# STAT 249 Sample Questions for Midterm Exam

1. (a) A computer's random digit ($\{0, 1, \ldots, 9\}$) generator is run twice. What is the probability that the second number generated is strictly bigger than the first one?

(b) Four cards are drawn with replacement from a standard deck. Find the probability that all four have different suits $\clubsuit\diamondsuit\spadesuit\heartsuit$.

2. (a) A and B are equally skilled at playing a certain game. They are playing a best-of-seven series so that the first player to win 4 games wins the series. If A wins the first game, and then B wins the next three games; what are B's chances of winning the series?

(b) Let $P(A|B) = 0.5$, $P(B) = 0.25$, and $P(A \cup B) = 0.75$. Find $P(\overline{B}|\overline{A})$.

3. On a multiple choice question with 5 possible responses, 90% of the class know the correct answer and the other 10% guess. If the instructor randomly selects an exam and find the right answer, what is the (conditional) probability that this student has guessed?

4. (a) Suppose you have a pair of loaded dice. Each die behaves in the following way:

$$P(1) = 1/21,\ P(2) = 2/21,\ P(3) = 3/21,\ P(4) = 4/21,\ P(5) = 5/21,\ P(6) = 6/21.$$

When you roll this pair of dice, how likely are you to get a total of 7?

(b) A fire-detection device utilizes three temperature cells acting independently of each other in such a manner that any one or more may activate the alarm. Each cell posseses a probability of $p = 0.8$ of activating the alarm when the temperature reaches $100^o$ Celsius or more. Find the probability that the alarm will function when the temperature reaches $100^o$.

5. (a) Suppose that 30% of the applicants for a certain industrial job posses advanced training in computer programming. Applicants are interviewed sequentially and are selected at random from the pool. What is the expected number of applicants who need to be interviewed in order to find the first one with advanced training?

(b) Suppose that a radio contains six transistors, two of which are defective. Three transistors are selected at random, removed from the radio, and inspected. Find the probability that at least one defective is observed.

**Solutions**

1. (a) The sample space consists of the 100 equally likely pairs $\{(0,0),(0,1),\ldots,(9,9)\}$. Ten of these pairs have two equal numbers, and the remaining 90 pairs split evenly according to whether the first number is strictly bigger than the second, or vice versa. Therefore the answer is 45/100.

(b) $P(\text{four different suits}) = (1)(3/4)(2/4)(1/4) = 6/64 = 0.09375$.

2. (a) The only way that B can lose is if A wins 3 in a row. This has a one out of eight chance of occurring. Therefore $P(\text{B wins}) = 7/8$.

(b) Using the multiplicative law, we get $P(A \cap B) = P(A|B)P(B) = (0.5)(0.25) = 0.125$. From the additive law we have

$$0.75 = P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + 0.25 - 0.125,$$

and solving for $P(A)$ gives 0.625. Therefore, we get the solution

$$P(\overline{B}|\overline{A}) = \frac{P(\overline{B} \cap \overline{A})}{P(\overline{A})} = \frac{1 - P(B \cup A)}{1 - P(A)} = \frac{0.25}{0.375} = \frac{2}{3}.$$

3.

$$\begin{aligned} P(\text{guess}|\text{correct}) &= \frac{P(\text{correct}|\text{guess})P(\text{guess})}{P(\text{correct}|\text{guess})P(\text{guess}) + P(\text{correct}|\text{know})P(\text{know})} \\ &= \frac{(1/5)(0.10)}{(1/5)(0.10) + (1)(0.90)} \\ &= 0.02174. \end{aligned}$$

4. (a)

$$\begin{aligned} P(X+Y=7) &= P(X=1,Y=6) + P(X=2,Y=5) + P(X=3,Y=4) \\ &\quad + P(X=4,Y=3) + P(X=5,Y=2) + P(X=6,Y=1) \\ &= (1/21)(6/21) + (2/21)(5/21) + (3/21)(4/21) \\ &\quad + (4/21)(3/21) + (5/21)(2/21) + (6/21)(1/21) \\ &= 56/21^2 = 0.126984. \end{aligned}$$

(b) Let $Y$ equal the number of cells activating the alarm when the temperature reaches $100^o$. Then $Y$ is a binomial random variable with parameters $(3, 0.8)$. The alarm will function if $Y = 1$, 2, or 3. Hence

$$P(\text{alarm functions}) = 1 - P(Y=0) = 1 - (0.2)^3 = 0.992.$$

5. (a) Let $Y$ be the number of applicants who need to be interviewed in order to find the first one with advanced training. Then $Y$ is a geometric random variable with the parameter 0.3. Hence the expected number is $E(Y) = 1/0.3 = 3.34$.

(b) Let $Y$ equal the number of defectives observed. Then $Y$ is a hypergeometric random variable with the probability mass function

$$p(y) = \frac{\binom{2}{y}\binom{4}{3-y}}{\binom{6}{3}}, \qquad y = 0, 1, 2.$$

Hence the desired probability equals $p(1) + p(2) = 3/5 + 1/5 = 4/5$.