ECON 302
Concordia University
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Fall 2018, Assignment 3 - Solutions

1. (30 pts) Consider a market with two firms, 1 and 2, producing an identical good. This good has market demand given by the inverse demand function $p(y) = 10 - 2y$, where p is price, and $y = y_1 + y_2$ is the market quantity. The firms have cost functions as follows: $C_i(y_i) = c_i y_i$, where $c_i = 2$ and y_i represents the amount of output produced by firm i for $i \in \{1, 2\}$.

(a) Suppose the two firms compete by producing the identical good simultaneously. Solve for their reaction functions and find the Cournot equilibrium outputs, market price and profit per firm. (8 pts)

Reaction functions:
Firm 1's profit function,
$\Pi_1(y_1, y_2) = p(y_1 + y_2)y_1 - C_1(y_1) = [10 - 2(y_1 + y_2)]y_1 - 2y_1$.
First order condition with respect to y_1,
$10 - 4y_1 - 2y_2 - 2 = 0 \Rightarrow y_1 = 2 - \frac{1}{2}y_2$.
Thus, firm 1's reaction function is,

$$y_1 = f_1(y_2) = 2 - \frac{1}{2}y_2$$

Firm 2's profit function,
$\Pi_2(y_1, y_2) = p(y_1 + y_2)y_2 - C_2(y_2) = [10 - 2(y_1 + y_2)]y_2 - 2y_2$.
First order condition with respect to y_2,
$10 - 2y_1 - 4y_2 - 2 = 0 \Rightarrow y_2 = 2 - \frac{1}{2}y_1$.
Thus, firm 2's reaction function is,

$$y_2 = f_2(y_1) = 2 - \frac{1}{2}y_1$$

Cournot equilibrium:
Since the two firms are identical (they have the same marginal cost of production), it should be $y_1 = y_2$.
Hence, by plugging $y_2 = y_1$ in firm 1's reaction function $y_1 = f_1(y_2) = 2 - \frac{1}{2}y_2$, we have,

$$y_1^* = y_2^* = \frac{4}{3} \text{ units}$$

Market price:
The market price is, $p(y_1^* + y_2^*) = 10 - 2(y_1^* + y_2^*) = 10 - 2\frac{8}{3} = \frac{14}{3}$.

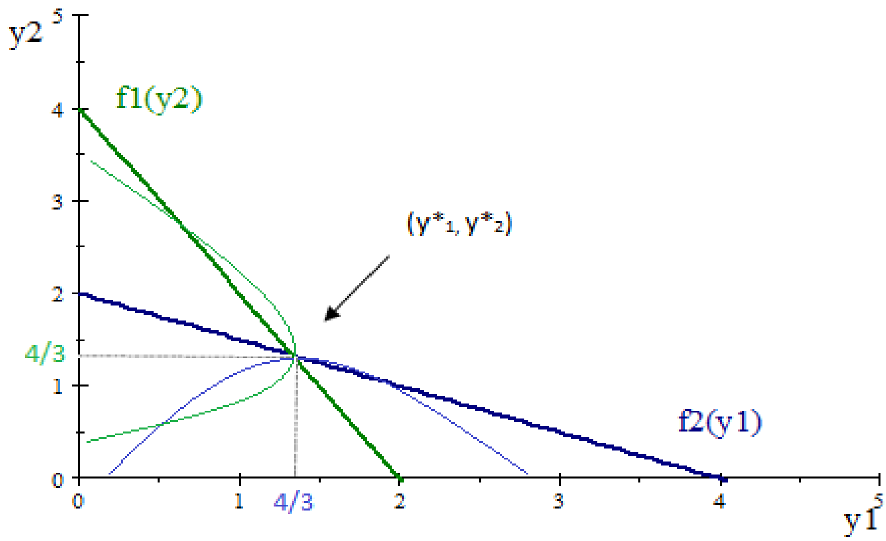
$$p(y^*) = \$\frac{14}{3}$$

Profit per firm:

Since firms are identical, they will have the same Cournot profits.
Hence, $\Pi_i(y_i^*) = p(y^*)y_i^* - C_i(y_i^*) = \frac{14}{3} * \frac{4}{3} - 2 * \frac{4}{3} = \frac{8}{3} * \frac{4}{3} = \frac{32}{9}$ for $i \in \{1, 2\}$.

$$\Pi_1(y_1^*) = \Pi_2(y_2^*) = \$\frac{32}{9}$$

(b) Graph their reaction functions and show the equilibrium point. Include isoprofit curves through the equilibrium point for both firms. (6 pts)



(c) Suppose the two firms cooperate, solve for the collusive outcome determining equilibrium outputs, market price and profit per firm. (6 pts)

Equilibrium output:
When the two firms collude, they maximize the total industry profits.
Total industry profits,
$\Pi(y_1, y_2) = p(y_1+y_2)y_1 - C_1(y_1) + p(y_1+y_2)y_2 - C_2(y_2) = [10-2(y_1+y_2)](y_1+y_2) - 2(y_1+y_2) = 10(y_1 + y_2) - 2(y_1 + y_2)^2 - 2(y_1 + y_2)$.
First order condition with respect to y_1, $10 - 4(y_1 + y_2) - 2 = 0$.
First order condition with respect to y_2, $10 - 4(y_1 + y_2) - 2 = 0$.
Hence, $y_1 + y_2 = 2$ and since firms are identical it should be that $y_1 = y_2$.

$$y_1^m = y_2^m = 1 \text{ units}$$

Market price:
The market price is, $p(y_1^m + y_2^m) = 10 - 2(y_1^m + y_2^m) = 6$.

$$p(y^m) = \$6$$

Profit per firm:

Since firms are identical, they will have the same profits.
Hence, $\Pi_i(y_i^m) = p(y^m)y_i^m - C_i(y_i^m) = 6 * 1 - 2 * 1 = 4$ for $i \in \{1, 2\}$.

$$\Pi_1(y_1^m) = \Pi_2(y_2^m) = \$4$$

(d) Using the information from parts (a) and (c), construct a 2×2 payoff matrix where the strategies available to each of the two firms are either to cooperate or deviate. (6 pts)

When firms cooperate (collude), each earns the collusion profit which is calculated in part (c), thus $\Pi_1(y_1^m) = \Pi_2(y_2^m) = \$4$.
When, for instance, firm 1 deviates while firm 2 produces the collusion output $y_2^m = 1$, then firm 1 produces based on his reaction function. That is,
$y_1^* = 2 - \frac{1}{2}y_2^m = 2 - \frac{1}{2}1 = 1.5$.
In that case, the market price would be, $p(y_1^* + y_2^m) = 10 - 2(y_1^* + y_2^m) = 10 - 2(1.5 + 1) = 10 - 5 = 5$.
Firm 1 earns, $\Pi_1(y_1^* + y_2^m) = p(y_1^* + y_2^m)y_1^* - C_1(y_1^*) = 5 * 1.5 - 2 * 1.5 = 3 * 1.5 = 4.5$.
Firm 2 earns, $\Pi_2(y_1^* + y_2^m) = p(y_1^* + y_2^m)y_2^m - C_2(y_2^m) = 5 * 1 - 2 * 1 = 3$.
When the roles are reversed (firm 2 deviates, while firm 1 produces the collusion output), so are the profits.
When both firms deviate, we have the Cournot equilibrium. Thus, each firm earns the Cournot equilibrium profit which is calculated in part (a), thus $\Pi_1(y_1^*) = \Pi_2(y_2^*) = \$\frac{32}{9}$.
The payoff matrix of the game is as follows:

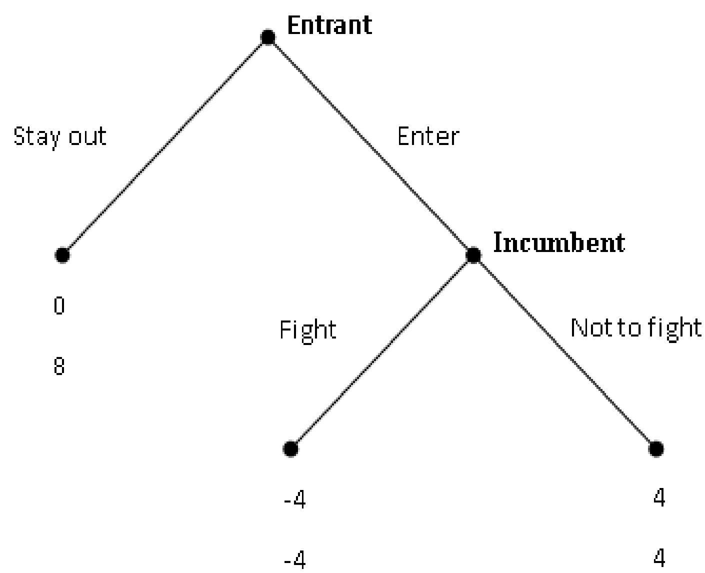
		Firm 2	
		y_2^m	y_2^*
Firm 1	y_1^m	$(4, 4)$	$(3, 4.5)$
	y_1^*	$(4.5, 3)$	$(\frac{32}{9}, \frac{32}{9})$

(e) What is the Nash equilibrium (or equilibria) of the game you constructed in part (d)? Is there any mixed strategy Nash equilibrium in this game? If yes, what is the mixed strategy Nash equilibrium? (4 pts)

The game in part (d) is a typical Prisoner's Dilemma game, where producing the Cournot equilibrium output, i.e. choosing $y_i^* = \frac{4}{3}$, is the dominant strategy for each firm.
Thus, this game has a unique Nash equilibrium where each firm plays its dominant strategy. That is, $(y_1^*, y_2^*) = (\frac{4}{3}, \frac{4}{3})$ is the unique Nash equilibrium and the payoffs for the firms are their Cournot profits, $\Pi_1(y_1^*) = \Pi_2(y_2^*) = \$\frac{32}{9}$.
Note that this is a pure strategy Nash equilibrium. In this game there is no mixed strategy Nash equilibrium because each firm has a dominant strategy which is to produce the Cournot equilibrium output. Given that firm 1 is choosing $y_1^* = \frac{4}{3}$ with a 100% probability, the best response of firm 2 is to choose $y_2^* = \frac{4}{3}$ with a 100% probability, and vice versa for firm 2.

2. (20 pts) A monopolist is facing a threat of entry by another firm. First the entrant decides whether or not to enter, and then the incumbent decides whether or not to cut its price (fight) in response. If the entrant decides to stay out, it gets a payoff of 0 and the incumbent gets a payoff of 8 as a monopolist. If the entrant decides to enter the market, two firms' payoffs depend on whether the incumbent fights or not. If the incumbent fights, then both firms end up with a negative payoff -4. If the incumbent decides not to fight, they divide the market profit and both get a payoff of 4.

(a) Present this game information in an extensive form. (10 pts)



(b) Solve for the backward induction outcome. (5 pts)

There is a unique backward induction outcome: The entrant chooses to enter, and the incumbent chooses not to fight. The payoffs are $(4, 4)$.

(c) Can entry deterrence arise in equilibrium in this game? Explain. (5 pts)

Entry deterrence cannot arise in equilibrium in this game. The incumbent's problem is that he cannot precommit himself to fighting if the other firm enters. If the other firm enters, the damage is done and the rational thing for the incumbent to do is not to fight. Insofar as the potential entrant recognizes this, he/she will correctly view any threats to fight as empty.

(3) (30 pts) Consider the following game. Players can choose either 'left' (l) or 'right' (r). The table provided below gives the payoffs to player A and B given any set of choices, where player A's payoff is the first number and player B's payoff is the second number.

		Player B	
		l	r
Player A	l	$4, 4$	$1, 6$
	r	$6, 1$	$-3, -3$

(a) Solve for the pure strategy Nash equilibria. (4 pts)

There are two pure strategy Nash equilibria: (l, r) and (r, l).

(b) Suppose player A chooses l with probability p and player B chooses l with probability q. Determine player A's best response to any choice of q by player B, and player B's best response to any choice of p by player A. Find the mixed strategy Nash equilibrium. (6 pts)

We have the following table:

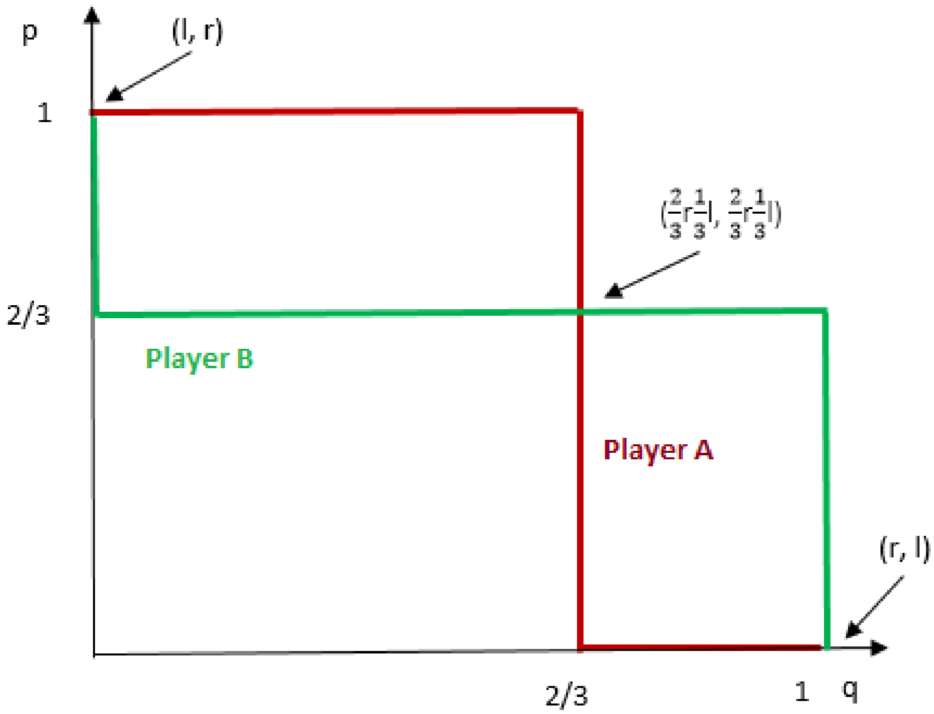
			Player B	
			q	$1-q$
			l	r
Player A	p	l	$4,4$	$1,6$
	$1-p$	r	$6,1$	$-3,-3$

Player A's Best Response:
Player A's payoff from choosing l is $4q + (1-q) = 3q + 1$ and his/her payoff from choosing r is $6q - 3(1-q) = 9q - 3$.
Player A's best response is l (i.e. $p = 1$) if $3q + 1 > 9q - 3 \Rightarrow q < 2/3$.
Player A's best response is r (i.e. $p = 0$) if $3q + 1 < 9q - 3 \Rightarrow q > 2/3$.
Player A's best response is to randomize between l and r (i.e. $0 \leq p \leq 1$) if $3q+1 = 9q-3 \Rightarrow q = 2/3$.

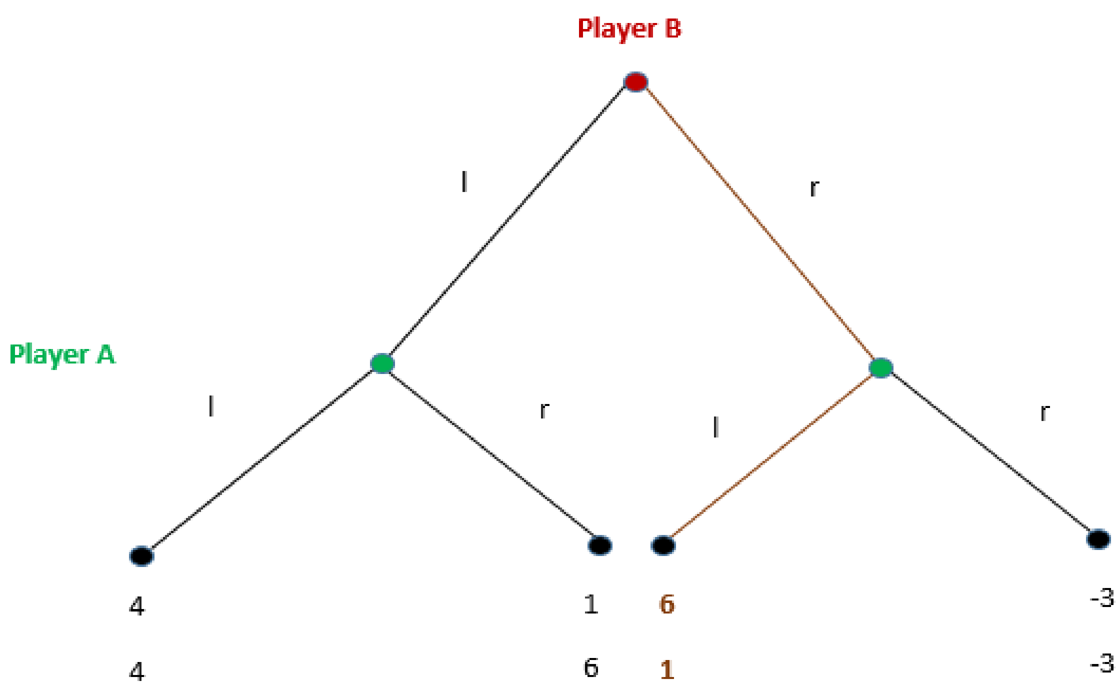
Player B's Best Response:
Player B's payoff from choosing l is $4p + (1-p) = 3p + 1$ and his/her payoff from choosing r is $6p - 3(1-p) = 9p - 3$.
Player B's best response is l (i.e. $q = 1$) if $3p + 1 > 9p - 3 \Rightarrow p < 2/3$.
Player B's best response is r (i.e. $q = 0$) if $3p + 1 < 9p - 3 \Rightarrow p > 2/3$.
Player B's best response is to randomize between l and r (i.e. $0 \leq q \leq 1$) if $3p+1 = 9p-3 \Rightarrow p = 2/3$.

Mixed strategy Nash equilibrium:
So the mixed strategy Nash equilibrium is: Player A chooses l with probability 2/3 and chooses r with probability 1/3; Player B chooses l with probability 2/3 and chooses r with probability 1/3. Thus, the mixed strategy Nash equilibrium is $(\frac{2}{3}l\frac{1}{3}r, \frac{2}{3}l\frac{1}{3}r)$.

(c) Show these best response functions and all the equilibria on a graph. (6 pts)



(d) Now suppose that player B moves first and player A moves second. Show the game information in an extensive form. What is the Nash equilibrium in this case? (8 pts)



We solve the problem using backward induction. The Nash equilibrium in this case is, player B chooses 'right' (r) and player A chooses 'left' (l).

(e) Would player A prefer the mixed strategy equilibrium in (*b*) or the sequential move equilibrium in (*d*)? Why? (6 pts)

			Player B	
			$\frac{2}{3}$	$\frac{1}{3}$
			l	r
Player A	$\frac{2}{3}$	l	4,4	1,6
	$\frac{1}{3}$	r	6,1	-3,-3

The mixed strategy equilibrium will give player A an expected payoff equal to, $\frac{2}{3} * \frac{2}{3} * 4 + \frac{2}{3} * \frac{1}{3} * 1 + \frac{1}{3} * \frac{2}{3} * 6 + \frac{1}{3} * \frac{1}{3} * (-3) = \frac{16}{9} + \frac{2}{9} + \frac{12}{9} - \frac{3}{9} = 3$.
The sequential move equilibrium will give player A a payoff equal to, 1.
Thus, player A would prefer the mixed strategy equilibrium since he/she will receive a higher payoff ($3 > 1$).

4. (20 pts) Consider the following two-player simultaneous-move game. Player A chooses either 'up' (u) or 'down' (d). Player B chooses either 'left' (l) or 'right' (r). The table provided below gives the payoffs to player A and B given any set of choices, where player A's payoff is the first number. There are payoffs provided for three versions of this simple game.

	Game 1	Game 2	Game 3
u, l	$1, 1$	$1, 5$	$3, 7$
u, r	$5, 0$	$10, 10$	$8, 2$
d, l	$0, 5$	$2, 2$	$9, 1$
d, r	$4, 4$	$5, 1$	$5, 6$

(a) For each of the three games, express the payoff information in the normal form (payoff matrix). (12 pts)

Game 1:

		Player B	
		l	r
Player A	u	$1, 1$	$5, 0$
	d	$0, 5$	$4, 4$

Game 2:

		Player B	
		l	r
Player A	u	$1, 5$	$10, 10$
	d	$2, 2$	$5, 1$

Game 3:

		Player B	
		l	r
Player A	u	$3,7$	$8,2$
	d	$9,1$	$5,6$

(b) For each of the three games, determine the pure strategy Nash equilibria. (8 pts)

Game 1:
The pure strategy Nash equilibrium is (u, l). This is a Nash equilibrium in dominant strategies.

Game 2:
There are two pure strategy Nash equilibria, (d, l) and (u, r). Both of them are possible equilibria, so it is hard to say what will happen.

Game 3:
There is no a pure strategy Nash equilibrium.