rise from the floors of limestone caves. They form over millenia as dripping water deposits calcium carbonate onto the cave floor.

Consider a cone-shaped stalagmite with a length always equal to five times its radius. Suppose the stalagmite's height increases at a rate of 0.13 millimetres per year. Write down two expressions for the rate of change of its volume: (i) with respect to its height h , and (ii) with respect to its radius r . (Hint: the volume V of a cone with radius r and height h is equal to $\frac{1}{3}\pi r^2 h$.)

7. Let P be the point $(1, 1)$.

(a) [**3 marks**] Write down an equation for the line of slope m which passes through P .

(b) [**2 marks**] The line you gave as an answer in part (a) has an x -intercept and a y -intercept. Find these. (They will be in terms of m .)

(c) [**3 marks**] As long as m is not equal to 1, the line you gave as an answer in part (a) cuts out a triangle whose vertices are at the origin, the x -intercept, and the y -intercept. Find m so that the area of this triangle is as small as possible.

8. Let $f(x) = \ln(4 - x^2)$.

(a) [2 marks] Find the domain of f .

(b) [2 marks] Find all the vertical and horizontal asymptotes of f , if there are any.

(c) [2 marks] Find all the x - and y -intercepts of f .

(d) [**3 marks**] Determine where f is increasing and where it is decreasing.

(e) [**3 marks**] Determine where f is concave up and where it is concave down.

(f) [4 marks] Sketch the graph of f , indicating all you have found in the previous parts of the question.

9. [4 bonus marks] Consider an aircraft flying north at 600 km/h, at an altitude of 4 km, passing directly overhead a car driving east at 100 km/h. Determine how fast the distance between them is changing one hour after the aircraft passes overhead the car. You do not have to simplify your answer.

This page may be used for rough work. It will not be marked.