

MAT 1362 – Fall 2019

Quiz 9 – DGD 1 (Tuesday) – Solutions

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QUESTION (4 points). Consider the function

$$f: \mathbb{R} \rightarrow \mathbb{R}_{\geq 0}, \quad f(x) = |x + 2|.$$

(a) Is f injective? Justify your answer.

Solution: No, f is not injective. For example

$$f(-1) = |1| = 1 \quad \text{and} \quad f(-3) = |-1| = 1,$$

but $-1 \neq -3$.

(b) Is f surjective? Justify your answer.

Solution: Yes, f is surjective. For any $y \in \mathbb{R}_{\geq 0}$, we have $y - 2 \in \mathbb{R}$ and

$$f(y - 2) = |(y - 2) + 2| = |y| = y,$$

since $y \geq 0$.

(c) Does f have a left inverse? If it does, give one and show that it is indeed a left inverse. Otherwise, justify why f does not have a left inverse.

Solution: No. A function has a left inverse if and only if it is injective. Since f is not injective, it does not have a left inverse

- (d) Does f have a right inverse? If it does, give one and show that it is indeed a right inverse. Otherwise, justify why f does not have a right inverse.

Solution: Yes. A function has a right inverse if and only if it is surjective. Since f is surjective, it has a right inverse. Define

$$g: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}, \quad g(y) = y - 2.$$

Then, as above, for all $y \in \mathbb{R}_{\geq 0}$, we have

$$(f \circ g)(y) = f(g(y)) = |(y - 2) + 2| = |y| = y,$$

since $y \geq 0$. Thus $f \circ g = \text{id}_{\mathbb{R}_{\geq 0}}$, and so g is a right inverse of f .