Here are some Math110 problems for practice only. This is not a complete overview of final exam material. The solutions will **not** be provided.

Part 1 (True/False):

1. A linearly independent set of vectors in $\mathbb{R}^n$ has at most n vectors.

2. If A is an $n \times n$ matrix and B is an $n \times p$ matrix such that $AB = 0$, then $B = 0$.

3. If A is an $n \times n$ matrix and $\text{rank}(A) = n$, then $\text{Det}(A) = 0$.

4. Every system of n linear equations in n unknowns can be solved by Cramer's Rule.

5. If A is a square matrix, then AA^T and A^TA are orthogonally diagonalizable.

6. For all vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in $\mathbb{R}^n$, $\boldsymbol{u} \cdot \boldsymbol{v} \geq 0$.

7. Any matrix can be transformed into reduced row echelon form by a finite sequence of elementary row operations.

8. If A is a 4×3 matrix and $\text{nullity}(A^T) = 2$, then $\text{rank}(A) = 2$.

9. If A and B are $n \times n$ matrices and B is obtained from A by a sequence of elementary row operations, then $\text{Det}(B) = \text{Det}(A)$.

10. If $\boldsymbol{v}$ is both in the row space and in the column space of a square matrix A, then $\boldsymbol{v} = \boldsymbol{0}$.

11. If matrices M and N have the same size, then $(M + N)^T = M^T + N^T$.

12. If matrices M and N are invertible, then MN is invertible, and $(MN)^{-1} = N^{-1}M^{-1}$.

13. If matrix M is invertible, then M^T is invertible.

Part 2:

14. Let $\boldsymbol{u} = \begin{bmatrix} 2 \\ 8 \\ 3 \\ -1 \end{bmatrix}$ and $\boldsymbol{v} = \begin{bmatrix} 1 \\ 7 \\ 2 \\ 9 \end{bmatrix}$. Compute $(\boldsymbol{u} - \boldsymbol{v}) \cdot (3\boldsymbol{v})$.

15. Find the projection of the vector $\boldsymbol{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ onto the vector $\boldsymbol{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

16. If the vector w is a linear combination of the vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 5 \\ -4 \\ 0 \end{bmatrix}$ with corresponding coefficients $c_1 = 3$, $c_2 = 2$, $c_3 = -1$, find vector w.

17. Suppose A and B are 4×4 matrices with $\det A = 5$ and $\det B = 3$. Find $\det(AB^2A^T)$.

18. Compute the determinant of the matrix $M = \begin{bmatrix} 2 & -3 & 0 & 1 \\ 5 & 4 & 2 & 0 \\ 1 & -1 & 0 & 3 \\ -2 & 1 & 0 & 0 \end{bmatrix}$. Find inverse of the matrix M (1) using cofactors, (2) by reducing augmented matrix, if M^{-1} exists.

19. Let $M = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & -2 \\ -2 & 0 & 8 \end{bmatrix}$. If $E_1 M = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & -2 \\ 0 & 6 & 10 \end{bmatrix}$, where E_1 is an elementary matrix, find E_1.

20. If $u_1 = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$, $u_2 = \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix}$, for which values of k is the vector $w = \begin{bmatrix} k \\ 2 \\ 1 \end{bmatrix}$ in the *span* of $\{u_1, u_2\}$? If W be the subspace of $\mathbb{R}^3$ spanned by vectors u_1, u_2, find a basis for $W^\perp$.

21. (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x - 2y + z \\ 5x + 3z \\ 2|z| \end{bmatrix}$. Determine if this trastformation is a linear trastformation.

    (b) Let $S : \mathbb{R}^3 \to \mathbb{R}^3$ be the transformation given by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} 3x - 2y + \sqrt{7}z \\ y - 5x + 3z \\ 2z + 0.1x \end{bmatrix}$. What is the standard matrix of the linear transformation S?

22. $B = \left\{ v_1 = \begin{bmatrix} 2/\sqrt{6} \\ 1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, v_3 = \begin{bmatrix} 1/\sqrt{3} \\ -1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix} \right\}$ is an orthonormal basis of $\mathbb{R}^3$. Find $[w]_B$, the coordinate vector of $w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ with respect to B.

23. Find the standard matrix that performs the **clockwise** rotation of vectors in $\mathbb{R}^2$ about the origin by $\pi/4$ radians, then reflects resulting vectors over line $y = -x$, and then projects resulting vectors on $y-$axis. Is this linear transformation invertible?

24. The vectors $x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$ form a basis for a subspace W of $\mathbb{R}^3$.

    Apply the Gram-Schmidt Process to obtain an orthonormal basis for W.

25. Consider the following matrix

    $$A = \begin{bmatrix} 1 & 1 & -4 & 3 & -6 \\ 1 & 0 & -1 & 3 & -4 \\ 2 & 1 & -5 & 6 & -10 \\ -1 & -2 & 7 & -3 & 8 \end{bmatrix}$$

    (a) Determine the rank and nullity of matrix A

    (b) Give a basis for the row space of A

    (c) Give a basis for the column space of A

    (d) Give a basis for the nullspace of A.

26. For the matrix $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$

    (a) find an invertible matrix P, P^{-1} and a diagonal matrix D such that $A = P^{-1}DP$.

    (b) Find formulae for calculating $B = A^k$ for all values of $k \geq 1$.

27. Let $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$.

    (a) Find the characteristic polynomial of A and the eigenvalues of A, and corresponding eigenvectiors.

    (b) Is A diagonalizable?

28. Compute the product $(5 + 5i)(2 + i)$.

29. Find the polar form of the complex number $z = 1 + i$. Find z^7.

30. What are the solutions to the equation $z^4 - 4i = 0$?