MAT1330 B : Instructor Dr. Elizabeth Maltais
Thursday, December 20, 2018 : Final Exam

Duration: 3 hours

Family name: ______________________________

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Please read the following instructions carefully.

- You have 3 hours to complete this exam.
- This is a closed book exam. Except for Faculty-approved calculators (models: Texas Instruments TI-30* and TI-34*, Casio FX-260* and Casio FX-300*), no notes, cell phones, smartwatches or related devices of any kind are permitted.
- There are 10 questions, each with point values as indicated. **You must show your work, your work must be legible, and when applicable you must record your answers in the boxes provided.**
- You may use the backs of pages to continue your answers and must indicate clearly when you have done so.
- Where it is possible to check your work, do so.
- Good luck!

SAMPLE EXAM

Marker's use only:

Question	Marks
1 (/3)	
2 (/5)	
3 (/4)	
4 (/4)	
5 (/5)	
6 (/4)	
7 (/9)	
8 (/4)	
9 (/6)	
10 (/6)	
Total (/50)	

Very important:

Cellular phones, unauthorized electronic devices or course notes are not allowed during this exam. Phones and devices must be turned off and put away in your bag. Do not keep them in your possession, such as in your pockets. If caught with such a device or document, the following may occur: academic fraud allegations will be filed which may result in you obtaining a 0 (zero) for the exam.

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Question 1. (3 points) Circle the correct answer for each question.

(a) (1 point) Which of the following is equal to the derivative of $f(x) = \ln(\sqrt{4x^2+9})$?

A. $f'(x) = \dfrac{2}{2x+3}$

B. $f'(x) = \sqrt{4x^2+9}\ln(\sqrt{4x^2+9})$

C. $f'(x) = \dfrac{8x}{\sqrt{4x^2+9}}$

D. $f'(x) = \dfrac{1}{4x^2+9}$

E. $f'(x) = \dfrac{4x}{4x^2+9}$

F. $f'(x) = \dfrac{8x}{2x+3}$

$f(x) = \ln(\sqrt{4x^2+9}) = \ln((4x^2+9)^{\frac{1}{2}}) = \frac{1}{2}\ln(4x^2+9)$

$f'(x) = \frac{1}{2}\left(\frac{1}{4x^2+9}\right)(8x) = \frac{8x}{2(4x^2+9)} = \frac{4x}{4x^2+9}$

(b) (1 point) The population of fish in a fish farm fluctuates according to the linear discrete time dynamical system (DTDS) $p_{t+1} = 1900 - 0.9p_t$, where t is measured in weeks. Which of the following is an equation for the general solution of this DTDS, based on an initial population of $p_0 = 3000$?

$f(x) = -0.9x + 1900$ Linear DTDS $F(x) = mx + b$ with $m = -0.9$, $b = 1900$

$x_0 = 3000$

A. $p_t = -(0.9)^t(2000) + 1000$

B. $p_t = (-0.9)^t(2000) + 1000$

C. $p_t = (0.9)^t(16000) + 19000$

D. $p_t = (-0.9)^t(-16000) + 19000$

E. $p_t = -(0.9)^t(3000) + 1000$

F. $p_t = (-0.9)^t(3000) + 19000$

$x^* = \frac{b}{1-m} = \frac{1900}{1-(-0.9)} = \frac{1900}{1.9} = 1000$

$x_t = m^t(x_0 - x^*) + x^*$

$\Rightarrow x_t = (-0.9)^t(3000-1000) + 1000$

$\Rightarrow x_t = (-0.9)^t(2000) + 1000$

(c) (1 point) If $f(x) = x^x$, then its first and second derivatives are :

$f(x) = x^x = e^{\ln(x^x)} = e^{x\ln(x)}$

A. $f'(x) = x^x(\ln(x)+1)$ and $f''(x) = x^x(\ln(x)+1)^2$

B. $f'(x) = \dfrac{x^{x-1}}{x-1}$ and $f''(x) = \dfrac{x^{x-2}}{(x-1)(x-2)}$

C. $f'(x) = x^x\left(\dfrac{1}{x}+1\right)$ and $f''(x) = x^x\left(\dfrac{1}{x}+1\right)^2 - x^{x-2}$

D. $f'(x) = x^x(\ln(x)+1)$ and $f''(x) = x^x(\ln(x)+1)^2 + x^{x-1}$

E. $f'(x) = x^x\ln(x)$ and $f''(x) = x^x(\ln(x))^2 + x^{x-1}$

F. $f'(x) = x^x$ and $f''(x) = x^x$

$\Rightarrow f'(x) = (e^{x\ln x})((1)(\ln x) + (x)(\frac{1}{x}))$

$\Rightarrow f'(x) = e^{x\ln x}(\ln x + 1)$

$\Rightarrow f'(x) = x^x(\ln x + 1)$

$f''(x) = \frac{d}{dx}(x^x)(\ln x+1) + (x^x)(\frac{1}{x}+0)$

$\Rightarrow f''(x) = (x^x(\ln x+1))(\ln x+1) + (x^x)(\frac{1}{x})$

$\Rightarrow f''(x) = x^x(\ln x+1)^2 + x^{x-1}$

Question 2. (2+3 = 5 points)
(a) (2 points) Suppose a curve is described implicitly by the equation $y^2\sin(x) = -e^x\cos(y)$. Find the equation of the tangent line to this curve at the point $\left(0, \frac{\pi}{2}\right)$.

$y^2\sin(x) = -e^x\cos(y)$

$\Rightarrow 2y\cdot y'\sin(x) + y^2\cdot\cos(x) = -e^x\cos(y) + (-e^x)\cdot(-\sin(y))\cdot y'$

$\Rightarrow 2y\cdot y'\sin(x) + y^2\cos(x) = -e^x\cos(y) + e^x\sin(y)\cdot y'$

Slope of tangent @ $(0,\frac{\pi}{2})$ is y' when $(x,y) = (0,\frac{\pi}{2})$

$\Rightarrow 2(\frac{\pi}{2})\cdot y'\cdot\sin(0) + (\frac{\pi}{2})^2\cos(0) = -e^0\cos(\frac{\pi}{2}) + e^0\sin(\frac{\pi}{2})\cdot y'$

$\Rightarrow \pi\cdot y'\cdot(0) + \frac{\pi^2}{4}(1) = (-1)(0) + (1)(1)\cdot y'$

$\Rightarrow 0 + \frac{\pi^2}{4} = y' \Rightarrow y' = \frac{\pi^2}{4}$ is slope of tangent line @ $(0,\frac{\pi}{2})$

eqn of tangent @ $(0,\frac{\pi}{2})$ is

$y - \frac{\pi}{2} = (\frac{\pi^2}{4})(x-0)$

$\Rightarrow y - \frac{\pi}{2} = \frac{\pi^2}{4}x$

$\Rightarrow \boxed{y = \frac{\pi^2}{4}x + \frac{\pi}{2}}$

(b) (3 points) The population of insects in a region varies according to humidity and temperature over the course of the year. One year, this population is described by the function

$$g(t) = 6(t^2+5)e^{-t/3}$$

for $0 \le t \le 10$ where $g(t)$ is the number of insects per m^2 at time t, measured in months since February 1st. Find the global maximum and the global minimum of this function on the interval $[0, 10]$. Round your answers to two decimal places.

$g'(t) = 6(2t)e^{-t/3} + 6(t^2+5)(e^{-t/3})(-\frac{1}{3}) = 12te^{-t/3} - 2(t^2+5)e^{-t/3} = -2e^{-t/3}(t^2-6t+5)$

Crit. #s $0 = g'(t)$
$0 = -2e^{-t/3}(t^2-6t+5)$
$0 = -2e^{-t/3}(t-1)(t-5)$
$t=1 \quad t=5$

$g(1) \approx 25.80$
$g(5) \approx 33.9976$ ← ABS MAX

Since $g(t)$ is continuous on $[0,10]$, we can apply E.V.T.

$g(0) = 30$
$g(10) \approx 22.474..$ ← ABS MIN

maximum = 34.00 insects/m^2

minimum = 22.47 insects/m^2

Question 3. (2+2=4 points) Compute the following limits, or else prove that they do not exist, using methods from algebra and Calculus. You must justify your steps with reference to theorems or definitions learned in class, correctly and legibly, and use correct notation, to earn full credit.

(a) $\lim_{x\to 2} \frac{|4-2x|}{x^2-4}$

As $x \to 2^-$, $x<2 \Rightarrow -2x > -4 \Rightarrow 4-2x > 4-4=0$ $\therefore$ as $x\to 2^-$, $|4-2x| = 4-2x$

$$\lim_{x\to 2^-} \frac{|4-2x|}{x^2-4} = \lim_{x\to 2^-}\frac{4-2x}{x^2-4} = \lim_{x\to 2^-}\frac{-2\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \lim_{x\to 2^-}\frac{-2}{x+2} = \frac{-2}{2+2} = -\frac{1}{2}$$

As $x \to 2^+$, $x>2 \Rightarrow -2x < -4 \Rightarrow 4-2x < 4-4=0$ $\therefore$ as $x\to 2^+$, $|4-2x| = -(4-2x)$

$$\lim_{x\to 2^+} \frac{|4-2x|}{x^2-4} = \lim_{x\to 2^+}\frac{-(4-2x)}{x^2-4} = \lim_{x\to 2^+}\frac{2\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \lim_{x\to 2^+}\frac{2}{x+2} = \frac{2}{2+2} = \frac{1}{2}$$

Since left limit $\neq$ right limit, the limil $\lim_{x\to 2}\frac{|4-2x|}{x^2-4}$ does not exist

(b) $\lim_{x\to 0} \frac{7x^2}{\sin^2(3x)}$ type $\frac{0}{0}$

* l'hopital's

$$= \lim_{x\to 0}\frac{14x}{2\sin(3x)\cdot\cos(3x)\cdot 3}$$

$$= \lim_{x\to 0}\frac{7x}{3\sin(3x)\cos(3x)}$$ type $\frac{0}{0}$

* l'hopital's

$$= \lim_{x\to 0}\frac{7}{3(\cos(3x)\cdot 3)\cos(3x) + 3\sin(3x)\cdot(-\sin(3x)\cdot 3)}$$

$$= \lim_{x\to 0}\frac{7}{9\cos^2(3x) - 9\sin^2(3x)}$$

* provided the new limit exists, or approaches ∞ or approaches $-\infty$.

$$= \frac{7}{9\cdot(1)^2 - 9\cdot(0)^2}$$

$$= \frac{7}{9}$$

Question 4. (4 points) Our goal is to estimate the value of $\ln(0.9)$ using a Taylor polynomial.

(a) (1 point) Write down the formula for the Taylor polynomial of degree 3 of a function $f(x)$ centered at $x = a$.

$$T_3(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

(b) (2 point) Give the cubic Taylor polynomial of the function $f(x) = \ln(1+x)$ centered at $x = 0$. Show your work.

$f(x) = \ln(1+x)$ $\quad$ $f(0) = \ln(1+0) = \ln(1) = 0$

$f'(x) = \frac{1}{1+x} = (1+x)^{-1}$ $\quad$ $f'(0) = \frac{1}{1+0} = 1$

$f''(x) = -(1+x)^{-2}$ $\quad$ $f''(0) = -(1+0)^{-2} = -1$

$f'''(x) = 2(1+x)^{-3}$ $\quad$ $f'''(0) = 2(1+0)^{-3} = 2$

$$T_3(x) = 0 + (1)(x-0) + \frac{(-1)}{2!}(x-0)^2 + \frac{(2)}{3!}(x-0)^3$$
$$= 0 + x - \frac{1}{2}x^2 + \frac{2}{3\cdot 2\cdot 1}x^3$$
$$= x - \frac{1}{2}x^2 + \frac{1}{3}x^3$$

$T_3(x) =$ $\boxed{x - \frac{1}{2}x^2 + \frac{1}{3}x^3}$

(c) (0.5 point) Again for $f(x) = \ln(1 + x)$, for what value of x do we have $f(x) = \ln(0.9)$?

$x =$ $\boxed{-0.1}$

$\ln(1+x) = \ln(0.9) \Rightarrow 1+x = 0.9 \Rightarrow x = -0.1$

(d) (0.5 point) Use your Taylor polynomial in part (b) to estimate $\ln(0.9)$, rounded to 6 decimal places.

$$\ln(0.9) = f(-0.1) \approx T_3(-0.1) = (-0.1) - \frac{1}{2}(-0.1)^2 + \frac{1}{3}(-0.1)^3 \approx -0.105333$$

Taylor polynomial estimate of $\ln(0.9) =$ $\boxed{-0.105333}$

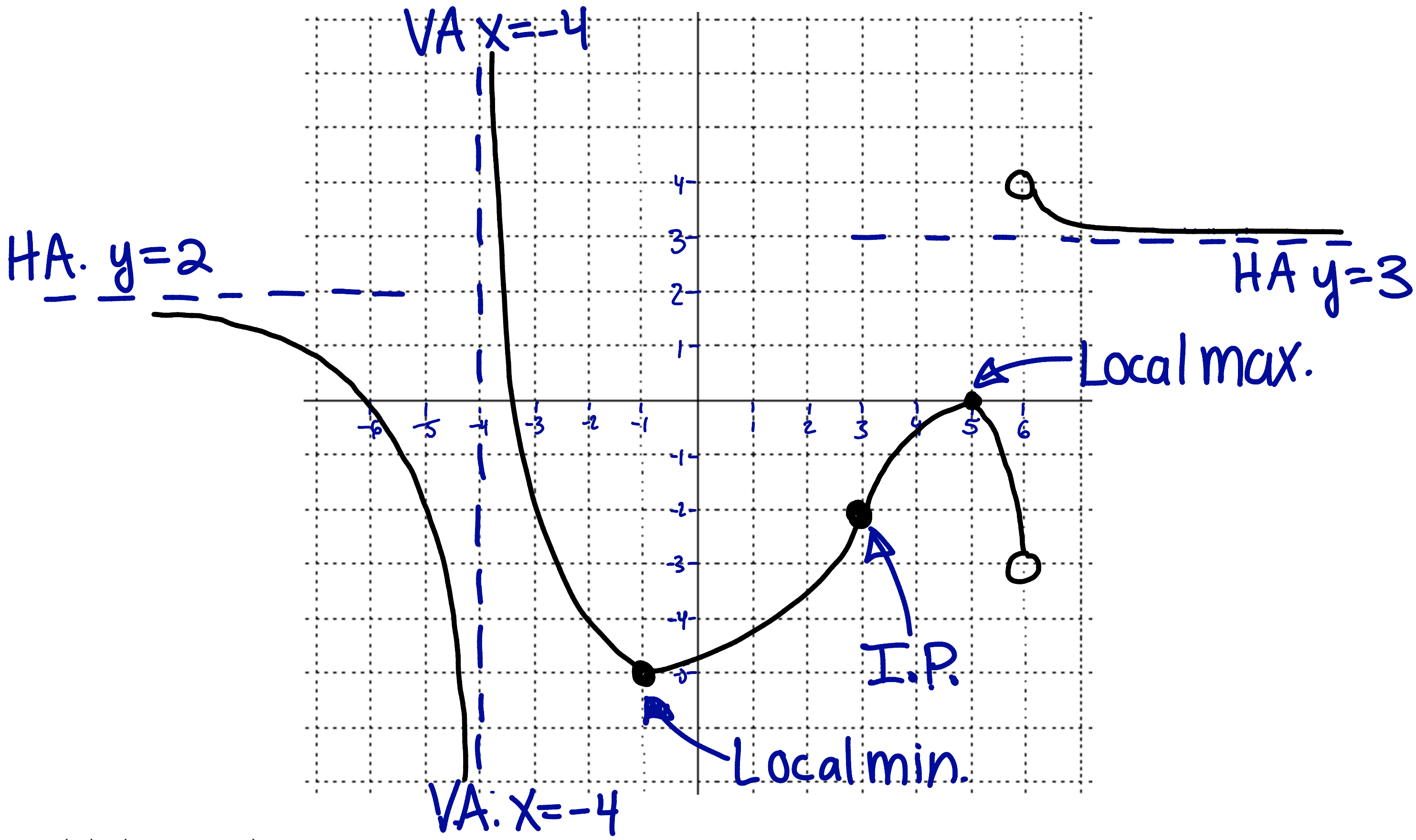
Question 5. (5 points) To sketch the graph of a continuous function f, whose domain is all real numbers except -4 and 6, we provide the following clues:

- $\lim_{x\to-\infty} f(x) = 2$
- $\lim_{x\to+\infty} f(x) = 3$
- $\lim_{x\to-4^-} f(x) = -\infty$
- $\lim_{x\to-4^+} f(x) = +\infty$
- $\lim_{x\to6^-} f(x) = -3$
- $\lim_{x\to6^+} f(x) = 4$

x	$x<-4$	$-4<x<-1$	-1	$-1<x<3$	3	$3<x<5$	5	$5<x<6$	$x>6$
$f'(x)$	$-$	$-$	0	$+$	$+$	$+$	0	$-$	$-$
$f''(x)$	$-$	$+$	$+$	$+$	0	$-$	$-$	$-$	$+$
$f(x)$			-5		-2		0		

$+$ indicates a positive value, $-$ a negative value

(a) (3 points) Sketch the graph of f on the coordinate grid below, explicitly labeling all local maxima, local minima, inflection points, and horizontal and vertical asymptotes.



(b) (2 points) For the two questions below, choose the best answer for each from the following selection of choices and write the letter in the box:

A. $(-1,3)\cup(3,5)$
B. $(-1,5)$
C. $(-\infty,-1)\cup(5,\infty)$
D. $(-4,3)\cup(6,\infty)$
E. $(-\infty,-4)\cup(3,6)$
F. $(-4,-1)\cup(-1,3)\cup(6,\infty)$

Where is f increasing? B Where is f concave up? D

Question 6. (1+1+1+1=4 points) Two master's students are studying the growth of a colony of yeast. They collect data in the lab around the clock for three days, and find that it can be fitted to the model:

$$r(t) = \frac{20}{1+50e^{-t}}, \quad t \geq 0$$

where $r(t)$ is the mass of the colony measured in mg and t is the time measured in days.

(a) (1 point) They want to know if the colony will reach a mass of 10 mg sometime in the next two days. Please use Intermediate Value Theorem to justify that $r(t) = 10$ has a solution in the interval $[3, 5]$.

$r(t)$ is a continuous function with domain $\mathbb{R}$

we have $r(3) = 5.73 < 10$, $r(5) = 14.96 > 10$ since $r(t)$ is continuous on $[3,5]$ and since $Y=10$ is between $r(3)$ and $r(5)$, the Intermediate Value Theorem guarantees that there exists a number $c \in (3,5)$ such that $f(c) = 10$

(b) (1 point) Regretfully, they don't know how to solve the equation $r(t) = 10$. But ever resourceful, they use Newton's method to approximate a root of $g(t) = r(t) - 10$. Given that

$$g(t) = \left(\frac{10-500e^{-t}}{1+50e^{-t}}\right) \quad \text{and} \quad g'(t) = \frac{1000e^{-t}}{(1+50e^{-t})^2}$$

please give the formula to compute the next iteration t_{n+1} of Newton's method from t_n.

$$t_{n+1} = t_n - \left[\frac{\left(\frac{10-500e^{-t_n}}{1+50e^{-t_n}}\right)}{\left(\frac{1000e^{-t_n}}{(1+50e^{-t_n})^2}\right)}\right]$$

(c) (1 point) Starting with an initial guess of $t_0 = 3$, they use Newton's method to determine t_1 and t_2. Give these answers to a precision of 2 decimal places.

$t_1 = \boxed{4.04}$

$t_2 = \boxed{3.91}$

(d) (1 point) You enter the lab and show them how to solve the equation $r(t) = 10$ algebraically, in the space below. You are a hero!

$$\frac{20}{1+50e^{-t}} = 10 \Rightarrow 20 = 10(1+50e^{-t}) \Rightarrow 2 = 1+50e^{-t}$$

$$\Rightarrow \frac{1}{50} = e^{-t}$$

$$\Rightarrow \ln\left(\frac{1}{50}\right) = -t \Rightarrow t = -\ln\left(\frac{1}{50}\right) = \ln(50) \approx 3.912\ldots$$

Question 7. (9 points + 1 point bonus) This question continues over three pages, but the work on each page is independent.

When there is at least 5 grams of a certain drug in a patient's body, the dynamics of its absorption is modelled by the following discrete-time dynamical system (DTDS):

$$x_{t+1} = \frac{{x_t}^2 - 5x_t}{3 + x_t} + d$$

where t is in hours and x_t denotes the amount of drug (in grams) present in the patient's body at time t (and we assume $x_t \geq 5$). The parameter $d \geq 0$ is a constant representing the dose (in grams) of the drug given to the patient at the end of each hour throughout treatment.

(a) (2 points) Give the updating function $f(x)$ of this DTDS and then find the fixed point of this DTDS in terms of the parameter d.

$f(x) = \boxed{\frac{x^2-5x}{3+x} + d}$

$x = f(x)$

$\Rightarrow x = \frac{x^2-5x}{3+x} + d$

$\Rightarrow x(3+x) = \left(\frac{x^2-5x}{3+x} + d\right)(3+x)$

$\Rightarrow x(3+x) = x^2 - 5x + d(3+x)$

$\Rightarrow 0 = x^2 - 5x + 3d + dx - x(3+x)$

$\Rightarrow 0 = \cancel{x^2} - 5x + 3d + dx - 3x - \cancel{x^2}$

$\Rightarrow 0 = -8x + dx + 3d$

$\Rightarrow 8x - dx = 3d$

$\Rightarrow x(8-d) = 3d$

$\Rightarrow x = \frac{3d}{8-d}$

fixed point $x^* = \boxed{\frac{3d}{8-d}}$

(b) (1 point) Find the range of values of the parameter $d > 0$ for which there will be a positive fixed point $x^* > 0$. Show your work!

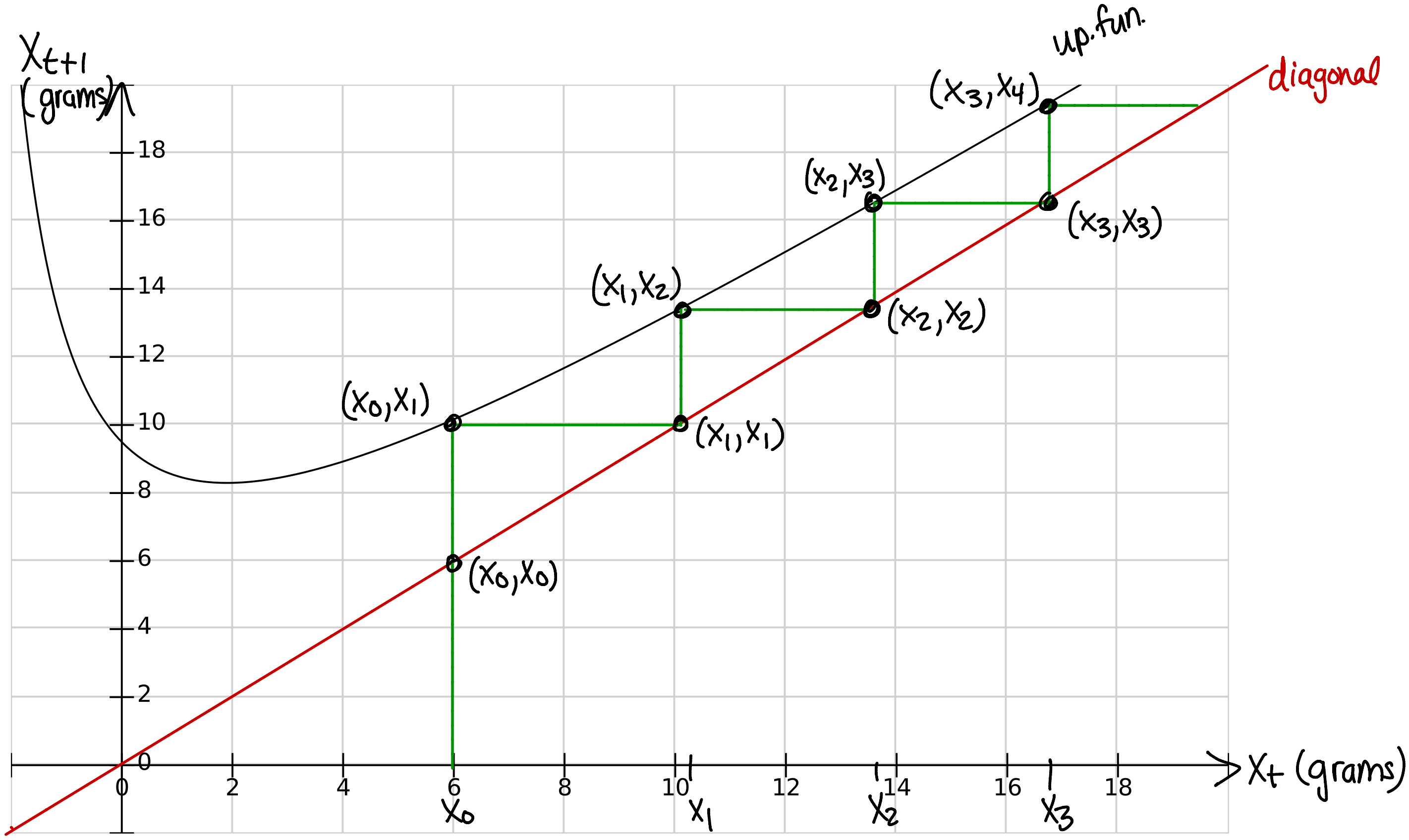
Since $d>0$, $3d>0$ so $\frac{3d}{8-d} > 0$ provided $8-d>0$ $\therefore$ we need $d<8$

$0 < d < \boxed{8}$

Question 7 continued.

(c) (2 points) Suppose the treatment is to give the patient an hourly dose of $d = 9.5$ g of the drug. We have graphed the updating function for you below. Starting from an initial drug level of $x_0 = 6$, cobweb for at least three steps. Label the axes (and write the units of measurement). Clearly indicate the values x_0, x_1, x_2, and x_3 on your graph.

Tip: use your student card as a straight edge.



(d) (1 point) Still assuming $d = 9.5$ g, and reflecting on your cobweb above, which of the following statements best describes what will happen with the drug levels in the patient's body in the long-term if the initial drug level is $x_0 = 6$? Circle one of the following options.

A. The level of drug in the patient's body will decrease until it is all eliminated in the long term.

B. The level of drug in the patient's body will become negative at some point, meaning the drug is being removed from surrounding tissues.

C. The level of drug in the patient's body will tend towards a fixed point in the long term and this fixed point is stable.

D. The level of drug in the patient's body will increase until it reaches toxic levels in the long term, before which we hope the treatment will be stopped.

E. The level of drug in the patient's body will be 9.5g in the long term because that is the dosage level of the treatment.

F. The level of drug in the patient's body will tend towards a fixed point in the long term and this fixed point is unstable.

Question 7 continued. Now, suppose the patient is not feeling well at the previous hourly dosage. The doctor has now reduced to an hourly dose of $d = 6$ g. In this case, the DTDS becomes

$$x_{t+1} = \frac{{x_t}^2 - 5x_t}{3 + x_t} + 6.$$

(e) (1 point) Write down the updating function and compute its derivative.

$$f(x) = \frac{x^2 - 5x}{3+x} + 6$$

$$f'(x) = \frac{(2x-5)(3+x) - (x^2-5x)(1)}{(3+x)^2} + 0$$

$$= \frac{6x + 2x^2 - 15 - 5x - x^2 + 5x}{(3+x)^2}$$

$$f'(x) = \frac{x^2 + 6x - 15}{(3+x)^2}$$

(f) (2 points) State the Stability Theorem, and use it determine the stability of the fixed point $x^* = 9$ of this DTDS.

Stability Theorem

Suppose $f(x)$ is a differentiable updating function and x^* is a fixed point.

- If $|f'(x^*)| < 1$, then the fixed pt x^* is stable
- If $|f'(x^*)| > 1$, then the fixed pt x^* is unstable

$x^* = 9$

$$|f'(x^*)| = |f'(9)| = \left|\frac{81 + 6(9) - 15}{(3+9)^2}\right| = |0.8333\ldots| = 0.83\ldots < 1$$

$\therefore x^* = 9$ is stable.

(g) (1 point bonus) For what range of values of d (rounded to two decimal places) does the original DTDS give a **positive** and **stable** fixed point? (Use the back of the page when you run out of space. Don't start this question until you've completed the rest of the exam!)

Question 8. (2+2=4 points)

(a) (2 points) Determine the function $g(x)$ which satisfies $g'(0) = 5$ and $g(0) = 3$ and is such that

$$g''(x) = 2\cos(x) + 3\sin(x).$$

Show your work.

$$\Rightarrow g'(x) = 2\sin(x) + 3(-\cos(x)) + C$$

$$\Rightarrow g'(x) = 2\sin(x) - 3\cos(x) + C$$

$$g'(0) = 5 \Rightarrow 5 = 2\sin(0) - 3\cos(0) + C \Rightarrow C = 8$$

$$\therefore g'(x) = 2\sin(x) - 3\cos(x) + 8$$

$$\Rightarrow g(x) = -2\cos(x) - 3\sin(x) + 8x + D$$

$$g(0) = 3 \Rightarrow 3 = -2\cos(0) - 3\sin(0) + 8(0) + D \Rightarrow D = 5$$

$g(x) =$ $\boxed{-2\cos(x) - 3\sin(x) + 8x + 5}$

(b) (2 points) Compute the indefinite integral $\int \frac{x^{1/3} + 2x^{-1/3}}{x^{2/3}}\, dx$.

$$\int \frac{x^{1/3} + 2x^{-1/3}}{x^{2/3}}\, dx = \int x^{-1/3} + 2x^{-1}\, dx$$

$$= \frac{3}{2}x^{2/3} + 2\ln|x| + C$$

Answer: $\boxed{\frac{3}{2}x^{2/3} + 2\ln|x| + C}$

Question 9. (6 points) Compute the following indefinite integrals. Your work must be legible, logical and correct to earn full marks. You may use the backs of pages if necessary.

(a) $\int x^2 \ln(x)\, dx$ $= (\ln(x))\left(\frac{1}{3}x^3\right) - \int \left(\frac{1}{x}\right)\left(\frac{1}{3}x^3\right)dx$

parts I
$u = \ln(x)$ $v' = x^2$
$u' = \frac{1}{x}$ $v = \frac{1}{3}x^3$

$$= \frac{1}{3}x^3\ln(x) - \frac{1}{3}\int x^2 dx$$

$$= \frac{1}{3}x^3\ln(x) - \frac{1}{3}\left(\frac{1}{3}x^3\right) + C$$

$$= \frac{1}{3}x^3\ln(x) - \frac{1}{9}x^3 + C$$

(b) $\int \frac{(e^x+2)e^x}{\sqrt{e^x+1}}\, dx$ $= \int \frac{(e^x+2)\cancel{e^x}}{\sqrt{u}} \cdot \frac{du}{\cancel{e^x}}$

U-substitution
$u = e^x + 1$
$\Rightarrow \frac{du}{dx} = e^x$
$\Rightarrow dx = \frac{du}{e^x}$

$$= \int \frac{e^x+2}{\sqrt{u}} du$$

since $u = e^x+1$, we have $u+1 = e^x+2$

$$= \int \frac{u+1}{\sqrt{u}} du$$

$$= \int u^{\frac{1}{2}} + u^{-\frac{1}{2}} du$$

$$= \frac{2}{3}u^{3/2} + 2u^{1/2} + C$$

$$= \frac{2}{3}(e^x+1)^{3/2} + 2\sqrt{e^x+1} + C$$

Question 10. (4+2=6 points)

(a) (2 points) Compute the following definite integral:

$$\int_1^2 4x^3\,dx = x^4 \Big|_1^2$$

$$= (2^4) - (1^4)$$

$$= 16 - 1$$

$$= 15$$

(b) (4 points) Identify each statement as TRUE (=always true) or FALSE (=not always true), by circling the correct choice.

True or False: There are angles θ for which $\arcsin(\sin(\theta)) \neq \theta$.

True or False: $\ln(a^b) = \ln(b)\ln(a)$

True or False: $|f(x)| = \begin{cases} f(x) & \text{if } x \geq 0 \\ -f(x) & \text{if } x < 0 \end{cases}$

True or False: $\dfrac{d}{dx}\ln(-x) = \dfrac{-1}{x}$ for all $x < 0$.

True or False: If f is a function with a maximum at $x = a$ then it must be the case that $f'(a) = 0$.

True or False: The Mean Value Theorem states that if f is defined and differentiable on $[a, b]$, then there is always a point $c \in [a, b]$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

True or False: If f is differentiable at $x = a$, then it is always true that f is continuous there.

True or False: Two antiderivatives of $\dfrac{1}{1 + x^2}$ are $\arctan(x)$ and $\ln(1 + x^2)$.