MAT 1362 – Fall 2019

Quiz 8 – DGD 1 (Tuesday) – Solutions

Professor: Alistair Savage

Date: November 12, 2019

QUESTION **(3 points)**. Consider the set

$$A = \left\{ 5 + \frac{8}{x} : x \in \mathbb{R}_{>0} \right\}.$$

Find the infimum of A (with proof) or prove that it does not exist.

Solution: We will show that $\inf(A) = 5$. First note that

$$\begin{aligned} x \in \mathbb{R}_{>0} &\implies x > 0 \\ &\implies \frac{1}{x} > 0 \\ &\implies \frac{8}{x} > 0 \\ &\implies 5 + \frac{8}{x} > 5. \end{aligned}$$

Hence 5 is a lower bound for A.

It remains to show that 5 is the *greatest* lower bound for A. Suppose, towards a contradiction, that b is a lower bound for A and $b > 5$. Then, for all $x \in \mathbb{R}_{>0}$, we have

$$\begin{aligned} b < 5 + \frac{8}{x} &\implies b - 5 < \frac{8}{x} \\ &\implies \frac{1}{b-5} > \frac{x}{8} \qquad \text{(since } b - 5 > 0\text{)} \end{aligned}$$

$$\implies \frac{8}{b-5} > x.$$

This implies that $\frac{8}{b-5}$ is an upper bound for $x \in \mathbb{R}_{>0}$, which is a contradiction (since $\mathbb{R}_{>0}$ has no upper bound).