Name: ______________________________

Student ID: ______________________________

CARLETON UNIVERSITY
DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 2004-C (Fall 2019)
Test 6 (Tuesday, November 26, 2019)

Time: 50 minutes (no cellphones, notes, books, talking). Use the back of the paper to show your work for long answer questions and only write the final solution in the space under the question.

MARKS

(5) 1. Use cylindrical coordinates to parametrize the paraboloid $z = x^2 + y^2 - 1$

$x = r\cos\theta$, $y = r\sin\theta$ $\Rightarrow \begin{cases} z = r^2 - 1 \\ r = \sqrt{z+1} \end{cases} \Rightarrow \begin{cases} x = \sqrt{z+1}\cos\theta \\ y = \sqrt{z+1}\sin\theta \\ z = z \end{cases}$ or $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = r^2 - 1 \end{cases}$

(5) 2. Find the following surface integral

$$\int\int_{\mathcal{S}} xz^2 dS,$$

where $\mathcal{S}$ is the part of the cylinder $x^2 + z^2 = 1$ above xy-plane bounded by the planes $y = 0$ and $y = 1$

$S: \ z = \sqrt{1-x^2}$

$z_x = \dfrac{-x}{\sqrt{1-x^2}}$

$z_y = 0$

$$\iint_S xz^2 dS = \int_{-1}^{1}\int_0^1 x(1-x^2)\sqrt{1+0+\frac{x^2}{1-x^2}}\, dy\, dx$$

$$= \int_{-1}^{1}\int_0^1 x\sqrt{1-x^2}\, dy\, dx = \text{easy to continue}$$

(5) 3. Find the flux of $\mathbf{F}(x, y, z) = (xz, yz, z^2)$ over the part of the paraboloid $z = x^2 + y^2$ cut off by $x^2 + y^2 = 1$.

$z = x^2 + y^2$

$z_x = 2x$

$z_y = 2y$

$$\iint_S (xz, yz, z^2)\, d\mathbf{S} = \iint_R -x(x^2+y^2)(2x) - y(x^2+y^2)(2y) + (x^2+y^2)^2\, dA$$

$$= \iint_R -2(x^2+y^2)(x^2+y^2) + (x^2+y^2)^2\, dA = \iint_R -(x^2+y^2)^2\, dA = \iint_R -r^3\, dr\, d\theta$$

(5) 4. Evaluate $\int_{\mathcal{C}} -ydx + xdy$ over the perimeter $\mathcal{C}$ of the closed curve $4x^2 + 9y^2 = 36$ oriented in a counterclockwise direction.

$$\iint_R Q_x - P_y\, dA = \int_C P dx + Q dy \Rightarrow \int_C -y dx + x dy = \iint_R 1 + 1\, dA = \iint_R 2\, dA$$

$= 2 \times$ the area of the ellipse. $= 2 \times 3 \times 2 \times \pi$

(5) 5. Let $\mathbf{F}(x, y, z) = (-y^2, x, z^2)$ and let $\mathcal{S}$ be that part of the plane $x+z = 2$ cut by the cylinder $x^2+y^2 = 1$. Use any method to evaluate the flux integral $I = \int\int_{\mathcal{S}}(\text{curl } \mathbf{F}) \cdot d\mathbf{S}$.

$C: r(t) = (\cos t, \sin t, 2 - \cos t)$

$r'(t) = (-\sin t, \cos t, \sin t)$

$$\iint_S \text{curl}\, \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot dr = \int_a^b F(r(t)) \cdot r'(t)\, dt$$

$$= \int_0^{2\pi} (-\sin^2 t, \cos t, (2-\cos t)^2) \cdot (-\sin t, \cos t, \sin t)\, dt = \int_0^{2\pi} \sin^3 t + 2\cos^2 t + 4\sin t - 4\sin t\cos t\, dt$$