

THE UNIVERSITY OF BRITISH COLUMBIA

<p style="text-align: center;"><b>MATH 104</b> <b>Mock Midterm 1</b> September 2018</p>
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TIME: 50 MINUTES

FULL NAME: _____ STUDENT # : _____
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SIGNATURE: \_\_\_\_\_

This Examination paper consists of 8 pages (including this one). Make sure you have all 8.

INSTRUCTIONS:

No memory aids allowed. No calculators allowed. No communication devices allowed.

MARKING:

<b>Q1</b>	/9
<b>Q2</b>	/9
<b>Q3</b>	/7
<b>Q4</b>	/5
<b>Q5</b>	/8
<b>Q6</b>	/12
<b>TOTAL</b>	/50

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NAMES OF INSTRUCTORS:

**Q1** [9 marks]

Find the following limits.

(a)  $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 4}$

(b)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$

(c) If  $\lim_{x \rightarrow 1} f(x) = 8$  and  $\lim_{x \rightarrow 1} g(x) = 3$ , then find  $\lim_{x \rightarrow 1} \sqrt[3]{f(x)g(x) + 3}$ .

**Q2** [9 marks]

Compute the derivatives of the following functions. DO NOT SIMPLIFY.

(a)  $f(x) = \frac{x^2 + 12x + e^3}{x + e^x}$

(b)  $g(t) = e^t(t^2 + x^2)$

(c)  $f(x) = (x^2 + x + 1)(x^3 + 1)$

**Q3** [7 marks]

(a) (2 marks) Carefully state the definition of the derivative of a function  $f(x)$  at a point  $x = a$ .

(b) (5 marks) Use the definition of the derivative from part (a) to compute  $f'(1)$  for  $f(x) = \frac{13}{x+7}$ . NO CREDIT will be given for any other method.

**Q4** [5 marks]

Find the values of the parameters  $a$  and  $b$  such that the function

$$f(x) = \begin{cases} 8x^3 + 4x^2a + 2xa^2 + a^3, & \text{if } x \leq 0, \\ 5bx + 8, & \text{if } 0 < x \leq 1, \\ x^2 + 12, & \text{if } x > 1, \end{cases}$$

is continuous at all the points in its domain. Is  $f$  differentiable at all points in its domain with these values of  $a$  and  $b$ ?

**Q5** [8 marks]

Find the equation of the tangent line to the curve  $y = f(x) = \frac{1}{x^{2/3}}$  that is parallel to the line  $y - 2x = \pi$ .

**Q6** [12 marks]

When EZ Electronics Company sells surge protectors at \$50 a piece, they produce and sell 3000 of them per month. For every \$1 increase in price, the number of surge protectors they sell decreases by 15. Assume that the fixed production costs are \$50,000 and the variable costs are \$30 per surge protector.

- (a) Find the linear demand function  $q = D(p)$ , where  $p$  is a price of a unit and  $q$  is the number of surge protectors made and sold. [Hint: The point  $(p, q) = (50, 3000)$  must lie on this line.]
- (b) Find the cost function  $C(q)$  as a function of  $q$ , and then express it as a function of  $p$ .
- (c) Find the revenue function  $R(q)$  as a function of  $q$ , and then express it as a function of  $p$ .

- (d) Sketch graphs of the cost function  $C(p)$  and the revenue function  $R(p)$  on the same set of axes, where the horizontal axis is the  $p$ -axis.
- (e) Find the *break-even points*. Give both the price  $p$  and corresponding quantity demanded  $q$  at each of these points.
- (f) If EZ Electronics Company is operating at the higher break-even point, should it increase or decrease the price of its surge protectors to increase its profits? Explain your answer.